Q1. Policy Evaluation

In this question, you will be working in an MDP with states S, actions A, discount factor γ, transition function T, and reward function R.

We have some fixed policy $\pi : S \to A$, which returns an action $a = \pi(s)$ for each state $s \in S$. We want to learn the Q function $Q^\pi(s, a)$ for this policy: the expected discounted reward from taking action a in state s and then continuing to act according to π: $Q^\pi(s, a) = \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma Q^\pi(s', \pi(s'))]$. The policy π will not change while running any of the algorithms below.

(a) Can we guarantee anything about how the values Q^π compare to the values Q^* for an optimal policy π^*?

- ◯ $Q^\pi(s, a) \le Q^*(s, a)$ for all s, a
- ◯ $Q^\pi(s, a) = Q^*(s, a)$ for all s, a
- ◯ $Q^\pi(s, a) \ge Q^*(s, a)$ for all s, a
- ◯ None of the above are guaranteed

(b) Suppose T and R are *unknown*. You will develop sample-based methods to estimate Q^π. You obtain a series of *samples* $(s_1, a_1, r_1), (s_2, a_2, r_2), \ldots (s_T, a_T, r_T)$ from acting according to this policy (where $a_t = \pi(s_t)$, for all t).

(i) Recall the update equation for the Temporal Difference algorithm, performed on each sample in sequence:

$$V(s_t) \leftarrow (1 - \alpha)V(s_t) + \alpha(r_t + \gamma V(s_{t+1}))$$

which approximates the expected discounted reward $V^\pi(s)$ for following policy π from each state s, for a learning rate α.

Fill in the blank below to create a similar update equation which will approximate Q^π using the samples.

You can use any of the terms $Q, s_t, s_{t+1}, a_t, a_{t+1}, r_t, r_{t+1}, \gamma, \alpha, \pi$ in your equation, as well as $\sum$ and max with any index variables (i.e. you could write $\max_a$, or $\sum_a$ and then use a somewhere else), but no other terms.

$$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha \left[\rule{6cm}{0.4pt}\right]$$

(ii) Now, we will approximate Q^π using a linear function: $Q(s, a) = \mathbf{w}^\top \mathbf{f}(s, a)$ for a weight vector $\mathbf{w}$ and feature function $\mathbf{f}(s, a)$.

To decouple this part from the previous part, use Q_{samp} for the value in the blank in part (i) (i.e. $Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha Q_{samp}$).

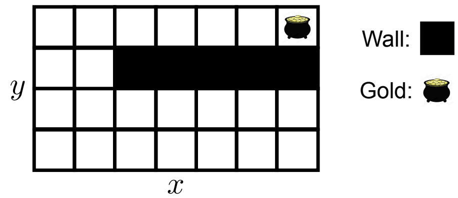
Which of the following is the correct sample-based update for $\mathbf{w}$?

- ◯ $\mathbf{w} \leftarrow \mathbf{w} + \alpha[Q(s_t, a_t) - Q_{samp}]$
- ◯ $\mathbf{w} \leftarrow \mathbf{w} - \alpha[Q(s_t, a_t) - Q_{samp}]$
- ◯ $\mathbf{w} \leftarrow \mathbf{w} + \alpha[Q(s_t, a_t) - Q_{samp}]\mathbf{f}(s_t, a_t)$
- ◯ $\mathbf{w} \leftarrow \mathbf{w} - \alpha[Q(s_t, a_t) - Q_{samp}]\mathbf{f}(s_t, a_t)$
- ◯ $\mathbf{w} \leftarrow \mathbf{w} + \alpha[Q(s_t, a_t) - Q_{samp}]\mathbf{w}$
- ◯ $\mathbf{w} \leftarrow \mathbf{w} - \alpha[Q(s_t, a_t) - Q_{samp}]\mathbf{w}$

(iii) The algorithms in the previous parts (part i and ii) are:

☐ model-based ☐ model-free

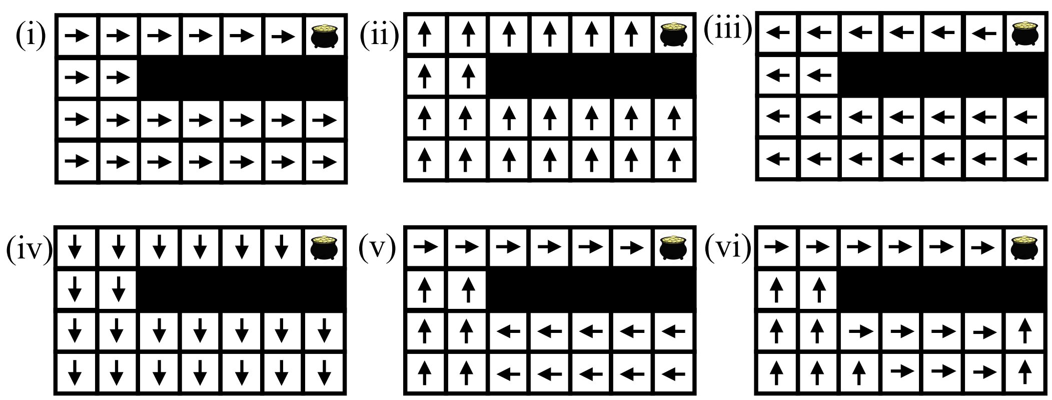
Q2. MDPs & RL



Consider the grid-world MDP above. The goal of the game is to reach the pot of gold. As soon as you land on the pot of gold you receive a reward and the game ends. Your agent can move around the grid by taking the following actions: North, South, East, West. Moving into a square that is not a wall is always successful. If you attempt to move into a grid location occupied by a wall or attempt to move off the board, you remain in your current grid location.

Our goal is to build a value function that assigns values to each grid location, but instead of keeping track of a separate number for each location, we are going to use features. Specifically, suppose we represent the value of state (x, y) (a grid location) as $V(x, y) = w^\top f(x, y)$. Here, $f(x, y)$ is a feature function that maps the grid location (x, y) to a vector of features and w is a weight vector that parameterizes our value function (note that entries in w can be any real number, positive or negative).

In the next few questions, we will look at various possible feature functions $f(x, y)$. We will think about the value functions that are representable using each set of features, and, further, think about which policies could be extracted from those value functions. Assume that when a policy is extracted from a value function, ties can be broken arbitrarily. In our definition of feature functions we will make use of the location of the pot of gold. Let the gold's location be (x^*, y^*). Keep in mind the policies (i), (ii), (iii), (iv), (v), and (vi) shown below.



(a) Suppose we use a single feature: the x-distance to the pot of gold. Specifically, suppose $f(x, y) = |x - x^*|$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector w is not allowed to be 0. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ○ (v) ○ (vi)

(b) Suppose we use a single feature: the y-distance to the pot of gold. Specifically, suppose $f(x, y) = |y - y^*|$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector w is not allowed to be 0. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ○ (v) ○ (vi)

(c) Suppose we use a single feature: the Manhattan distance to the pot of gold. Specifically, suppose $f(x, y) = |x - x^*| + |y - y^*|$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector w is not allowed to be 0. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ○ (v) ○ (vi)

(d) Suppose we use a single feature: the length of the shortest path to the pot of gold. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector w is not allowed to be 0. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ○ (v) ○ (vi)

(e) Suppose we use two features: the x-distance to the pot of gold and the y-distance to the pot of gold. Specifically, suppose $f(x, y) = (|x - x^*|, |y - y^*|)$. Which of the policies could be extracted from a value function that is representable using this feature function? Assume the weights vector w must have at least one non-zero entry. Fill in all that apply.

○ (i) ○ (ii) ○ (iii) ○ (iv) ○ (v) ○ (vi)