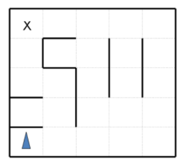
CS 188
Fall 2022

# Regular Discussion 2 Solutions

## 1 Search and Heuristics

Imagine a car-like agent wishes to exit a maze like the one shown below:




The agent is directional and at all times faces some direction $d \in (N, S, E, W)$. With a single action, the agent can *either* move forward at an adjustable velocity $v$ *or* turn. The turning actions are *left* and *right*, which change the agent's direction by 90 degrees. Turning is only permitted when the velocity is zero (and leaves it at zero). The moving actions are *fast* and *slow*. *Fast* increments the velocity by 1 and *slow* decrements the velocity by 1; in both cases the agent then moves a number of squares equal to its NEW adjusted velocity (see example below). A consequence of this formulation is that it is impossible for the agent to move in the same nonzero velocity for two consecutive timesteps. Any action that would result in a collision with a wall crashes the agent and is illegal. Any action that would reduce $v$ below 0 or above a maximum speed $V_{\max}$ is also illegal. The agent's goal is to find a plan which parks it (stationary) on the exit square using as few actions (time steps) as possible.

As an example: if at timestep $t$ the agent's current velocity is 2, by taking the *fast* action, the agent first increases the velocity to 3 and move 3 squares forward, such that at timestep $t + 1$ the agent's current velocity will be 3 and will be 3 squares away from where it was at timestep $t$. If instead the agent takes the *slow* action, it first decreases its velocity to 1 and then moves 1 square forward, such that at timestep $t + 1$ the agent's current velocity will be 1 and will be 1 squares away from where it was at timestep $t$. If, with an instantaneous velocity of 1 at timestep $t + 1$, it takes the slow action again, the agent's current velocity will become 0, and it will not move at timestep $t + 1$. Then at timestep $t + 2$, it will be free to turn if it wishes. Note that the agent could not have turned at timestep $t + 1$ when it had a current velocity of 1, because it has to be stationary to turn.

**(a)** If the grid is $M$ by $N$, what is the size of the state space? Justify your answer. You should assume that all configurations are reachable from the start state.

The size of the state space is $4MN(V_{max} + 1)$. The state representation is (direction facing, $x, y$, speed). Note that the speed can take any value in $\{0, ..., V_{max}\}$.

**(b)** Is the Manhattan distance from the agent's location to the exit's location admissible? Why or why not?

No, Manhattan distance is not an admissible heuristic. The agent can move at an average speed of greater than 1 (by first speeding up to $V_{max}$ and then slowing down to 0 as it reaches the goal), and so can reach the goal in less time steps than there are squares between it and the goal. A specific example: A timestep 0, the agent's starts stationary at square 0 and the target is 9 squares away at square 9. At timestep 0, the agent takes the *fast* action and ends up at square 1 with a velocity of 1. At timestep 1, the agent takes the *fast* action and ends up at square 3 with a velocity of 2. At timestep 2, the agent takes the *fast* action and ends up at square 6 with a velocity of 3. At timestep 3, the agent takes the *slow* action and ends up at square 8 with a velocity of 2. At timestep 4, the agent takes the *slow* action and ends up at square 9 with a velocity of 1. At timestep 5, the agent takes the *slow* action and stays at square 9 with a velocity of 0. Therefore, the agent can move 9 squares by taking 6 actions.

**(c)** State and justify a non-trivial admissible heuristic for this problem which is not the Manhattan distance to the exit.

There are many answers to this question. Here are a few, in order of weakest to strongest:

(a) The number of turns required for the agent to face the goal.

(b) Consider a relaxation of the problem where there are no walls, the agent can turn, change speed arbitrarily, and maintain constant velocity. In this relaxed problem, the agent would move with $V_{max}$, and then suddenly stop at the goal, thus taking $d_{manhattan}/V_{max}$ time.

(c) We can improve the above relaxation by accounting for the acceleration and deceleration dynamics. In this case the agent will have to accelerate from 0 from the start state, maintain a constant velocity of $V_max$, and slow down to 0 when it is about to reach the goal. Note that this heuristic will always return a greater value than the previous one, but is still not an overestimate of the true cost to reach the goal. We can say that this heuristic *dominates* the previous one.

In particular, let us assume that $d_{manhattan}$ is greater than and equal to the distance it takes to accelerate to and decelerate from $V_{max}$ (In the case that $d_{manhattan}$ is smaller than this distance, we can still use $d_{manhattan}/V_{max}$ as a heuristic). We can break up the $d_{manhattan}$ into three parts: $d_{accel}$, $d_{V_{max}}$, and $d_{decel}$. The agent travels a distance of $d_{accel}$ when it accelerates from 0 to $V_{max}$ velocity. The agent travels a distance of $d_{decel}$ when it decelerates from $V_{max}$ to 0 velocity. In between acceleration and deceleration, the agent travels a distance of $d_{V_{max}} = d_{manhattan} - d_{accel} - d_{decel}$. $d_{accel} = 1 + 2 + 3 + V_{max} = \frac{(V_{max})(V_{max}+1)}{2}$ and $d_{decel} = (V_{max} - 1) + (V_{max} - 2) + ... + 1 + 0 = \frac{(V_{max})(V_{max}-1)}{2}$. So $d_{V_{max}} = d_{manhattan} - \frac{(V_{max})(V_{max}+1)}{2} - \frac{(V_{max})(V_{max}-1)}{2} = d_{manhattan} - V_{max}^2$. It takes $V_{max}$ steps to travel the initial $d_{accel}$, $\frac{d_{manhattan} - V_{max}^2}{V_{max}}$ steps to travel the middle $d_{V_{max}}$ and $V_{max}$ steps to travel the last $d_{decel}$. Therefore, our heuristic is

$$\begin{cases} \frac{d_{manhattan}}{V_{max}}, & \text{if } d_{manhattan} \leq V_{max}^2 = d_{accel} + d_{decel} \\ \frac{d_{manhattan}}{V_{max}} + V_{max}, & \text{if } d_{manhattan} > V_{max}^2 = d_{accel} + d_{decel} \end{cases}$$

**(d)** If we used an inadmissible heuristic in A* graph search, would the search be complete? Would it be optimal?

If the heuristic function is bounded, then A* graph search would visit all the nodes eventually, and would find a path to the goal state if there exists one. An inadmissible heuristic does not guarantee optimality as it can make the good optimal goal look as though it is very far off, and take you to a suboptimal goal.

**(e)** If we used an *admissible* heuristic in A* graph search, is it guaranteed to return an optimal solution? What if the heuristic was consistent? What if we were using A* tree search instead of A* graph search?

Although admissible heuristics guarantee optimality for A* *tree* search, they do not necessarily guarantee optimality for A* *graph* search; they are only guaranteed to return an optimal solution if they are consistent as well.

**(f)** When might we want to use an inadmissible heuristic over an admissible heuristic?

The time to solve an A* search problem is a function of two factors: the number of nodes expanded, and the time spent per node. An inadmissible heuristic may be faster to compute, leading to a solution that is obtained faster due to less time spent per node. It can also be a closer estimate to the actual cost function (even though at times it will overestimate!), thus expanding less nodes. We lose the guarantee of optimality by using an inadmissible heuristic. But sometimes we may be okay with finding a suboptimal solution to a search problem.

# CSPs

CSPs are defined by three factors:

1. *Variables* - CSPs possess a set of $N$ variables $X_1, ..., X_N$ that can each take on a single value from some defined set of values.
2. *Domain* - A set $\{x_1, ..., x_d\}$ representing all possible values that a CSP variable can take on.
3. *Constraints* - Constraints define restrictions on the values of variables, potentially with regard to other variables.

CSPs are often represented as constraint graphs, where nodes represent variables and edges represent constraints between them.

- *Unary Constraints* - Unary constraints involve a single variable in the CSP. They are not represented in constraint graphs, instead simply being used to prune the domain of the variable they constrain when necessary.
- *Binary Constraints* - Binary constraints involve two variables. They're represented in constraint graphs as traditional graph edges.
- *Higher-order Constraints* - Constraints involving three or more variables can also be represented with edges in a CSP graph.

In **forward checking**, whenever a value is assigned to a variable $X_i$, forward checking prunes the domains of unassigned variables that share a constraint with $X_i$ that would violate the constraint if assigned. The idea of forward checking can be generalized into the principle of **arc consistency**. For arc consistency, we interpret each undirected edge of the constraint graph for a CSP as two directed edges pointing in opposite directions. Each of these directed edges is called an **arc**. The arc consistency algorithm works as follows:

- Begin by storing all arcs in the constraint graph for the CSP in a queue $Q$.
- Iteratively remove arcs from $Q$ and enforce the condition that in each removed arc $X_i \longrightarrow X_j$, for every remaining value $v$ for the tail variable $X_i$, there is at least one remaining value $w$ for the head variable $X_j$ such that $X_i = v, X_j = w$ does not violate any constraints. If some value $v$ for $X_i$ would not work with any of the remaining values for $X_j$, we remove $v$ from the set of possible values for $X_i$.
- If at least one value is removed for $X_i$ when enforcing arc consistency for an arc $X_i \longrightarrow X_j$, add arcs of the form $X_k \longrightarrow X_i$ to $Q$, for all unassigned variables $X_k$. If an arc $X_k \longrightarrow X_i$ is already in $Q$ during this step, it doesn't need to be added again.
- Continue until $Q$ is empty, or the domain of some variable is empty and triggers a backtrack.

We've delineated that when solving a CSP, we fix some ordering for both the variables and values involved. In practice, it's often much more effective to compute the next variable and corresponding value "on the fly" with two broad principles, **minimum remaining values** and **least constraining value**:

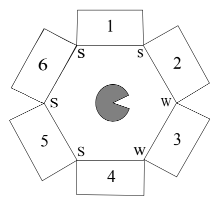
- *Minimum Remaining Values (MRV)* - When selecting which variable to assign next, using an MRV policy chooses whichever unassigned variable has the fewest valid remaining values (the *most constrained variable*).
- *Least Constraining Value (LCV)* - Similarly, when selecting which value to assign next, a good policy to implement is to select the value that prunes the fewest values from the domains of the remaining unassigned variables.

# 2 CSPs: Trapped Pacman

Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost (G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.

The one sign of what lies behind the corridors is the wind: a pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all. Unfortunately, Pacman cannot measure the strength of the breeze at a specific corridor. Instead, he can stand *between* two adjacent corridors and feel the max of the two breezes. For example, if he stands between a pit and an exit he will sense a strong (S) breeze, while if he stands between an exit and a ghost, he will sense a weak (W) breeze. The measurements for all intersections are shown in the figure below.

Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will *not* both be exits.




Pacman models this problem using variables $X_i$ for each corridor $i$ and domains P, G, and E.

**(a)** State the binary and/or unary constraints for this CSP (either implicitly or explicitly).

Binary:

$X_1 = P$ or $X_2 = P$, $X_2 = E$ or $X_3 = E$,
$X_3 = E$ or $X_4 = E$, $X_4 = P$ or $X_5 = P$,
$X_5 = P$ or $X_6 = P$, $X_1 = P$ or $X_6 = P$,
$\forall i, j$ s.t. $\text{Adj}(i, j)\ \neg(X_i = E$ and $X_j = E)$

Unary:

$X_2 \neq P$,
$X_3 \neq P$,
$X_4 \neq P$

Note: This is just one of many solutions. The answers below will be based on this formulation.

**(b)** Suppose we assign $X_1$ to $E$. Perform forward checking after this assignment. Also, enforce unary constraints.

| | | | |
|---|---|---|---|
| $X_1$ | | | E |
| $X_2$ | | | |
| $X_3$ | | G | E |
| $X_4$ | | G | E |
| $X_5$ | P | G | E |
| $X_6$ | P | | |

**(c)** Suppose forward checking returns the following set of possible assignments:

| | | | |
|---|---|---|---|
| $X_1$ | P | | |
| $X_2$ | | G | E |
| $X_3$ | | G | E |
| $X_4$ | | G | E |
| $X_5$ | P | | |
| $X_6$ | P | G | E |

According to MRV, which variable or variables could the solver assign first?

$X_1$ or $X_5$ (tie breaking)

**(d)** Assume that Pacman knows that $X_6 = G$. List all the solutions of this CSP or write *none* if no solutions exist.

(P,E,G,E,P,G)
(P,G,E,G,P,G)