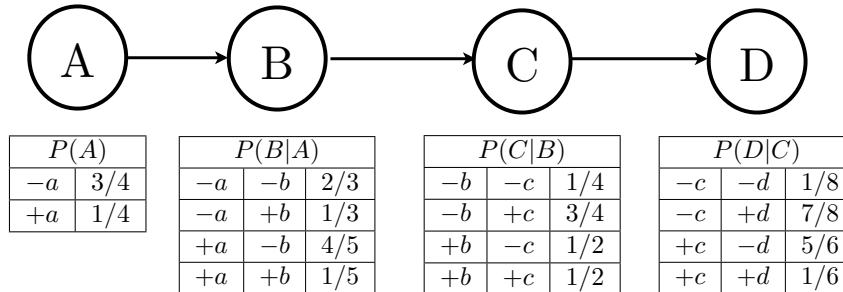


Q1. Bayes' Nets Sampling

Assume the following Bayes' net, and the corresponding distributions over the variables in the Bayes' net:



(a) You are given the following samples:

- $(+a, +b, -c, -d)$
- $(+a, -b, +c, -d)$
- $(-a, +b, +c, -d)$
- $(-a, -b, +c, -d)$

- $(+a, -b, -c, +d)$
- $(+a, +b, +c, -d)$
- $(-a, +b, -c, +d)$
- $(-a, -b, +c, -d)$

(i) If these samples came from doing Prior Sampling, calculate our sample estimate of  $P(+c)$ .

(ii) Now we will estimate  $P(+c | +a, -d)$ . Above, clearly cross out the samples that would **not** be used when doing Rejection Sampling for this task, and write down the sample estimate of  $P(+c | +a, -d)$ .

(b) Using Likelihood Weighting Sampling to estimate  $P(-a | +b, -d)$ , the following samples were obtained. What is the weight of each sample?

Sample	Weight
$-a \quad +b \quad +c \quad -d$	_____
$+a \quad +b \quad +c \quad -d$	_____
$+a \quad +b \quad -c \quad -d$	_____
$-a \quad +b \quad -c \quad -d$	_____

(c) From the weighted samples, estimate  $P(-a | +b, -d)$ .

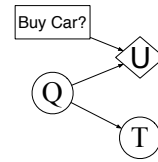
(d) Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is  $A = +a$ . Which sequence(s) below could have been generated by Gibbs Sampling?

Sequence 1	Sequence 2	Sequence 3	Sequence 4
1: $+a \quad -b \quad -c \quad +d$	1: $+a \quad -b \quad -c \quad +d$	1: $+a \quad -b \quad -c \quad +d$	1: $+a \quad -b \quad -c \quad +d$
2: $+a \quad -b \quad -c \quad +d$	2: $+a \quad -b \quad -c \quad -d$	2: $+a \quad -b \quad -c \quad -d$	2: $+a \quad -b \quad -c \quad -d$
3: $+a \quad -b \quad +c \quad +d$	3: $-a \quad -b \quad -c \quad +d$	3: $+a \quad +b \quad -c \quad -d$	3: $+a \quad +b \quad -c \quad +d$

## 2 Decision Networks and VPI

A buyer is deciding whether to buy a certain used car. The car may be good quality ( $Q = +q$ ) or bad quality ( $Q = -q$ ). A test ( $T$ ) costs \$50 and can help to figure out the quality of the car. There are only two outcomes for the test:  $T = \text{pass}$  or  $T = \text{fail}$ . The car costs \$1,500, and its market value is \$2,000 if it is good quality; if not, \$700 in repairs will be needed to make it good quality. The buyer's estimate is that the car has 70% chance of being good quality.



- (a) Calculate the expected net gain from buying the car, given no test.
- (b) Tests can be described by the probability that the car will pass or fail the test given that the car is good or bad quality. We know:  $P(T = \text{pass}|Q = +q) = 0.9$  and  $P(T = \text{pass}|Q = -q) = 0.2$   
Calculate the probability that the car will pass (or fail) its test, and then the probability that it is good (or bad) quality given each possible test outcome.

- (c) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

- (d) Calculate the value of (perfect) information of the test. Should the buyer pay for a test?