1 HMMs

Consider the following Hidden Markov Model. $O_1$ and $O_2$ are supposed to be shaded.

Suppose that we observe $O_1 = a$ and $O_2 = b$. Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = a, O_2 = b)$ one step at a time.

(a) Compute $P(W_1, O_1 = a)$.

(b) Using the previous calculation, compute $P(W_2, O_1 = a)$.

(c) Using the previous calculation, compute $P(W_2, O_1 = a, O_2 = b)$.

(d) Finally, compute $P(W_2|O_1 = a, O_2 = b)$. 
2 Particle Filtering

Let’s use Particle Filtering to estimate the distribution of $P(W_2|O_1 = a, O_2 = b)$. Here’s the HMM again. $O_1$ and $O_2$ are supposed to be shaded.

We start with two particles representing our distribution for $W_1$.

$P_1 : W_1 = 0$

$P_2 : W_1 = 1$

Use the following random numbers to run particle filtering:

$$[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]$$

(a) **Observe**: Compute the weight of the two particles after evidence $O_1 = a$.

(b) **Resample**: Using the random numbers, resample $P_1$ and $P_2$ based on the weights.

(c) **Predict**: Sample $P_1$ and $P_2$ from applying the time update.

(d) **Update**: Compute the weight of the two particles after evidence $O_2 = b$.

(e) **Resample**: Using the random numbers, resample $P_1$ and $P_2$ based on the weights.

(f) What is our estimated distribution for $P(W_2|O_1 = a, O_2 = b)$?