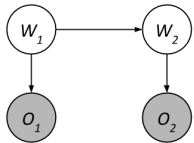


## 1 Particle Filtering

Let's use Particle Filtering to estimate the distribution of  $P(W_2|O_1 = a, O_2 = b)$ . Here's the HMM again.  $O_1$  and  $O_2$  are supposed to be shaded.



$W_1$	$P(W_1)$
0	0.3
1	0.7

$W_t$	$W_{t+1}$	$P(W_{t+1} W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

$W_t$	$O_t$	$P(O_t W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

We start with two particles representing our distribution for  $W_1$ .

$P_1 : W_1 = 0$

$P_2 : W_1 = 1$

Use the following random numbers to run particle filtering:

[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]

(a) **Observe:** Compute the weight of the two particles after evidence  $O_1 = a$ .

(b) **Resample:** Using the random numbers, resample  $P_1$  and  $P_2$  based on the weights.

(c) **Predict:** Sample  $P_1$  and  $P_2$  from applying the time update.

(d) **Update:** Compute the weight of the two particles after evidence  $O_2 = b$ .

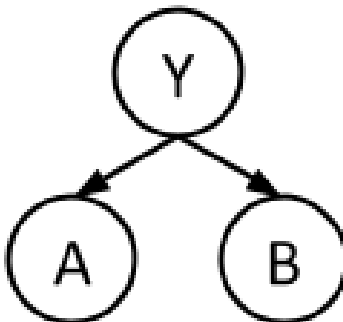
(e) **Resample:** Using the random numbers, resample  $P_1$  and  $P_2$  based on the weights.

(f) What is our estimated distribution for  $P(W_2|O_1 = a, O_2 = b)$ ?

## 2 Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels  $Y$  as a function of input features  $A$  and  $B$ .  $Y$ ,  $A$ , and  $B$  are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

$A$	1	1	1	1	0	1	0	1	1	1
$B$	1	0	0	1	1	1	1	0	1	1
$Y$	1	1	0	0	0	1	1	0	0	0



1. What are the maximum likelihood estimates for the tables  $P(Y)$ ,  $P(A|Y)$ , and  $P(B|Y)$ ?

$Y$	$P(Y)$
0	
1	

$A$	$Y$	$P(A Y)$
0	0	
1	0	
0	1	
1	1	

$B$	$Y$	$P(B Y)$
0	0	
1	0	
0	1	
1	1	

2. Consider a new data point ( $A = 1, B = 1$ ). What label would this classifier assign to this sample?

3. Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for  $P(A|Y)$  given Laplace Smoothing with  $k = 2$ .

$A$	$Y$	$P(A Y)$
0	0	
1	0	
0	1	
1	1	