CS 188
Fall 2022

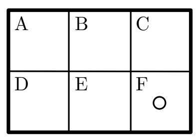
Midterm Review MDPs Solutions

Q1. MDP

Pacman is using MDPs to maximize his expected utility. In each environment:

- Pacman has the standard actions {North, East, South, West} unless blocked by an outer wall
- There is a reward of 1 point when eating the dot (for example, in the grid below, $R(C, South, F) = 1$)
- The game ends when the dot is eaten

(a) Consider a the following grid where there is a single food pellet in the bottom right corner (F). The **discount** factor is 0.5. There is no living reward. The states are simply the grid locations.



(i) What is the optimal policy for each state?

State	$\pi(state)$
A	East or South
B	East or South
C	South
D	East
E	East

(ii) What is the optimal value for the state of being in the upper left corner (A)? Reminder: the discount factor is 0.5.

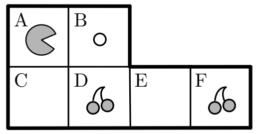
$V^*(A) =$ 0.25

k	V(A)	V(B)	V(C)	V(D)	V(E)	V(F)
0	0	0	0	0	0	0
1	0	0	1	0	1	0
2	0	0.5	1	0.5	1	0
3	0.25	0.5	1	0.5	1	0
4	0.25	0.5	1	0.5	1	0

(iii) Using value iteration with the value of all states equal to zero at k=0, for which iteration k will $V_k(A) = V^*(A)$?

$k =$ 3 (see above)

(b) Consider a new Pacman level that begins with cherries in locations D and F. Landing on a grid position with cherries is worth 5 points and then the cherries at that position disappear. There is still one dot, worth 1 point. The game still only ends when the dot is eaten.



(i) With no discount ($\gamma = 1$) and a living reward of -1, what is the optimal policy for the states in this level's state space?

State	$\pi(state)$
A, D_{Cherry}=true, F_{Cherry}=true	South
A, D_{Cherry}=true, F_{Cherry}=false	South
A, D_{Cherry}=false, F_{Cherry}=true	East
A, D_{Cherry}=false, F_{Cherry}=false	East
C, D_{Cherry}=true, F_{Cherry}=true	East
C, D_{Cherry}=true, F_{Cherry}=false	East
C, D_{Cherry}=false, F_{Cherry}=true	East
C, D_{Cherry}=false, F_{Cherry}=false	North/East
D, D_{Cherry}=false, F_{Cherry}=true	East
D, D_{Cherry}=false, F_{Cherry}=false	North
E, D_{Cherry}=true, F_{Cherry}=true	East
E, D_{Cherry}=true, F_{Cherry}=false	West
E, D_{Cherry}=false, F_{Cherry}=true	East
E, D_{Cherry}=false, F_{Cherry}=false	West
F, D_{Cherry}=true, F_{Cherry}=false	West
F, D_{Cherry}=false, F_{Cherry}=false	West

(ii) With no discount ($\gamma = 1$), what is the range of living reward values such that Pacman eats exactly one cherry when starting at position A?

Valid range for the living reward is (-2.5,-1.25).

Let x equal the living reward.

The reward for eating zero cherries {A,B} is $x + 1$ (one step plus food).

The reward for eating exactly one cherry {A,C,D,B} is $3x + 6$ (three steps plus cherry plus food).

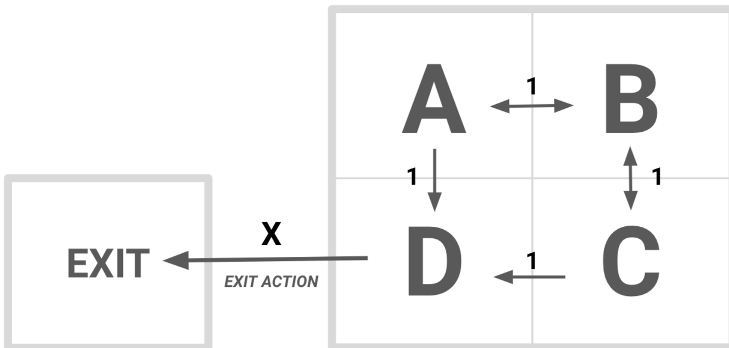
The reward for eating two cherries {A,C,D,E,F,E,D,B} is $7x + 11$ (seven steps plus two cherries plus food).

x must be greater than -2.5 to make eating at least one cherry worth it ($3x + 6 > x + 1$).

x must be less than -1.25 to eat less than one cherry ($3x + 6 > 7x + 11$).

Q2. Strange MDPs

In this MDP, the available actions at **state A, B, C** are *LEFT, RIGHT, UP,* and *DOWN* unless there is a wall in that direction. The only action at **state D** is the *EXIT ACTION* and gives the agent a **reward of** x. The **reward for non-exit actions is always 1**.



(a) Let all actions be deterministic. Assume $\gamma = \frac{1}{2}$. Express the following in terms of x.

$V^*(D) = x$

$V^*(C) = max(1 + 0.5x, 2)$

$V^*(A) = max(1 + 0.5x, 2)$

$V^*(B) = max(1 + 0.5(1 + 0.5x), 2)$

The 2 comes from the utility being an infinite geometric sum of discounted reward $= \frac{1}{(1-\frac{1}{2})} = 2$

(b) Let any non-exit action be successful with probability $= \frac{1}{2}$. Otherwise, the agent stays in the same state with reward $= 0$. The *EXIT ACTION* from the **state D** is still deterministic and will always succeed. Assume that $\gamma = \frac{1}{2}$.

For which value of x does $Q^*(A, DOWN) = Q^*(A, RIGHT)$? Box your answer and justify/show your work.

$Q^*(A, DOWN) = Q^*(A, RIGHT)$ implies $V^*(A) = Q^*(A, DOWN) = Q^*(A, RIGHT)$

$$V^*(A) = Q^*(A, DOWN) = \frac{1}{2}(0 + \frac{1}{2}V^*(A)) + \frac{1}{2}(1 + \frac{1}{2}x) = \frac{1}{2} + \frac{1}{4}(V^*(A)) + \frac{1}{4}x \tag{1}$$

$$V^*(A) = \frac{2}{3} + \frac{1}{3}x \tag{2}$$

$$V^*(A) = Q^*(A, RIGHT) = \frac{1}{2}(0 + \frac{1}{2}V^*(A)) + \frac{1}{2}(1 + \frac{1}{2}V^*(B)) = \frac{1}{2} + \frac{1}{4}V^*(A) + \frac{1}{4}V^*(B) \tag{3}$$

$$V^*(A) = \frac{2}{3} + \frac{1}{3}V^*(B) \tag{4}$$

Because $Q^*(B, LEFT)$ and $Q^*(B, DOWN)$ are symmetric decisions, $V^*(B) = Q^*(B, LEFT)$.

$$V^*(B) = \frac{1}{2}(0 + \frac{1}{2}V^*(B)) + \frac{1}{2}(1 + \frac{1}{2}V^*(A)) = \frac{1}{2} + \frac{1}{4}V^*(B) + \frac{1}{4}V^*(A) \tag{5}$$

$$V^*(B) = \frac{2}{3} + \frac{1}{3}V^*(A) \tag{6}$$

Combining (2), (4), and (6) gives us: x = 1

(c) We now add one more layer of complexity. Turns out that the reward function is not guaranteed to give a particular reward when the agent takes an action. Every time an agent transitions from one state to another, once the agent reaches the new state s', a fair 6-sided dice is rolled. If the dices lands with value x, the agent receives the reward $R(s, a, s') + x$. The sides of dice have value $1, 2, 3, 4, 5$ and 6.

Write down the new bellman update equation for $V_{k+1}(s)$ in terms of $T(s, a, s')$, $R(s, a, s')$, $V_k(s')$, and γ.

$V_{k+1}(s) = \max_a \sum_{s'} T(s, a, s')[\frac{1}{6}(\sum_{i=1}^{6} R(s, a, s') + i) + \gamma V_k(s')]$

$= \max_a \sum_{s'} T(s, a, s')(R(s, a, s') + 3.5 + \gamma V_k(s'))$