Q1. Power Pellets

Consider a Pacman game where Pacman can eat 3 types of pellets:

- Normal pellets (n-pellets), which are worth one point.
- Decaying pellets (d-pellets), which are worth $\max(0, 5 - t)$ points, where $t$ is time.
- Growing pellets (g-pellets), which are worth $t$ points, where $t$ is time.

The location and type of each pellet is fixed. The pellet’s point value stops changing once eaten. For example, if Pacman eats one g-pellet at $t = 1$ and one d-pellet at $t = 2$, Pacman will have won $1 + 3 = 4$ points.

Pacman needs to find a path to win at least 10 points but he wants to minimize distance travelled. The cost between states is equal to distance travelled.

(a) Which of the following must be including for a minimum, sufficient state space?

- Pacman’s location
- Location and type of each pellet
- How far Pacman has travelled
- Current time
- How many pellets Pacman has eaten and the point value of each eaten pellet
- Total points Pacman has won
- Which pellets Pacman has eaten

A state space should include which pellets are left on the board, the current value of pellets, Pacman’s location, and the total points collected so far. With this in mind:

1. The starting location and type of each pellet are not included in the state space as this is something that does not change during the search. This is analogous to how the walls of a Pacman board are not included in the state space.
2. How far Pacman has travelled does not need to be explicitly tracked by the state, since this will be reflected in the cost of a path.
3. Pacman does need the current time to determine the value of pellets on the board.
4. The number of pellets Pacman has eaten is extraneous.
5. Pacman must track the total number of points won for the goal test.
6. Pacman must know which pellets remain on the board, which is the complement of the pellets he has eaten.

(b) Which of the following are admissible heuristics? Let $x$ be the number of points won so far.

- Distance to closest pellet, except if in the goal state, in which case the heuristic value is 0.
- Distance needed to win $10 - x$ points, determining the value of all pellets as if they were n-pellets.
- Distance needed to win $10 - x$ points, determining the value of all pellets as if they were g-pellets (i.e. all pellet values will be $t$).
- Distance needed to win $10 - x$ points, determining the value of all pellets as if they were d-pellets (i.e. all pellet values will be $\max(0, 5 - t)$).
- Distance needed to win $10 - x$ points assuming all pellets maintain current point value (g-pellets stop
increasing in value and d-pellets stop decreasing in value)

☐ None of the above

(1) Admissible; to get 10 points Pacman will always have to travel at least as far as the distance to the closest pellet, so this will always be an underestimate.

(2) Not admissible; if all the pellets are actually g-pellets, assuming they are n-pellets will lead to Pacman collecting more pellets in more locations, and thus travel further.

(3) Ambiguous; if pellets are n-pellets or d-pellets, Pacman will generally have to go further, except at the beginning of the game when d-pellets are worth more, in which case this heuristic will over-estimate the cost to the goal. However, if Pacman is allowed to stay in place with no cost, then this heuristic is admissible because the heuristic will instead calculate all pellet values as 10. This option was ignored in scoring.

(4) Not admissible; if pellets are n-pellets or g-pellets, Pacman would have an overestimate.

(5) Not admissible; if pellets are g-pellets, then using the current pellet value might lead Pacman to collect more locations, and thus travel further than necessary.

(c) Instead of finding a path which minimizes distance, Pacman would like to find a path which minimizes the following:

\[ C_{\text{new}} = a \cdot t + b \cdot d \]

where \( t \) is the amount of time elapsed, \( d \) is the distance travelled, and \( a \) and \( b \) are non-negative constants such that \( a + b = 1 \). Pacman knows an admissible heuristic when he is trying to minimize time (i.e. when \( a = 1, b = 0 \)), \( h_t \), and when he is trying to minimize distance, \( h_d \) (i.e. when \( a = 0, b = 1 \)).

Which of the following heuristics is guaranteed to be admissible when minimizing \( C_{\text{new}} \)?

☐ mean\((h_t, h_d)\)
☐ min\((h_t, h_d)\)
☐ max\((h_t, h_d)\)
☐ \( a \cdot h_t + b \cdot h_d \)

☐ None of the above

For this question, think about the inequality \( C_{\text{new}} = a \cdot t + b \cdot d \geq a \cdot h_t + b \cdot h_d \). We can guarantee a heuristic \( h_{\text{new}} \) is admissible if \( h_{\text{new}} \leq a \cdot h_t + b \cdot h_d \).

(1) If \( a = b \), 0.5 \( h_t + 0.5 \cdot h_d \) is not guaranteed to be less than \( a \cdot h_t + b \cdot h_d \), so this will not be admissible.

(2) \( \min(h_t, h_d) = a \cdot \min(h_t, h_d) + b \cdot \min(h_t, h_d) \leq a \cdot h_t + b \cdot h_d \)

(3) \( \max(h_t, h_d) \) will be greater than \( a \cdot h_t + b \cdot h_d \) unless \( h_t = h_d \), so this will not be admissible.

(4) Admissible.
Q2. Coin Stars

In a new online game called Coin Stars, all players are walking around an M x N grid to collect **hidden coins**, which only appear when you’re on top of them. There are also power pellets scattered across the board, which are visible to all players. If you walk onto a square with a power pellet, your power level goes up by 1, and the power pellet disappears. Players will also attack each other if one player enters a square occupied by another player. In an attack, the player with a higher power level will steal all the coins from the other player. If they have equal power levels, nothing happens. Each turn, players go in order to move in one of the following directions: {N, S, E, W}.

In this problem, you and your friend Amy are playing Coin Stars against each other. You are player 1, and your opponent Amy is player 2. Our state space representation includes the locations of the power pellets \((x_{pj}, y_{pj})\) and the following player information: (1) Each player’s location \((x_i, y_i)\); (2) Each player’s power level \(l_i\); (3) Each player’s coin count \(c_i\).

(a) Suppose a player wins by collecting more coins at the end of a number of rounds, so we can formulate this as a minimax problem with the value of the node being \(c_1 - c_2\). Consider the following game tree where you are the maximizing player (maximizing the your net advantage, as seen above) and the opponent is the minimizer. Assuming both players act optimally, if a branch can be pruned, fill in its square completely, otherwise leave the square unmarked.

(b) Suppose that instead of the player with more coins winning, every player receives payout equal to the number of coins they’ve collected. Can we still use a multi-layer minimax tree (like the one above) to find the optimal action?

- Yes, because the update in payout policy does not affect the minimax structure of the game.
- Yes, but not for the reason above
- No, because we can no longer model the game under the updated payout policy with a game tree.

No, because the game is no longer zero-sum: your opponent obtaining more coins does not necessarily make you worse off, and vice versa. We can still model this game with a game-tree, where each node contains a tuple of two values, instead of a single value. But this means the tree is no longer a minimax tree.

An example of using the minimax tree but not optimizing the number of coins collected: when given a choice between gathering 3 coins or stealing 2 coins from the opponent, the minimax solution with \(c_1 - c_2\) will steal the 2 coins (net gain of 4), even though this will cause it to end up with fewer coins.