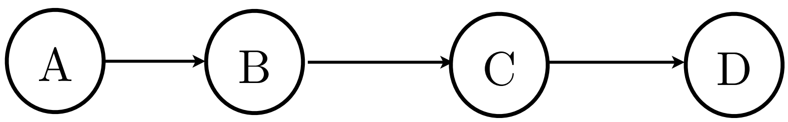
CS 188
Fall 2022

Regular Discussion 8 Solutions

Q1. Bayes' Nets Sampling

Assume the following Bayes' net, and the corresponding distributions over the variables in the Bayes' net:



$P(A)$	
$-a$	$3/4$
$+a$	$1/4$

$P(B\|A)$		
$-a$	$-b$	$2/3$
$-a$	$+b$	$1/3$
$+a$	$-b$	$4/5$
$+a$	$+b$	$1/5$

$P(C\|B)$		
$-b$	$-c$	$1/4$
$-b$	$+c$	$3/4$
$+b$	$-c$	$1/2$
$+b$	$+c$	$1/2$

$P(D\|C)$		
$-c$	$-d$	$1/8$
$-c$	$+d$	$7/8$
$+c$	$-d$	$5/6$
$+c$	$+d$	$1/6$

(a) You are given the following samples:

$(+a, +b, -c, -d)$
$(+a, -b, +c, -d)$
$\cancel{(-a, +b, +c, -d)}$
$\cancel{(-a, -b, +c, -d)}$

$\cancel{(+a, -b, -c, +d)}$
$(+a, +b, +c, -d)$
$\cancel{(-a, +b, -c, +d)}$
$\cancel{(-a, -b, +c, -d)}$

(i) If these samples came from doing Prior Sampling, calculate our sample estimate of $P(+c)$. 5/8

(ii) Now we will estimate $P(+c \mid +a, -d)$. Above, clearly cross out the samples that would **not** be used when doing Rejection Sampling for this task, and write down the sample estimate of $P(+c \mid +a, -d)$. 2/3

(b) Using Likelihood Weighting Sampling to estimate $P(-a \mid +b, -d)$, the following samples were obtained. What is the weight of each sample?

Sample				**Weight**
$-a$	$+b$	$+c$	$-d$	$P(+b \mid -a)P(-d \mid +c) = 1/3 * 5/6 = 5/18 = 0.277$
$+a$	$+b$	$+c$	$-d$	$P(+b \mid +a)P(-d \mid +c) = 1/5 * 5/6 = 5/30 = 1/6 = 0.17$
$+a$	$+b$	$-c$	$-d$	$P(+b \mid +a)P(-d \mid -c) = 1/5 * 1/8 = 1/40 = 0.025$
$-a$	$+b$	$-c$	$-d$	$P(+b \mid -a)P(-d \mid -c) = 1/3 * 1/8 = 1/24 = 0.042$

(c) From the weighted samples, estimate $P(-a \mid +b, -d)$.

$$\frac{5/18+1/24}{5/18+5/30+1/40+1/24} = 0.625$$

(d) Recall that during Gibbs Sampling, samples are generated through an iterative process.

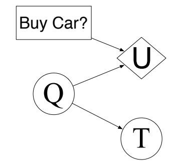
Assume that the only evidence that is available is $A = +a$. Which sequence(s) below could have been generated by Gibbs Sampling?

Sequence 1					Sequence 2					Sequence 3					Sequence 4				
1 :	$+a$	$-b$	$-c$	$+d$	1 :	$+a$	$-b$	$-c$	$+d$	1 :	$+a$	$-b$	$-c$	$+d$	1 :	$+a$	$-b$	$-c$	$+d$
2 :	$+a$	$-b$	$-c$	$+d$	2 :	$+a$	$-b$	$-c$	$-d$	2 :	$+a$	$-b$	$-c$	$-d$	2 :	$+a$	$-b$	$-c$	$-d$
3 :	$+a$	$-b$	$+c$	$+d$	3 :	$-a$	$-b$	$-c$	$+d$	3 :	$+a$	$+b$	$-c$	$-d$	3 :	$+a$	$+b$	$-c$	$+d$

Sequence 1 and Sequence 3. Gibbs sampling updates one variable at a time and never changes the evidence. The first and third sequences have at most one variable change per row, and hence could have been generated from Gibbs sampling. In sequence 2, the evidence variable is changed. In sequence 4, the second and third samples have both B and D changing.

2 Decision Networks and VPI

A buyer is deciding whether to buy a certain used car. The car may be good quality ($Q = +q$) or bad quality ($Q = -q$). A test (T) costs \$50 and can help to figure out the quality of the car. There are only two outcomes for the test: $T = \text{pass}$ or $T = \text{fail}$. The car costs \$1,500, and its market value is \$2,000 if it is good quality; if not, \$700 in repairs will be needed to make it good quality. The buyer's estimate is that the car has 70% chance of being good quality.



(a) Calculate the expected net gain from buying the car, given no test.

$$\begin{aligned} EU(\text{buy}) &= P(Q = +q) \cdot U(+q, \text{buy}) + P(Q = -q) \cdot U(-q, buy) \\ &= .7 \cdot 500 + 0.3 \cdot -200 = 290 \end{aligned}$$

(b) Tests can be described by the probability that the car will pass or fail the test given that the car is good or bad quality. We know: $P(T = \text{pass}|Q = +q) = 0.9$ and $P(T = \text{pass}|Q = -q) = 0.2$

Calculate the probability that the car will pass (or fail) its test, and then the probability that it is good (or bad) quality given each possible test outcome.

$$\begin{aligned} P(T = \text{pass}) &= \sum_q P(T = \text{pass}, Q = q) \\ &= P(T = \text{pass}|Q = +q)P(Q = +q) + P(T = \text{pass}|Q = -q)P(Q = -q) \\ &= 0.69 \\ P(T = \text{fail}) &= 0.31 \\ P(Q = +q|T = \text{pass}) &= \frac{P(T = \text{pass}|Q = +q)P(Q = +q)}{P(T = \text{pass})} \\ &= \frac{0.9 \cdot 0.7}{0.69} = \frac{21}{23} \approx 0.91 \\ P(Q = +q|T = \text{fail}) &= \frac{P(T = \text{fail}|Q = +q)P(Q = +q)}{P(T = \text{fail})} \\ &= \frac{0.1 \cdot 0.7}{0.31} = \frac{7}{31} \approx 0.22 \end{aligned}$$

(c) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

$$\begin{aligned} EU(\text{buy}|T = \text{pass}) &= P(Q = +q|T = \text{pass})U(+q, \text{buy}) + P(Q = -q|T = \text{pass})U(-q, \text{buy}) \\ &\approx 0.91 \cdot 500 + 0.09 \cdot (-200) \approx 437 \\ EU(\text{buy}|T = \text{fail}) &= P(Q = +q|T = \text{fail})U(+q, \text{buy}) + P(Q = -q|T = \text{fail})U(-q, \text{buy}) \\ &\approx 0.22 \cdot 500 + 0.78 \cdot (-200) = -46 \\ EU(\neg\text{buy}|T = \text{pass}) &= 0 \\ EU(\neg\text{buy}|T = \text{fail}) &= 0 \end{aligned}$$

Therefore: $MEU(T = \text{pass}) = 437$ (with buy) and $MEU(T = \text{fail}) = 0$ (using $\neg$buy)

(d) Calculate the value of (perfect) information of the test. Should the buyer pay for a test?

$$\begin{aligned} VPI(T) &= \left(\sum_t P(T = t)MEU(T = t)\right) - MEU(\varnothing) \\ &= 0.69 \cdot 437 + 0.31 \cdot 0 - 290 \approx 11.53 \end{aligned}$$

You shouldn't pay for it, since the cost is \$50.