Announcements

- **Homework 2** due **tomorrow (Sept 16)** at 11:59pm PT

- **Project 2** due **next Thursday (Sept 22)** at 11:59pm PT
Recap: Why Pacman Starves (d=2)

- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!
Recap: Why Pacman Starves (d=2)
Recap: Why Pacman Starves (d=2)
Uncertain Outcomes
Worst-Case vs. Average Case

Idea: Uncertain outcomes controlled by chance, not an adversary!
Expectimax Search

- Why wouldn’t we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip

- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes

- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children

- Later, we’ll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes
Video of Demo Minimax vs Expectimax (Exp)
def value(state):
    if the state is a terminal state: return the state’s utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)

def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v

def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
Expectimax Pseudocode

def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v

v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10
Expectimax Example
Expectimax Pruning?
Depth-Limited Expectimax

Estimate of true expectimax value (which would require a lot of work to compute)
Probabilities
Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

Example: Traffic on freeway
- Random variable: $T$ = whether there’s traffic
- Outcomes: $T$ in \{none, light, heavy\}
- Distribution: $P(T=\text{none}) = 0.25$, $P(T=\text{light}) = 0.50$, $P(T=\text{heavy}) = 0.25$

Some laws of probability (more later):
- Probabilities are always non-negative
- Probabilities over all possible outcomes sum to one

As we get more evidence, probabilities may change:
- $P(T=\text{heavy}) = 0.25$, $P(T=\text{heavy} \mid \text{Hour=8am}) = 0.60$
- We’ll talk about methods for reasoning and updating probabilities later
The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes.

Example: How long to get to the airport?

Time:  
- 20 min
- 30 min
- 60 min

Probability:  
- 0.25
- 0.50
- 0.25

35 min
In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state:
- Model could be a simple uniform distribution (roll a die)
- Model could be sophisticated and require a great deal of computation
- We have a chance node for any outcome out of our control: opponent or environment
- The model might say that adversarial actions are likely!

For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes.

Having a probabilistic belief about another agent’s action does not mean that the agent is flipping any coins!
Quiz: Informed Probabilities

- Let’s say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise.
- Question: What tree search should you use?

**Answer: Expectimax!**

- To figure out EACH chance node’s probabilities, you have to run a simulation of your opponent.
- This kind of thing gets very slow very quickly.
- Even worse if you have to simulate your opponent simulating you...
- ... except for minimax, which has the nice property that it all collapses into one game tree.
Modeling Assumptions
The Dangers of Optimism and Pessimism

Dangerous Optimism
Assuming chance when the world is adversarial

Dangerous Pessimism
Assuming the worst case when it’s not likely
Assumptions vs. Reality

<table>
<thead>
<tr>
<th></th>
<th>Adversarial Ghost</th>
<th>Random Ghost</th>
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<tbody>
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<td>Minimax Pacman</td>
<td>Won 1/5</td>
<td>Avg. Score: -303</td>
</tr>
<tr>
<td>Expectimax Pacman</td>
<td>Won 5/5</td>
<td>Avg. Score: 503</td>
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Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

[Demos: world assumptions (L7D3,4,5,6)]
Video of Demo World Assumptions
Adversarial Ghost – Expectimax Pacman
Video of Demo World Assumptions
Random Ghost – Minimax Pacman
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<tr>
<td></td>
<td>Avg. Score: 483</td>
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Results from playing 5 games

[Demos: world assumptions (L7D3,4,5,6)]
Other Game Types
Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children
Example: Backgammon

- Dice rolls increase $b$: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$

- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier...

- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play

- 1st AI world champion in any game!

Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically...

```
1,6,6  7,1,2  6,1,2  7,2,1  5,1,7  1,5,2  7,7,1  5,2,5
```
Probabilities and Randomness in Algorithm Design
Overcoming Resource Limits with Randomness

- Even with alpha-beta pruning and limited depth, large $b$ is an issue (recall best-case time complexity is $b^{m/2}$)
  - Possible for chess: with alpha-beta, $35^{(8/2)} \approx 1M$; depth 8 is quite good
  - Difficult for Go: $300^{(8/2)} \approx 8 \text{ billion}$

- Monte Carlo Tree Search (MCTS) combines two important ideas:
  - *Evaluation by rollouts* – play multiple games to termination from a state $s$ (using a simple, fast or random policy) and count wins and losses
  - *Selective search* – explore parts of the tree that will help improve the decision at the root, regardless of depth
Rollouts

- For each rollout:
  - Repeat until terminal:
    - Play a move according to a fixed, fast rollout policy
  - Record the result
- Fraction of wins correlates with the true value of the position!
- Having a “better” rollout policy helps
MCTS Version 0

- Do $N$ rollouts from each child of the root, record fraction of wins
- Pick the move that gives the best outcome by this metric
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- Pick the move that gives the best outcome by this metric
Allocate rollouts to more promising nodes
MCTS Version 0.9

- Allocate rollouts to more promising nodes
- Allocate rollouts to more promising nodes
- Allocate rollouts to more uncertain nodes
Upper Confidence Bounds (UCB) heuristics

- UCB1 formula combines “promising” and “uncertain”:

\[ UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\text{PARENT}(n))}{N(n)}} \]

- \( N(n) = \) number of rollouts from node \( n \)
- \( U(n) = \) total utility of rollouts (e.g., # wins) for Player(Parent(n))
  - Keep track of both for each node
Repeat until out of time:

- **Selection**: recursively apply UCB to choose a path down to a leaf node $n$
- **Expansion**: add a new child $c$ to $n$ and
- **Simulation**: run a rollout from $c$
- **Backpropagation**: update $U$ and $N$ counts from $c$ back up to the root

[Example adapted from Introduction to Monte Carlo Tree Search. Bradberry. 2015]
MCTS Version 2.0: UCT

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- Choose the action leading to the child with highest \( N \)
Why is there no min or max?????

- "Value" of a node, $U(n)/N(n)$, is a weighted sum of child values!
- Idea: as $N \to \infty$, the vast majority of rollouts are concentrated in the best child(ren), so weighted average $\to$ max/min
- Theorem: as $N \to \infty$ UCT selects the minimax move
  - (but $N$ never approaches infinity!)
MCTS Application: AlphaGo

- Monte Carlo Tree Search with additions including:
  - Rollout policy is a neural network trained with reinforcement learning and expert human moves
  - In combination with rollout outcomes, use a trained value function to better predict node’s utility

Next Time: MDPs!