Search Continued

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[These slides adapted from Dan Klein and Pieter Abbeel; ai.berkeley.edu]
Recap: Search
Depth-First (Tree) Search
Breadth-First (Tree) Search
Cost-Sensitive Search
BFS finds the shortest path in terms of number of actions. It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.
Uniform Cost Search
Uniform Cost Search

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs $C^*$ and arcs cost at least $\epsilon$, then the “effective depth” is roughly $C^*/\epsilon$
  - Takes time $O(b^{C^*/\epsilon})$ (exponential in effective depth)

- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\epsilon})$

- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes! (if no solution, still need depth != $\infty$)

- Is it optimal?
  - Yes! (Proof via A*)
Uniform Cost Issues

- Remember: UCS explores increasing cost contours

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location

- We’ll fix that soon!

[Demo: empty grid UCS (L2D5)]
[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]
Video of Demo Empty UCS
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)
The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object
Up next: Informed Search

- Uninformed Search
  - DFS
  - BFS
  - UCS

- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search
  - Graph Search
A heuristic is:
- A function that estimates how close a state is to a goal
- Designed for a particular search problem
- Pathing?
- Examples: Manhattan distance, Euclidean distance for pathing
Example: Heuristic Function

$h(x)$
Greedy Search
Greedy Search

- Expand the node that seems closest...

- Is it optimal?
  - No. Resulting path to Bucharest is not the shortest!
Greedy Search

- **Strategy**: expand a node that you think is closest to a goal state
  - **Heuristic**: estimate of distance to nearest goal for each state

- **A common case**:
  - Best-first takes you straight to the (wrong) goal

- **Worst-case**: like a badly-guided DFS
A* Search
A* Search
Combining UCS and Greedy

- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Greedy** orders by goal proximity, or *forward cost* $h(n)$

- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

Example: Teg
When should A* terminate?

- Should we stop when we enqueue a goal?
  - No: only stop when we dequeue a goal
Is A* Optimal?

- What went wrong?
  - Actual bad goal cost < estimated good goal cost
  - We need estimates to be less than actual costs!

![Diagram](image)
Admissible Heuristics
Idea: Admissibility

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe.

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs.
A heuristic $h$ is **admissible** (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal.

**Examples:**

- Coming up with admissible heuristics is most of what’s involved in using A* in practice.
Optimality of A* Tree Search
Optimality of A* Tree Search

Assume:
- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:
- A will exit the fringe before B
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$

$$f(n) = g(n) + h(n)$$  \text{Definition of f-cost}
$$f(n) \leq g(A)$$  \text{Admissibility of h}
$$g(A) = f(A)$$  \text{h = 0 at a goal}
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor \( n \) of A is on the fringe, too (maybe A!)
- Claim: \( n \) will be expanded before B
  1. \( f(n) \) is less or equal to \( f(A) \)
  2. \( f(A) \) is less than \( f(B) \)

\( g(A) < g(B) \) B is suboptimal
\( f(A) < f(B) \) \( h = 0 \) at a goal
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$
  2. $f(A)$ is less than $f(B)$
  3. $n$ expands before B

$f(n) \leq f(A) < f(B)$
Proof:
- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$
  2. $f(A)$ is less than $f(B)$
  3. $n$ expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal
Properties of A*

Uniform-Cost

A*
UCS vs A* Contours

- Uniform-cost expands equally in all “directions”

- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]
Video of Demo Contours (Empty) -- UCS
Video of Demo Contours (Empty) -- Greedy
Video of Demo Contours (Empty) – A*
Video of Demo Contours (Pacman Small Maze) – A*
Comparison

Greedy

Uniform Cost

A*
Video of Demo Pacman (Tiny Maze) – UCS / A*
Video of Demo Empty Water Shallow/Deep – Guess Algorithm
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

- Inadmissible heuristics are often useful too
Example: 8 Puzzle

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

Start State

<table>
<thead>
<tr>
<th>7</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Actions

<table>
<thead>
<tr>
<th>3</th>
<th>7</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

Goal State

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>
8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h(\text{start}) = 8$
- This is a relaxed-problem heuristic

**Start State**

**Goal State**

<table>
<thead>
<tr>
<th>Average nodes expanded when the optimal path has...</th>
<th>4 steps</th>
<th>8 steps</th>
<th>12 steps</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UCS</strong></td>
<td>112</td>
<td>6,300</td>
<td>$3.6 \times 10^6$</td>
</tr>
<tr>
<td><strong>TILES</strong></td>
<td>13</td>
<td>39</td>
<td>227</td>
</tr>
</tbody>
</table>

Statistics from Andrew Moore
What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

Total Manhattan distance

Why is it admissible?

\[ h(\text{start}) = 3 + 1 + 2 + ... = 18 \]

<table>
<thead>
<tr>
<th>TILES</th>
<th>Average nodes expanded when the optimal path has...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>...4 steps</td>
</tr>
<tr>
<td>13</td>
<td>39</td>
</tr>
<tr>
<td>12</td>
<td>25</td>
</tr>
</tbody>
</table>
How about using the *actual cost* as a heuristic?
- Would it be admissible?
- Would we save on nodes expanded?
- What’s wrong with it?

With A*: a trade-off between quality of estimate and work per node
- As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself.
Graph Search
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.
Graph Search

- In BFS, for example, we shouldn’t bother expanding the circled nodes (why?)
Graph Search

- Idea: never expand a state twice

- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but...
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

- Important: store the closed set as a set, not a list

- Can graph search wreck completeness? Why/why not?

- How about optimality?
A* Graph Search Gone Wrong?

State space graph

Search tree

Closed Set: S B C A
Consistency of Heuristics

- **Main idea:** estimated heuristic costs \( \leq \) actual costs
  - Admissibility: heuristic cost \( \leq \) actual cost to goal
    \[ h(A) \leq \text{actual cost from A to G} \]
  - Consistency: heuristic “arc” cost \( \leq \) actual cost for each arc
    \[ h(A) - h(C) \leq \text{cost(A to C)} \]

- **Consequences of consistency:**
  - The f value along a path never decreases
    \[ h(A) \leq \text{cost(A to C)} + h(C) \]
  - A* graph search is optimal
Optimality of A* Search

- With a admissible heuristic, Tree A* is optimal.
- With a consistent heuristic, Graph A* is optimal.
  - See slides, also video lecture from past years for details.
- With h=0, the same proof shows that UCS is optimal.
A*: Summary
A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems
All these search algorithms are the same except for fringe strategies

- Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
- Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
- Can even code one implementation that takes a variable queuing object
Search and Models

- Search operates over models of the world
  - The agent doesn’t actually try all the plans out in the real world!
  - Planning is all “in simulation”
- Your search is only as good as your models…
Search Gone Wrong?