CS 188
Fall 2022

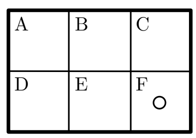
Midterm Review MDPs

Q1. MDP

Pacman is using MDPs to maximize his expected utility. In each environment:

- Pacman has the standard actions {North, East, South, West} unless blocked by an outer wall
- There is a reward of 1 point when eating the dot (for example, in the grid below, $R(C, South, F) = 1$)
- The game ends when the dot is eaten

(a) Consider a the following grid where there is a single food pellet in the bottom right corner (F). The **discount** factor is 0.5. There is no living reward. The states are simply the grid locations.



(i) What is the optimal policy for each state?

State	$\pi(state)$
A	
B	
C	
D	
E	

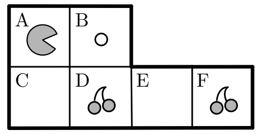
(ii) What is the optimal value for the state of being in the upper left corner (A)? Reminder: the discount factor is 0.5.

$V^*(A) =$

(iii) Using value iteration with the value of all states equal to zero at k=0, for which iteration k will $V_k(A) = V^*(A)$?

$k =$

(b) Consider a new Pacman level that begins with cherries in locations D and F. Landing on a grid position with cherries is worth 5 points and then the cherries at that position disappear. There is still one dot, worth 1 point. The game still only ends when the dot is eaten.

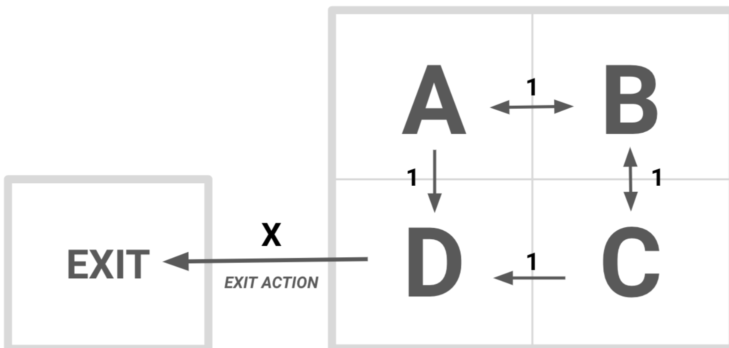


(i) With no discount ($\gamma = 1$) and a living reward of -1, what is the optimal policy for the states in this level's state space?

(ii) With no discount ($\gamma = 1$), what is the range of living reward values such that Pacman eats exactly one cherry when starting at position A?

Q2. Strange MDPs

In this MDP, the available actions at **state A, B, C** are *LEFT, RIGHT, UP,* and *DOWN* unless there is a wall in that direction. The only action at **state D** is the *EXIT ACTION* and gives the agent a **reward of** x. The **reward for non-exit actions is always 1**.



(a) Let all actions be deterministic. Assume $\gamma = \frac{1}{2}$. Express the following in terms of x.

$V^*(D) =$ $V^*(C) =$

$V^*(A) =$ $V^*(B) =$

(b) Let any non-exit action be successful with probability $= \frac{1}{2}$. Otherwise, the agent stays in the same state with reward $= 0$. The *EXIT ACTION* from the **state D** is still deterministic and will always succeed. Assume that $\gamma = \frac{1}{2}$.

For which value of x does $Q^*(A, DOWN) = Q^*(A, RIGHT)$? Box your answer and justify/show your work.

(c) We now add one more layer of complexity. Turns out that the reward function is not guaranteed to give a particular reward when the agent takes an action. Every time an agent transitions from one state to another, once the agent reaches the new state s', a fair 6-sided dice is rolled. If the dices lands with value x, the agent receives the reward $R(s, a, s') + x$. The sides of dice have value $1, 2, 3, 4, 5$ and 6.

Write down the new bellman update equation for $V_{k+1}(s)$ in terms of $T(s, a, s')$, $R(s, a, s')$, $V_k(s')$, and γ.