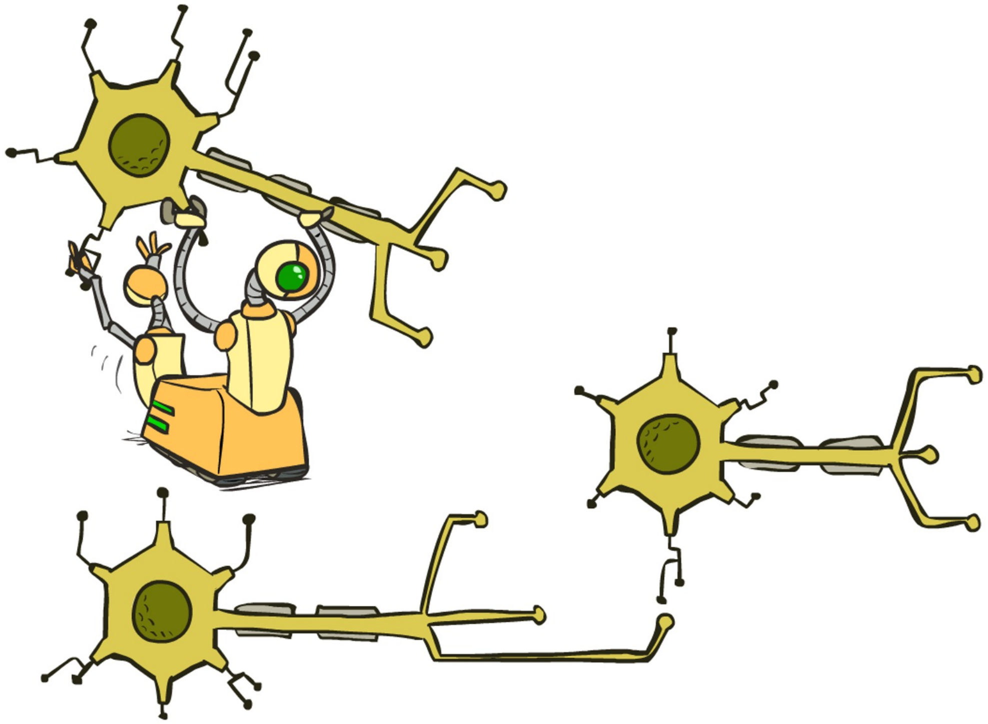
Announcements

- Slides for Catherine Olsson's guest lecture posted
  - Please reach out to catherio@anthropic.com with any questions!
- Another guest lecture by Miles Brundage after thanksgiving break (Dec 3)

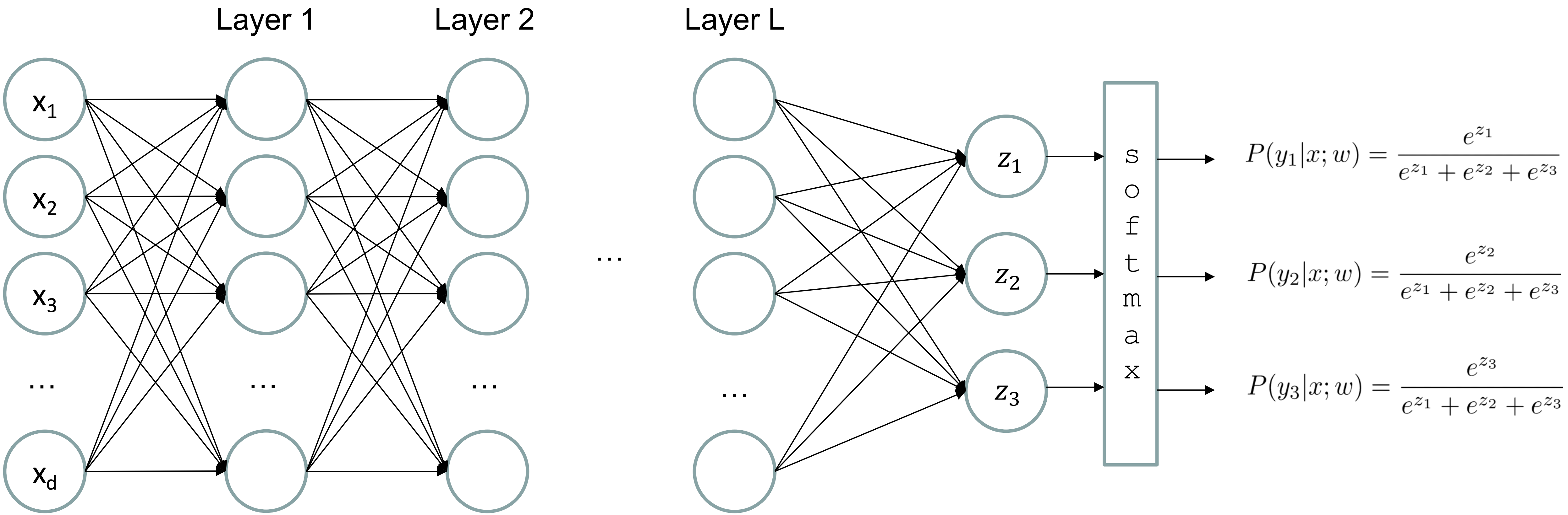
CS 188: Artificial Intelligence

Neural Networks and Optimization

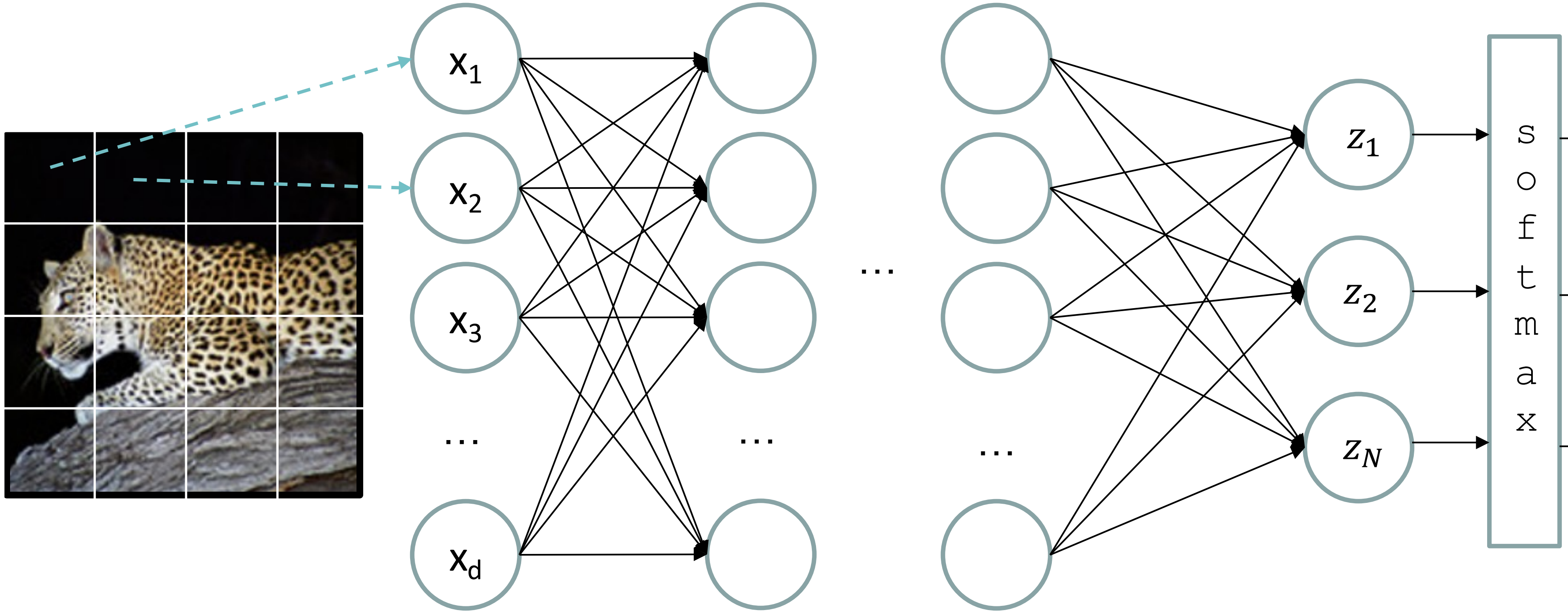


[These slides were created by Dan Klein, Pieter Abbeel, Anca Dragan for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

Recap: Deep Neural Network



Recap: Deep Neural Network



$$P(y_1|x;w) = \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}$$

$$P(y_2|x;w) = \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}$$

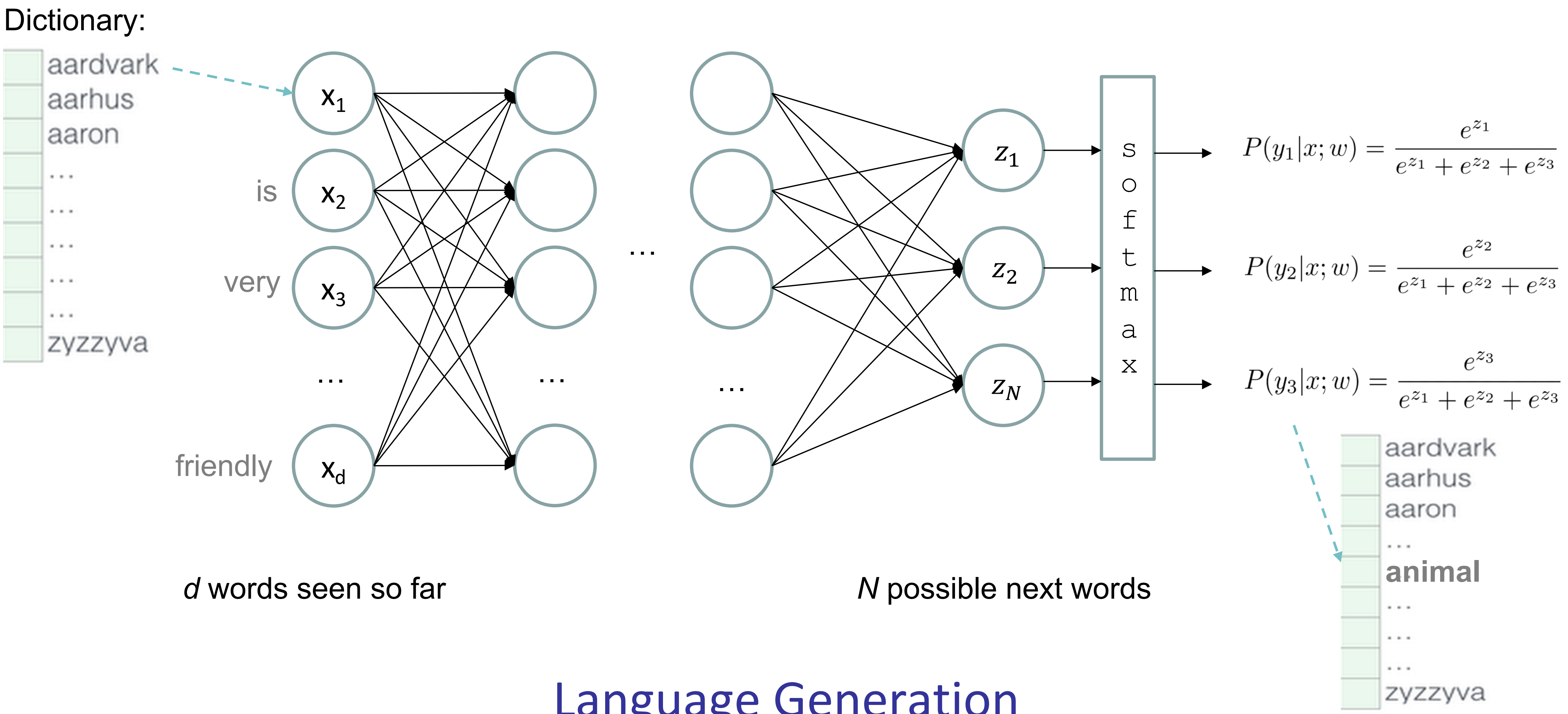
$$P(y_3|x;w) = \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}$$

d pixels in image

N possible classes

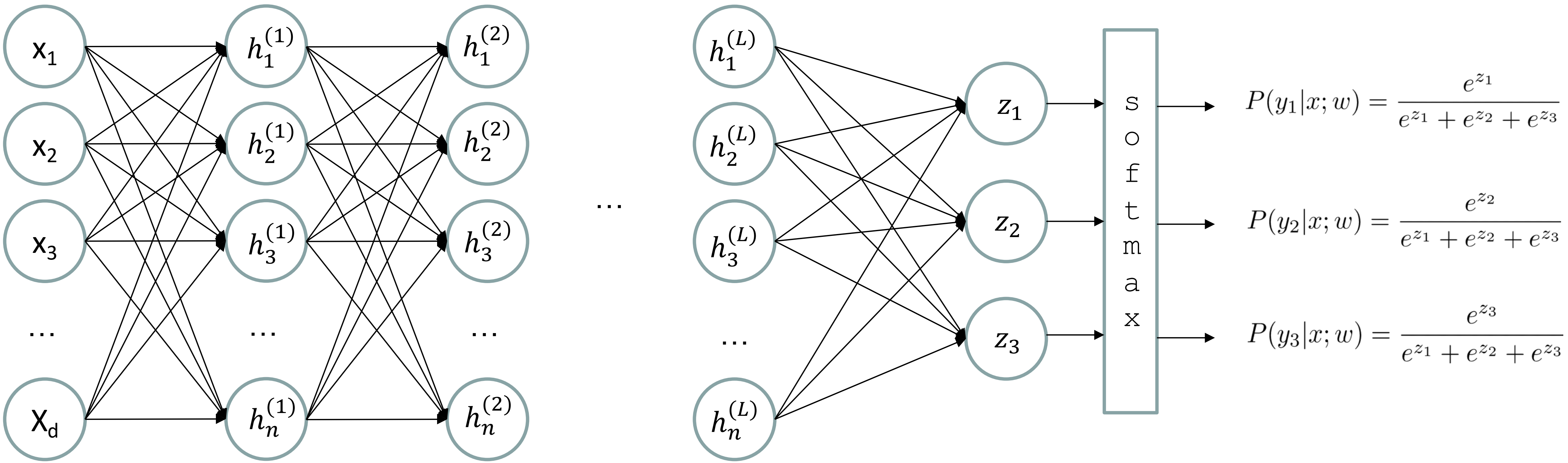
Image Classification

Recap: Deep Neural Network



Language Generation

Recap: Deep Neural Network



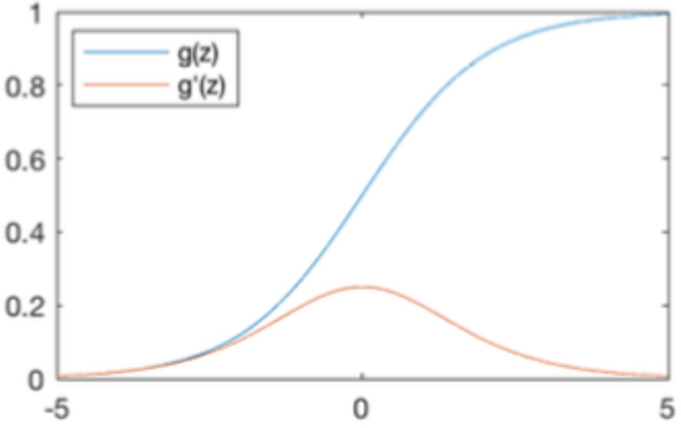
$$h_i^{(l)} = \phi\left(\sum_j w_{ji}^{(l)} \cdot h_j^{(l-1)}\right)$$

ϕ **= activation function**

- Neural network with L layers
- $h^{(l)}$: activations at layer l
- $w^{(l)}$: weights taking activations from layer l-1 to layer l

Recap: Common Activation Functions ϕ

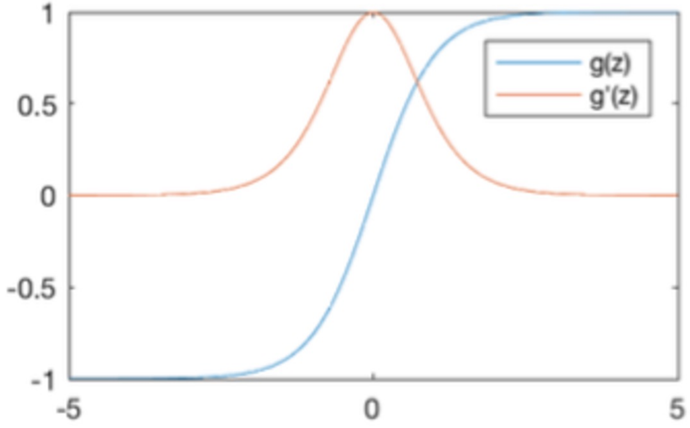
Sigmoid Function



$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

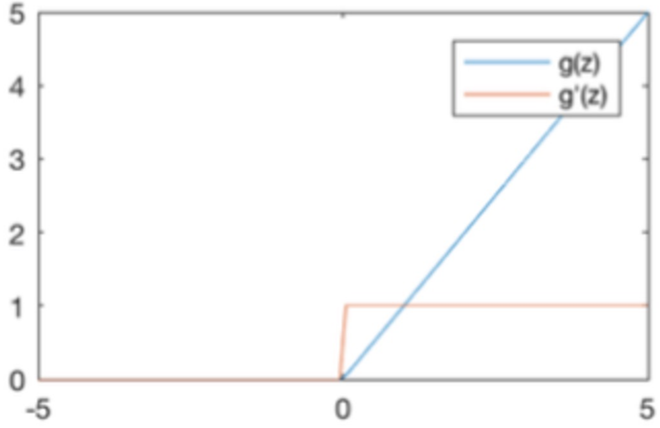
Hyperbolic Tangent



$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

Rectified Linear Unit (ReLU)

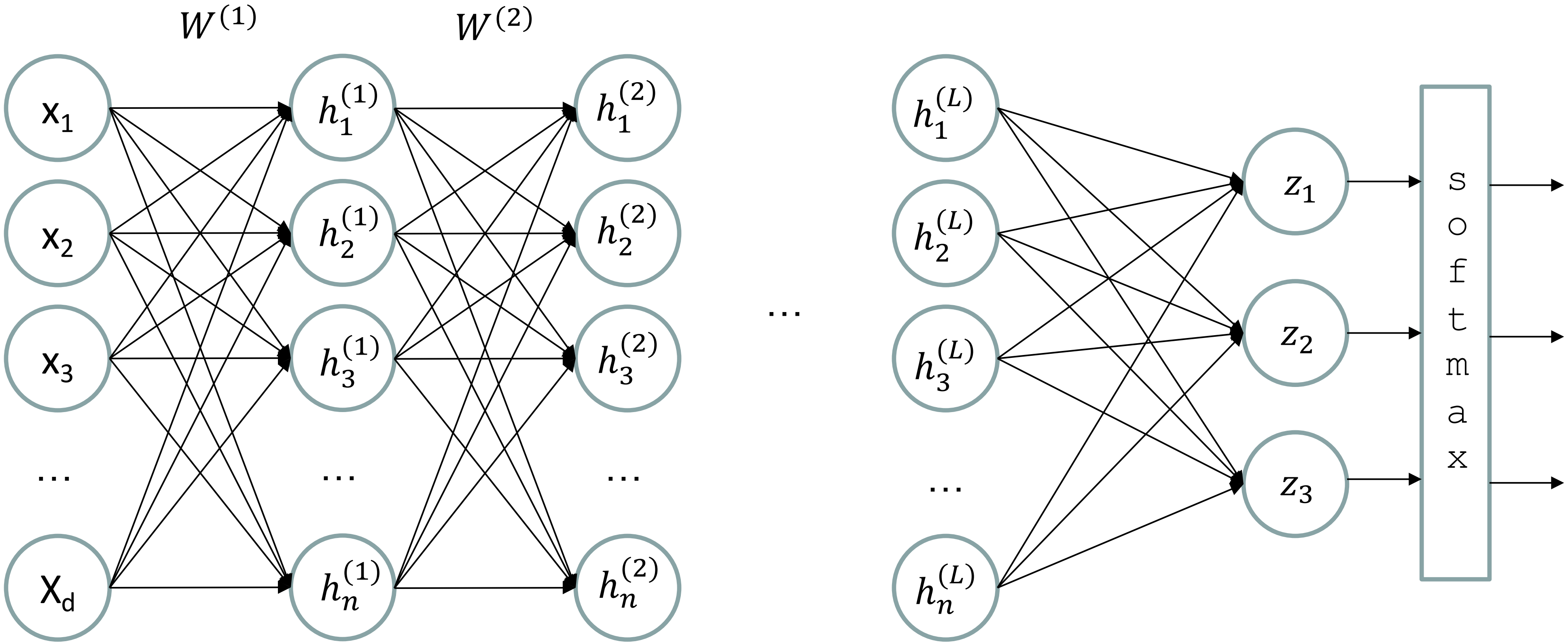


$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

[source: MIT 6.S191 introtodeeplearning.com]

Recap: Deep Neural Network



$$P(y_1|x;w) = \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}$$

$$P(y_2|x;w) = \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}$$

$$P(y_3|x;w) = \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}$$

Shape (1,n) vector

Shape (n,n) matrix

$$h^{(l)} = \phi(h^{(l-1)} \times W^{(l)})$$

Shape (1,n) vector

Neural Network Shapes

Take d-dimensional input vector x and calculate first hidden unit vector $h^{(1)}$

Shape (1,n) vector — Shape (d,n) matrix

$$h^{(1)} = \phi(x \times W^{(1)})$$

Shape (1,d) vector

Calculate next hidden unit vector $h^{(l)}$ from previous $h^{(l-1)}$

Shape (1,n) vector — Shape (n,n) matrix

$$h^{(l)} = \phi(h^{(l-1)} \times W^{(l)})$$

Shape (1,n) vector

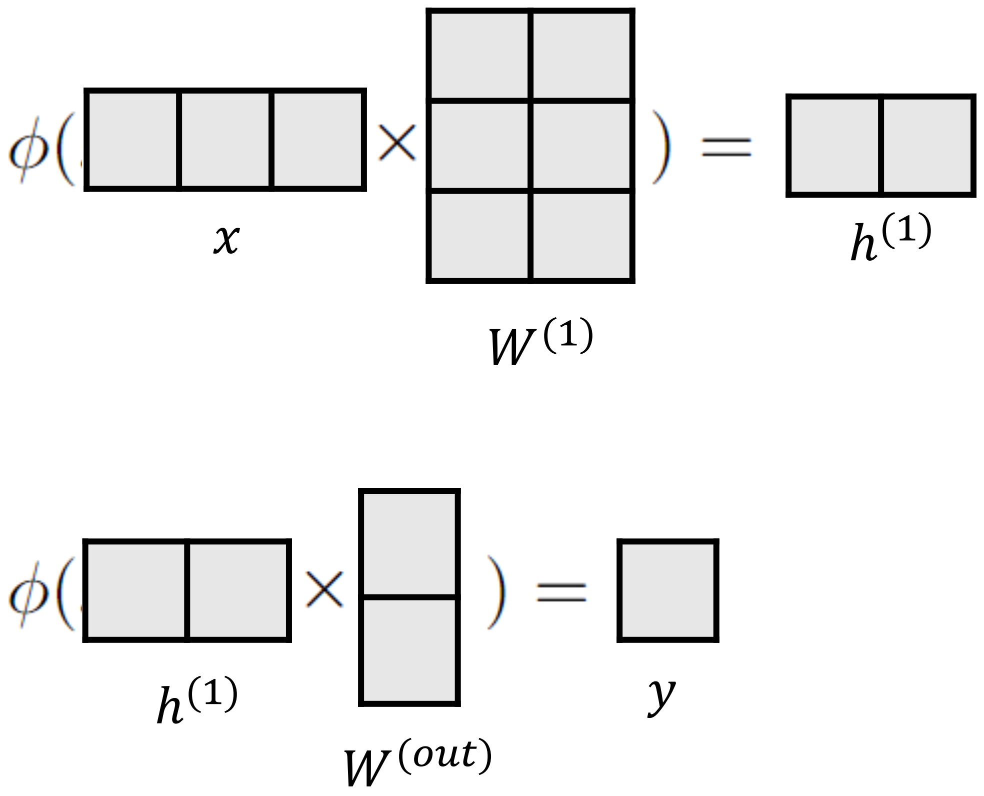
Calculate final k-dimensional vector z (and pass to softmax to get p(y|x))

Shape (1,k) vector — Shape (n,k) matrix

$$z = \phi(h^{(L)} \times W^{(out)})$$

Shape (1,n) vector

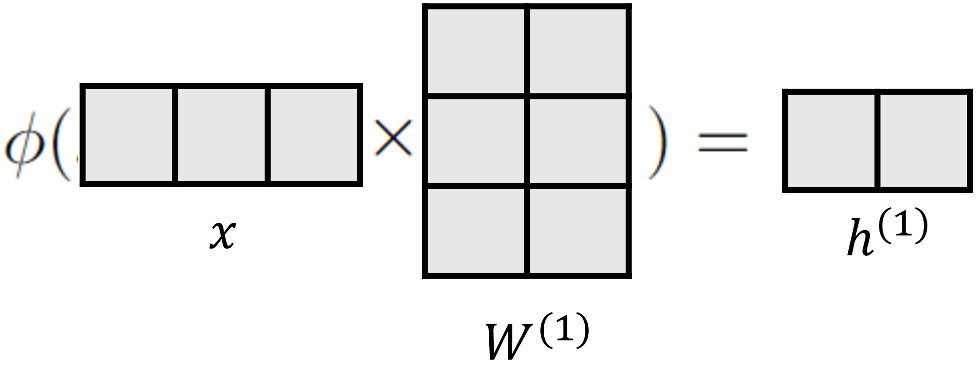
Example: Sizes of neural networks

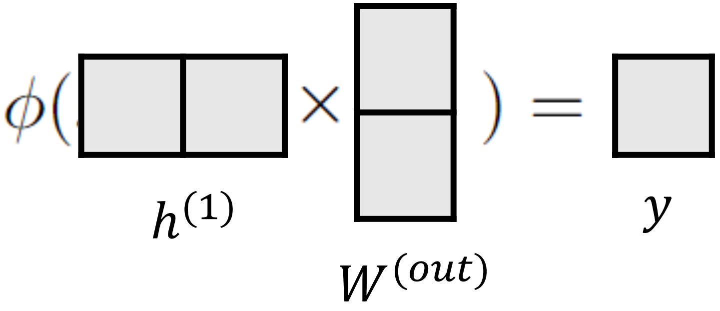


We have a neural network with the matrices drawn.

1. How many layers are in the network?
2. How many input dimensions d?
3. How many hidden neurons n?
4. How many output dimensions k?

Example: Sizes of neural networks





We have a neural network with the matrices drawn.

1. How many layers are in the network?
   1
2. How many input dimensions d?
   3
3. How many hidden neurons n?
   2
4. How many output dimensions k?
   1

Neural Networks Properties

- Theorem (Universal Function Approximators). A two-layer neural network with a sufficient number of neurons can approximate any continuous function to any desired accuracy.

Universal Function Approximation Theorem*

Hornik theorem 1: Whenever the activation function is *bounded and nonconstant*, then, for any finite measure μ, standard multilayer feedforward networks can approximate any function in $L^p(\mu)$ (the space of all functions on R^k such that $\int_{R^k} |f(x)|^p d\mu(x) < \infty$) arbitrarily well, provided that sufficiently many hidden units are available.

Hornik theorem 2: Whenever the activation function is *continuous, bounded and nonconstant*, then, for arbitrary compact subsets $X \subseteq R^k$, standard multilayer feedforward networks can approximate any continuous function on X arbitrarily well with respect to uniform distance, provided that sufficiently many hidden units are available.

- In words: Given any continuous function f(x), if a 2-layer neural network has enough hidden units, then there is a choice of weights that allow it to closely approximate f(x).

Cybenko (1989) “Approximations by superpositions of sigmoidal functions”
Hornik (1991) “Approximation Capabilities of Multilayer Feedforward Networks”
Leshno and Schocken (1991) ”Multilayer Feedforward Networks with Non-Polynomial Activation Functions Can Approximate Any Function”

Universal Function Approximation Theorem*

Math. Control Signals Systems (1989) 2: 303–314

Mathematics of Control, Signals, and Systems

Approximation by Superpositions of a Sigmoidal Function*

G. Cybenko†

Abstract. In this paper we demonstrate that finite linear combinations of compositions of a fixed, univariate function and a set of affine functionals can uniformly approximate any continuous function of n real variables with support in the unit hypercube; only mild conditions are imposed on the univariate function. Our results settle an open question about representability in the class of single hidden layer neural networks. In particular, we show that arbitrary decision regions can be arbitrarily well approximated by continuous feedforward neural networks with only a single internal, hidden layer and any continuous sigmoidal nonlinearity. The paper discusses approximation properties of other possible types of nonlinearities that might be implemented by artificial neural networks.

Key words. Neural networks, Approximation, Completeness.

1. Introduction

A number of diverse application areas are concerned with the representation of general functions of an n-dimensional real variable, $x \in \mathbb{R}^n$, by finite linear combinations of the form

$$\sum_{j=1}^{N} \alpha_j \sigma(y_j^{\mathrm{T}} x + \theta_j), \qquad (1)$$

where $y_j \in \mathbb{R}^n$ and $\alpha_j, \theta \in \mathbb{R}$ are fixed. (y^{T} is the transpose of y so that $y^{\mathrm{T}}x$ is the inner product of y and x.) Here the univariate function σ depends heavily on the context of the application. Our major concern is with so-called sigmoidal σ's:

$$\sigma(t) \to \begin{cases} 1 & \text{as } t \to +\infty, \\ 0 & \text{as } t \to -\infty. \end{cases}$$

Such functions arise naturally in neural network theory as the activation function of a neural node (or *unit* as is becoming the preferred term) [L1], [RHM]. The main result of this paper is a demonstration of the fact that sums of the form (1) are dense in the space of continuous functions on the unit cube if σ is any continuous sigmoidal

* Date received: October 21, 1988. Date revised: February 17, 1989. This research was supported in part by NSF Grant DCR-8619103, ONR Contract N000-86-G-0202 and DOE Grant DE-FG02-85ER25001.

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ORIGINAL CONTRIBUTION

Approximation Capabilities of Multilayer Feedforward Networks

KURT HORNIK

Technische Universität Wien, Vienna, Austria

(*Received 30 January 1990; revised and accepted 25 October 1990*)

Abstract—*We show that standard multilayer feedforward networks with as few as a single hidden layer and arbitrary bounded and nonconstant activation function are universal approximators with respect to $L^p(\mu)$ performance criteria, for arbitrary finite input environment measures μ, provided only that sufficiently many hidden units are available. If the activation function is continuous, bounded and nonconstant, then continuous mappings can be learned uniformly over compact input sets. We also give very general conditions ensuring that networks with sufficiently smooth activation functions are capable of arbitrarily accurate approximation to a function and its derivatives.*

Keywords—Multilayer feedforward networks, Activation function, Universal approximation capabilities, Input environment measure, $L^p(\mu)$ approximation, Uniform approximation, Sobolev spaces, Smooth approximation.

1. INTRODUCTION

The approximation capabilities of neural network architectures have recently been investigated by many authors, including Carroll and Dickinson (1989), Cybenko (1989), Funahashi (1989), Gallant and White (1988), Hecht-Nielsen (1989), Hornik, Stinchcombe, and White (1989, 1990), Irie and Miyake (1988), Lapedes and Farber (1988), Stinchcombe and White (1989, 1990). (This list is by no means complete.)

If we think of the network architecture as a rule for computing values at l output units given values at k input units, hence implementing a class of mappings from R^k to R^l, we can ask how well arbitrary mappings from R^k to R^l can be approximated by the network, in particular, if as many hidden units as required for internal representation and computation may be employed.

How to measure the accuracy of approximation depends on how we measure closeness between functions, which in turn varies significantly with the specific problem to be dealt with. In many applications, it is necessary to have the network perform *simultaneously* well on all input samples taken from some compact input set X in R^k. In this case, closeness is measured by the uniform distance between functions on X, that is,

$$\rho_{\mu,X}(f, g) = \sup_{x \in X} |f(x) - g(x)|.$$

In other applications, we think of the inputs as random variables and are interested in the *average performance* where the average is taken with respect to the input environment measure μ, where $\mu(R^k) < \infty$. In this case, closeness is measured by the $L^p(\mu)$ distances

$$\rho_{p,\mu}(f, g) = \left[\int_{R^k} |f(x) - g(x)|^p \, d\mu(x) \right]^{1/p},$$

$1 \le p < \infty$, the most popular choice being $p = 2$, corresponding to mean square error.

Of course, there are many more ways of measuring closeness of functions. In particular, in many applications, it is also necessary that the *derivatives* of the approximating function implemented by the network closely resemble those of the function to be approximated, up to some order. This issue was first taken up in Hornik et al. (1990), who discuss the sources of need of smooth functional approximation in more detail. Typical examples arise in robotics (learning of smooth movements) and signal processing (analysis of chaotic time series); for a recent application to problems of nonparametric inference in statistics and econometrics, see Gallant and White (1989).

All papers establishing certain approximation ca-

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MULTILAYER FEEDFORWARD NETWORKS WITH NON-POLYNOMIAL ACTIVATION FUNCTIONS CAN APPROXIMATE ANY FUNCTION

by

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Note: Important to use non-linear activation functions

- **With** non-linear activation ϕ for intermediate output:

$$y = \phi(w_1 h_1 + w_2 h_2)$$
$$= \phi(w_1 \phi(w_{11}x_1 + w_{21}x_2 + w_{31}x_3) + w_2 \phi(w_{12}x_1 + w_{22}x_2 + w_{32}x_3))$$

- **Without** intermediate activations ϕ:

$$y = \phi(w_1(w_{11}x_1 + w_{21}x_2 + w_{31}x_3) + w_2(w_{12}x_1 + w_{22}x_2 + w_{32}x_3))$$
$$= \phi((w_1 w_{11} + w_2 w_{12})x_1 + (w_1 w_{21} + w_2 w_{22})x_2 + (w_1 w_{31} + w_2 w_{32})x_3)$$
$$= \phi(ax_1 + bx_2 + cx_3)$$ ← same as not including a hidden layer!

Deep Neural Network Training

- Training the deep neural network is just like logistic regression - Maximize log of likelihood of the data:

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)} | x^{(i)}; w)$$

  - For each training example (i), maximize probability of label y(i) given input x(i)
  - Parameter w tends to be a much, much larger vector

- How do we maximize w?
  - Numerical optimization (i.e. hill climbing)

Hill Climbing

- Recall from CSPs lecture: simple, general idea
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

- What's particularly tricky when hill-climbing for logistic regression or neural networks?
  - Optimization over a continuous space
    - Infinitely many neighbors!
    - How to do this efficiently?

Review: Derivatives and Gradients

- What is the derivative of the function $g(x) = x^2 + 3$?

$$\frac{dg}{dx} = 2x$$

- What is the derivative of g(x) at x=5?

$$\frac{dg}{dx}\Big|_{x=5} = 10$$

Review: Derivatives and Gradients

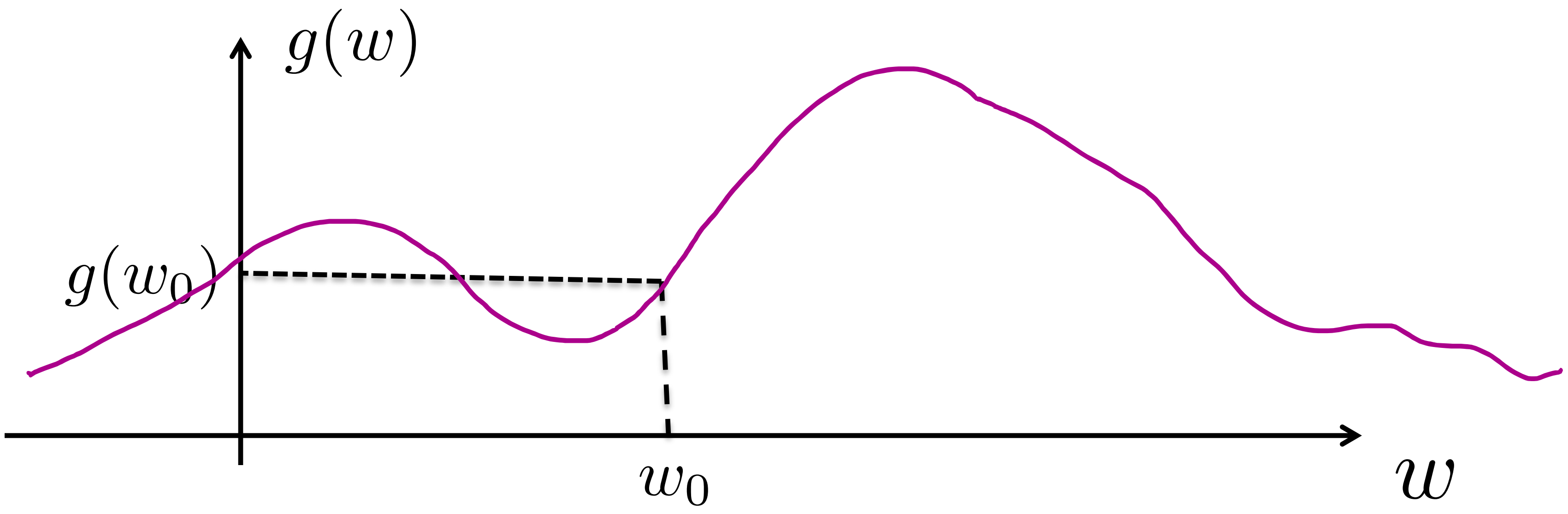
- What is the gradient of the function $g(x, y) = x^2 y$?
  - Recall: Gradient is a vector of partial derivatives with respect to each variable

$$\nabla g = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2xy \\ \\ x^2 \end{bmatrix}$$

- What is the derivative of g(x, y) at x=0.5, y=0.5?

$$\nabla g|_{x=0.5, y=0.5} = \begin{bmatrix} 2(0.5)(0.5) \\ (0.5^2) \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

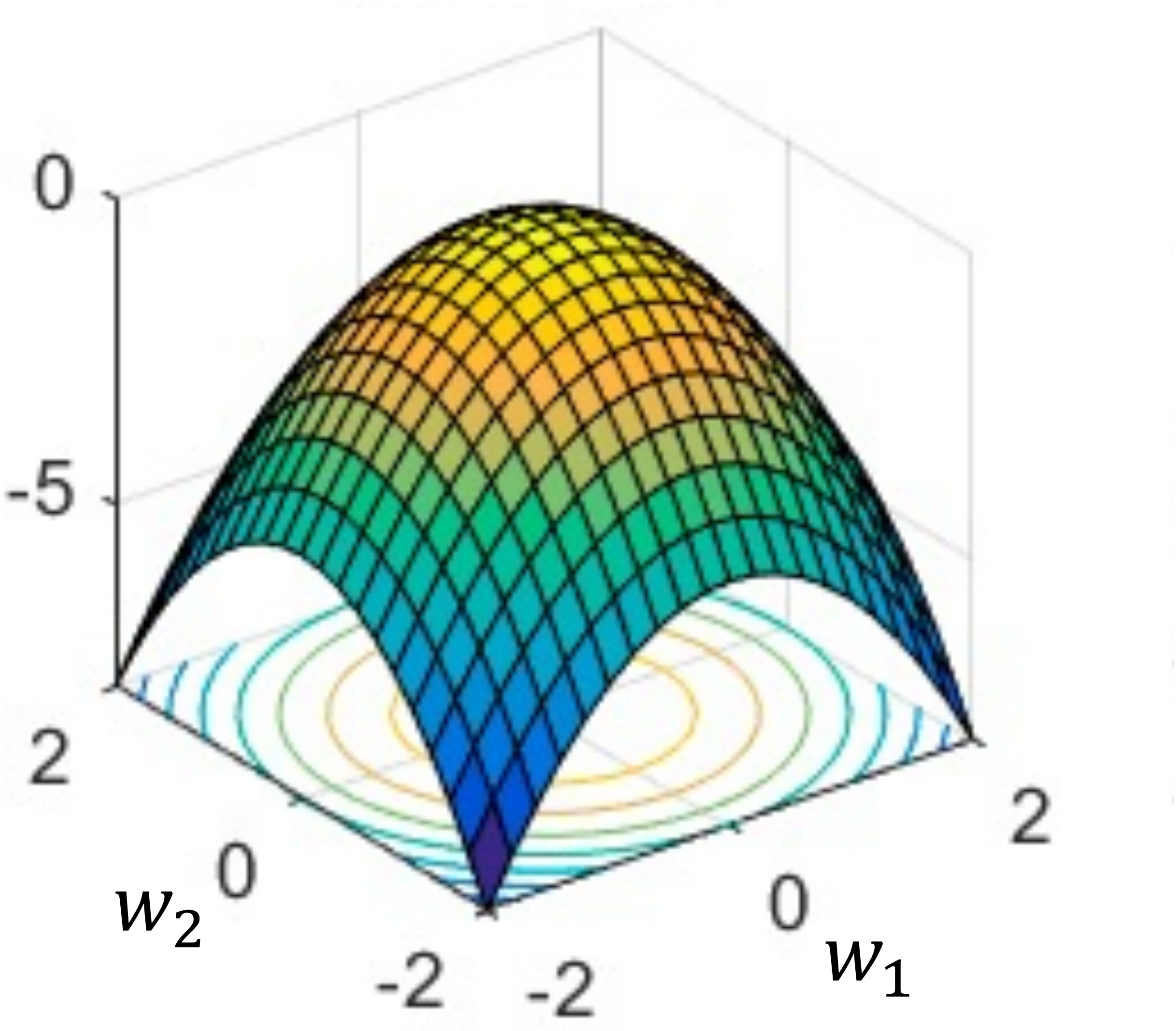
1-D Optimization



- Could evaluate $g(w_0 + h)$ and $g(w_0 - h)$
  - Then step in best direction

- Or, evaluate derivative: $\dfrac{\partial g(w_0)}{\partial w} = \lim_{h \to 0} \dfrac{g(w_0 + h) - g(w_0 - h)}{2h}$
  - Tells which direction to step into

2-D Optimization



Source: offconvex.org

Gradient Ascent

- Perform update in uphill direction for each coordinate
- The steeper the slope (i.e. the higher the derivative) the bigger the step for that coordinate
- E.g., consider: $g(w_1, w_2)$
  - Updates:

    $$w_1 \leftarrow w_1 + \alpha * \frac{\partial g}{\partial w_1}(w_1, w_2)$$

    $$w_2 \leftarrow w_2 + \alpha * \frac{\partial g}{\partial w_2}(w_1, w_2)$$

  - Updates in vector notation:

    $$w \leftarrow w + \alpha * \nabla_w g(w)$$

    with: $\nabla_w g(w) = \begin{bmatrix} \frac{\partial g}{\partial w_1}(w) \\ \frac{\partial g}{\partial w_2}(w) \end{bmatrix}$ **= gradient**

Gradient Ascent

- Idea:
  - Start somewhere
  - Repeat: Take a step in the gradient direction

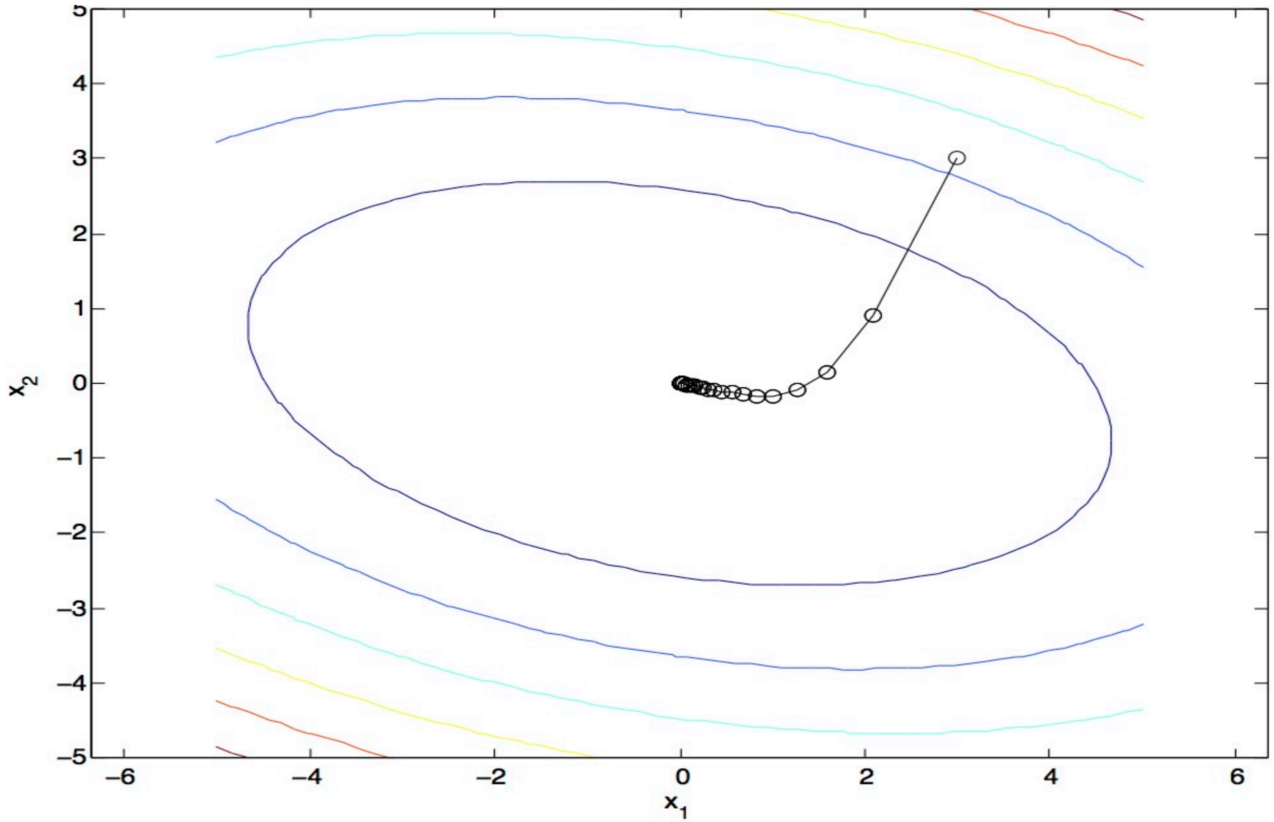


Figure source: Mathworks

Gradient Ascent

- Idea:
  - Start somewhere
  - Repeat: Take a step in the gradient direction

Not guaranteed to find global maximum:

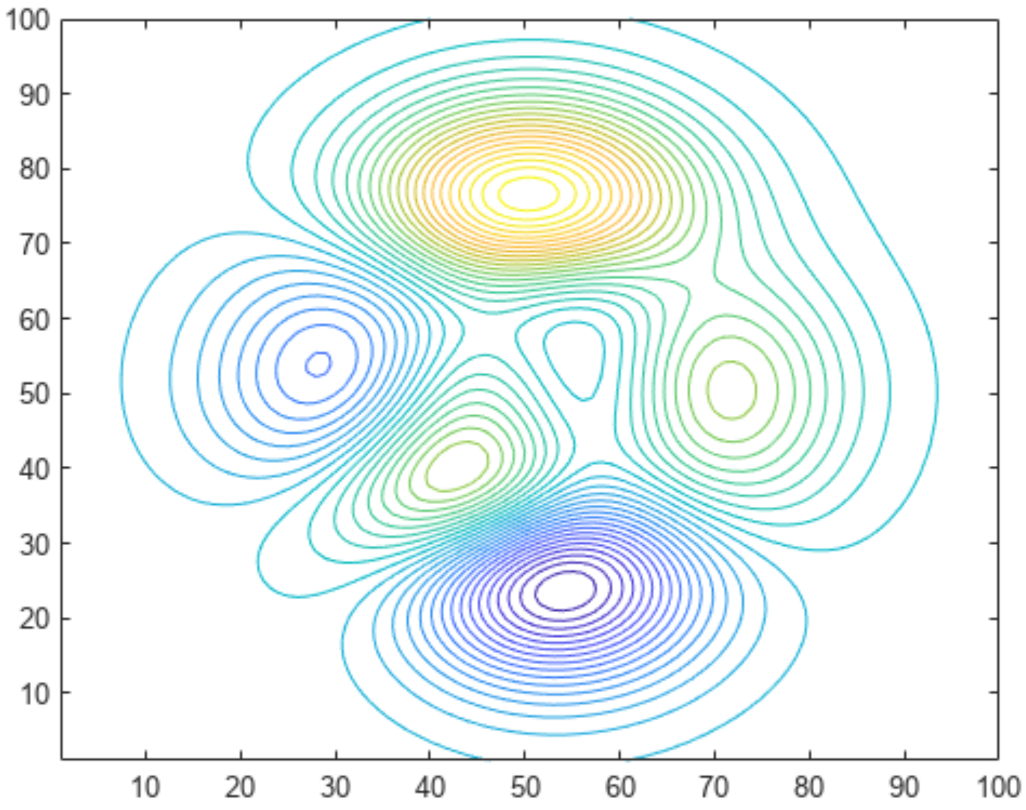


Figure source: Mathworks

Gradient in n dimensions

$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial w_1} \\ \frac{\partial g}{\partial w_2} \\ \cdots \\ \frac{\partial g}{\partial w_n} \end{bmatrix}$$

Optimization Procedure: Gradient Ascent

```
Init w
for iter = 1, 2, …
```

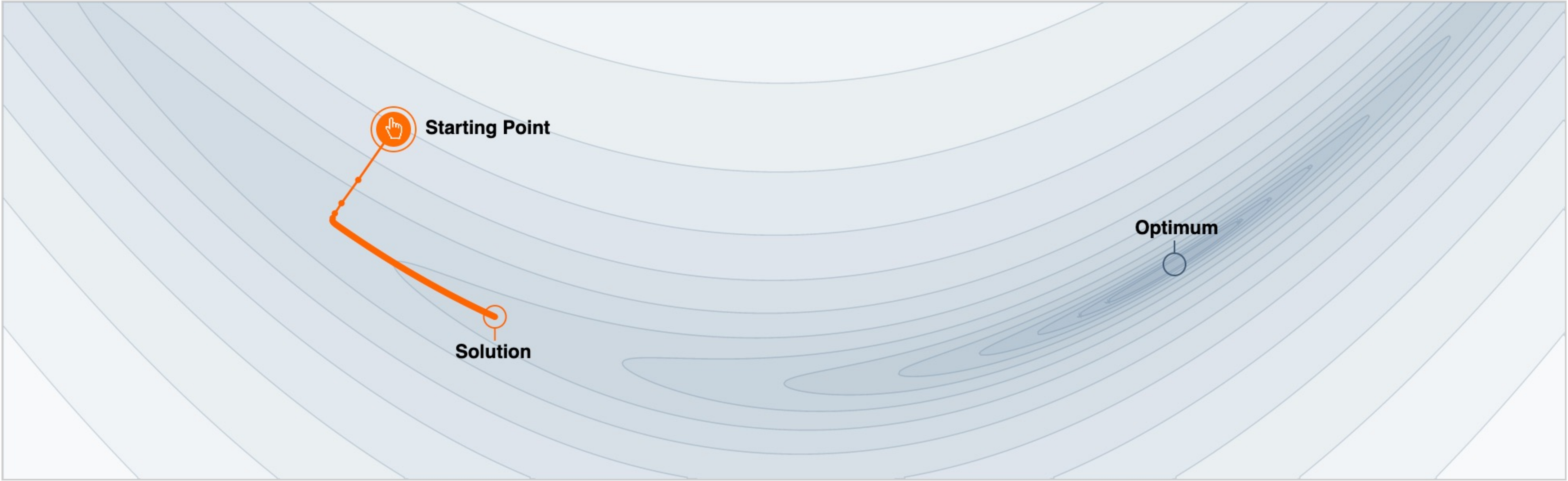
$$w \leftarrow w + \alpha \cdot \nabla g(w)$$

- α: learning rate --- tweaking parameter that needs to be chosen carefully
- How? Try multiple choices
  - Crude rule of thumb: update changes w about 0.1 – 1 %

Learning Rate

Choice of learning rate α is a hyperparameter

Example: α=0.001 (too small)

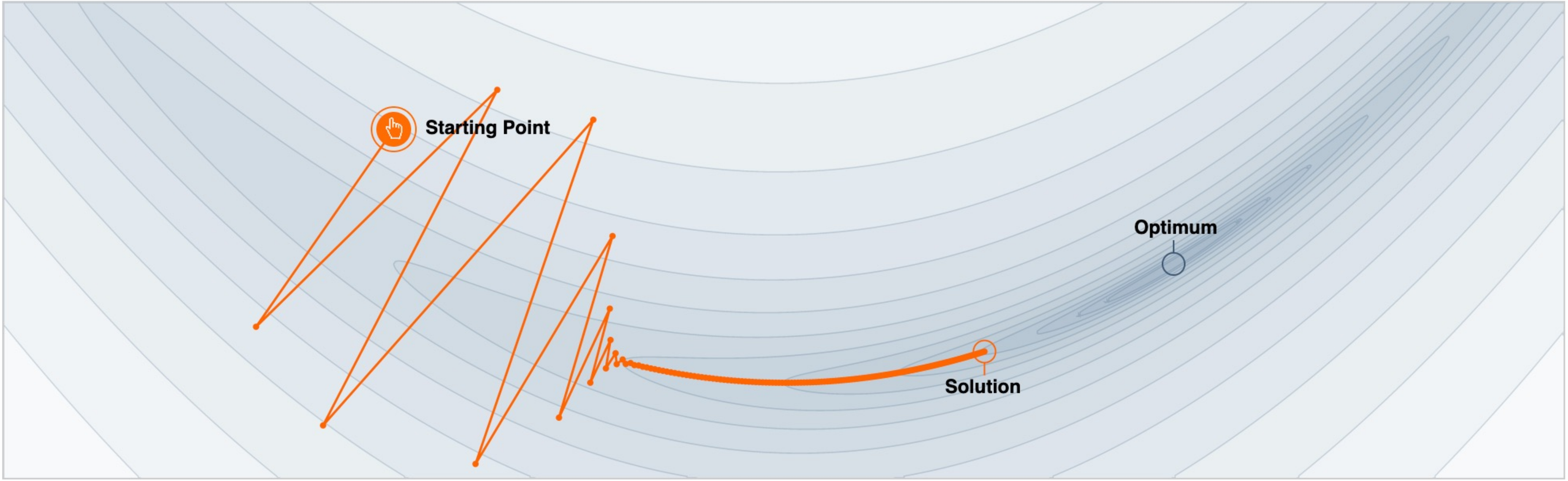


Source: https://distill.pub/2017/momentum/

Learning Rate

Choice of step size α is a hyperparameter

Example: α=0.004 (too large)



Source: https://distill.pub/2017/momentum/

Gradient Ascent with Momentum*

- Often use *momentum* to improve gradient ascent convergence

Gradient Ascent:

```
Init w
for iter = 1, 2, …
```
$$w \leftarrow w + \alpha \cdot \nabla g(w)$$

Gradient Ascent with momentum:

```
Init w
for iter = 1, 2, …
```
$$z \leftarrow \beta \cdot z + \nabla g(w)$$
$$w \leftarrow w + \alpha \cdot z$$

- One interpretation: *w* moves like a particle with mass
- Another: *exponential moving average* on gradient

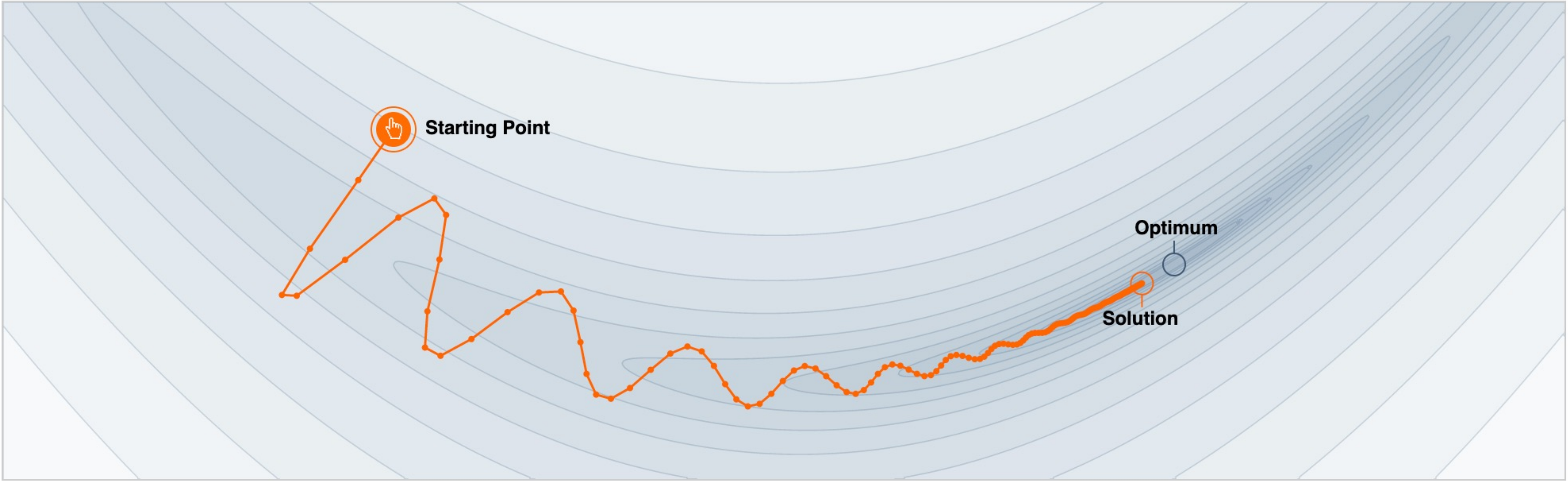
Gradient Ascent with Momentum*

Example: α=0.001 and β=0.0

Source: https://distill.pub/2017/momentum/

Gradient Ascent with Momentum*

Example: α=0.001 and β=0.9



Source: https://distill.pub/2017/momentum/

Batch Gradient Ascent on the Log Likelihood Objective

$$\max_w \quad ll(w) = \max_w \quad \underbrace{\sum_i \log P(y^{(i)}|x^{(i)};w)}_{g(w)}$$

- init w
- for iter = 1, 2, …

$$w \leftarrow w + \alpha * \sum_i \nabla \log P(y^{(i)}|x^{(i)};w)$$

Sum rule for derivatives: derivative of [a(w) + b(w)] = derivative of a(w) + derivative of b(w)

Stochastic Gradient Ascent on the Log Likelihood Objective

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

Observation: once gradient on one training example has been computed, might as well incorporate before computing next one

- init w
- for iter = 1, 2, …
  - pick random j

$$w \leftarrow w + \alpha * \nabla \log P(y^{(j)}|x^{(j)}; w)$$

Mini-Batch Gradient Ascent on the Log Likelihood Objective

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

Observation: gradient over small set of training examples (=mini-batch) can be computed in parallel, might as well do that instead of a single one

- `init` w
- `for iter = 1, 2, …`
  - `pick random subset of training examples J`

$$w \leftarrow w + \alpha * \sum_{j \in J} \nabla \log P(y^{(j)}|x^{(j)}; w)$$

How about computing all the derivatives?

Derivatives tables:

$$\frac{d}{dx}(a)=0$$

$$\frac{d}{dx}(x)=1$$

$$\frac{d}{dx}(au)=a\frac{du}{dx}$$

$$\frac{d}{dx}(u+v-w)=\frac{du}{dx}+\frac{dv}{dx}-\frac{dw}{dx}$$

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{1}{v}\frac{du}{dx}-\frac{u}{v^2}\frac{dv}{dx}$$

$$\frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sqrt{u})=\frac{1}{2\sqrt{u}}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u}\right)=-\frac{1}{u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u^n}\right)=-\frac{n}{u^{n+1}}\frac{du}{dx}$$

$$\frac{d}{dx}[f(u)]=\frac{d}{du}[f(u)]\frac{du}{dx}$$

$$\frac{d}{dx}[\ln u]=\frac{d}{dx}[\log_e u]=\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left[\log_a u\right]=\log_a e\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}e^u=e^u\frac{du}{dx}$$

$$\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$$

$$\frac{d}{dx}\left(u^v\right)=vu^{v-1}\frac{du}{dx}+\ln u \quad u^v\frac{dv}{dx}$$

$$\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$$

[source: http://hyperphysics.phy-astr.gsu.edu/hbase/Math/derfunc.html

How about computing all the derivatives?

- But neural net f is never one of those?
  - No problem: CHAIN RULE:

If $$f(x) = g(h(x))$$

Then $$f'(x) = g'(h(x))h'(x)$$

Derivatives can be computed by following well-defined procedures

Automatic Differentiation

Automatic differentiation software

- e.g. TensorFlow, PyTorch, Jax
- Only need to program the function g(x,y,w)
- Can automatically compute all derivatives w.r.t. all entries in w
- This is typically done by caching info during forward computation pass of f, and then doing a backward pass = “backpropagation”
- Autodiff / Backpropagation can often be done at computational cost comparable to the forward pass

Need to know this exists

How this is done? Details outside of scope of CS188, but we’ll show a basic example

Backpropagation*

- Gradient of $g(w_1, w_2, w_3) = w_1^4 w_2 + 5w_3$ at $w_1 = 2$, $w_2 = 3$, $w_3 = 2$
- Think of *g* as a composition of many functions
  - Then, we can use the chain rule to compute the gradient
- g = b + c

$$\frac{\partial g}{\partial b} = 1, \frac{\partial g}{\partial c} = 1$$

- $b = a \times w_2$

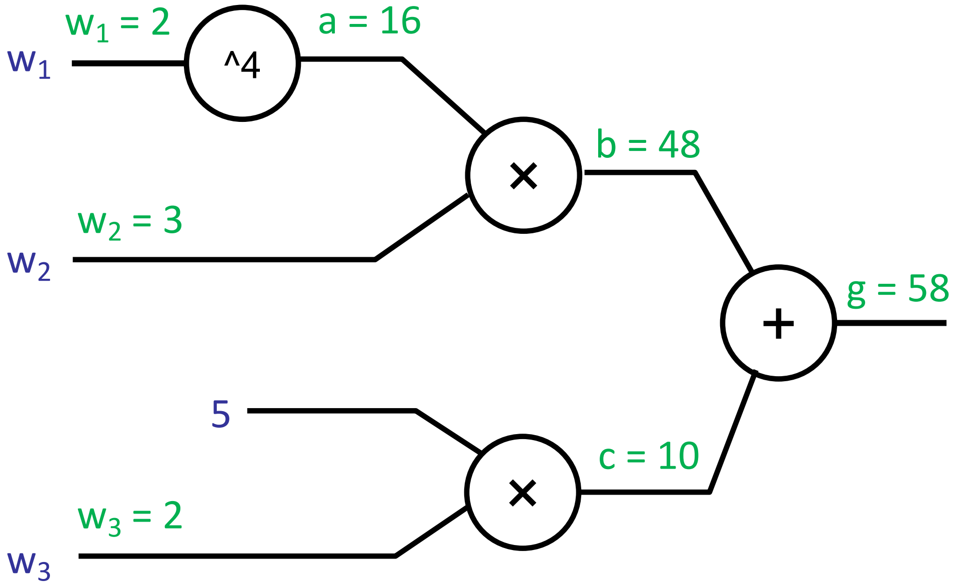
$$\frac{\partial g}{\partial a} = \frac{\partial g}{\partial b}\frac{\partial b}{\partial a} = 1 \cdot w_2 = 3 \qquad \frac{\partial g}{\partial w_2} = \frac{\partial g}{\partial b}\frac{\partial b}{\partial w_2} = 1 \cdot a = 16$$

- $a = w_1^4$

$$\frac{\partial g}{\partial w_1} = \frac{\partial g}{\partial a}\frac{\partial a}{\partial w_1} = 3 \cdot 4w_1^3 = 96$$

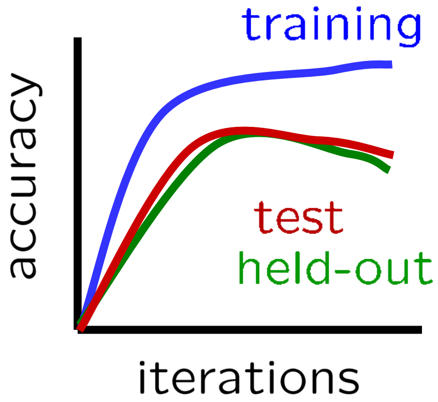
- $c = 5w_1$

$$\frac{\partial g}{\partial w_3} = \frac{\partial g}{\partial c}\frac{\partial c}{\partial w_3} = 1 \cdot 5 = 5$$



Preventing Overfitting in Optimization

Early stopping:



Weight regularization

Weight Regularization

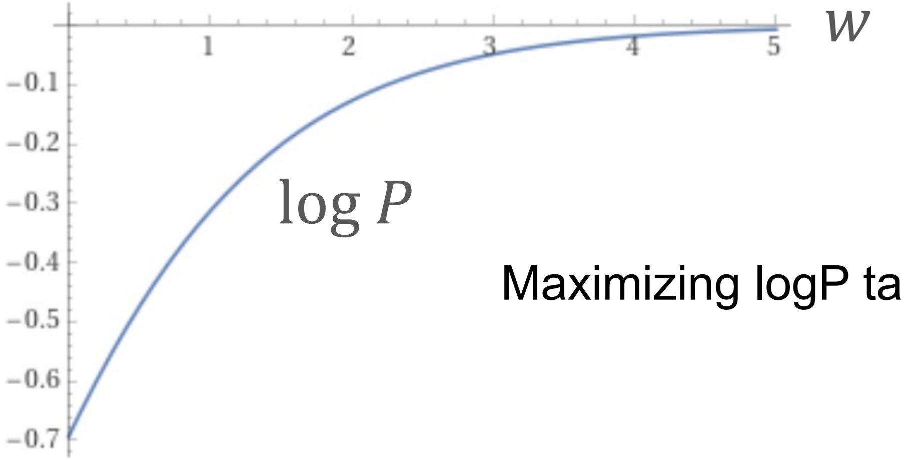
What can go wrong when we maximize log-likelihood?

Example: logistic regression with only one datapoint: f(x)=1, y=+1

$$\max_w \sum_i \log P(y^{(i)} | x^{(i)}; w)$$

- $P(y = +1 | x; w) = \frac{1}{1+e^{-w \cdot f(x)}}$

$$\max_w \log\left(\frac{1}{1+e^{-w}}\right)$$



Maximizing logP takes w to infinity

w can grow very large and lead to overfitting and learning instability

Weight Regularization

What can go wrong when we maximize log-likelihood?

$$\max_w \sum_i \log P(y^{(i)}|x^{(i)};w)$$

w can grow very large

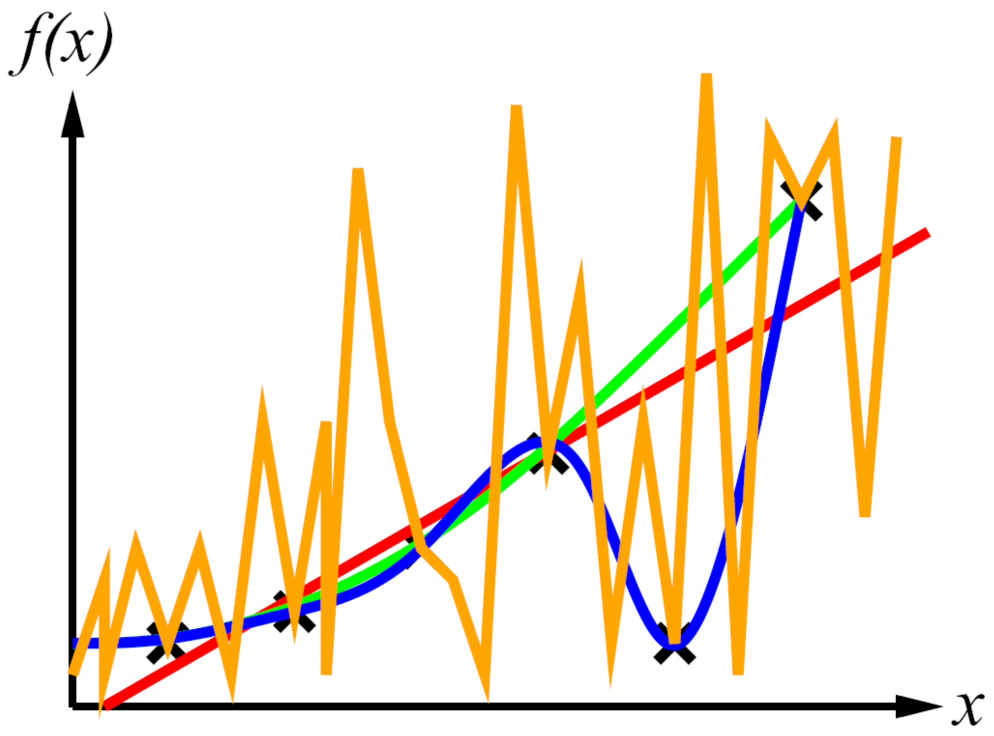
Solution: add an objective term to penalize weight magnitude

$$\max_w \sum_i \log P(y^{(i)}|x^{(i)};w) - \frac{\lambda}{2}\sum_j w_j^2$$

λ is a hyperparameter (typically 0.1 to 0.0001 or smaller)

Consistency vs. Simplicity

- Example: curve fitting (regression, function approximation)



- Consistency vs. simplicity
- Ockham's razor

Consistency vs. Simplicity

- Usually algorithms prefer consistency by default (why?)

- Several ways to operationalize “simplicity”
  - Reduce the hypothesis/model space
    - Assume more: e.g. independence assumptions, as in naïve Bayes
    - Fewer features or neurons
    - Other limits on model structure
  - Regularization
    - Laplace Smoothing: cautious use of small counts
    - Small weight vectors in neural networks (stay close to zero-mean prior)
    - Hypothesis space stays big, but harder to get to the outskirts

Fun Neural Net Demo Site

Demo-site:

http://playground.tensorflow.org/

Neural Networks: Summary of Key Ideas

Optimize probability of label given input

$$\max_w \quad ll(w) = \max_w \quad \sum_i \log P(y^{(i)}|x^{(i)}; w)$$

Continuous optimization

Gradient ascent:

- Compute steepest uphill direction = gradient (= just vector of partial derivatives)
- Take step in the gradient direction
- Repeat (until held-out data accuracy starts to drop = "early stopping")

Deep neural nets

Last layer = still logistic regression

Now also many more layers before this last layer

- = computing the features
- the features are learned rather than hand-designed

Universal function approximation theorem

- `If` neural net is large enough
- `Then` neural net can represent any continuous mapping from input to output with arbitrary accuracy
- But remember: need to avoid overfitting / memorizing the training data ? early stopping!

Automatic differentiation gives the derivatives efficiently (how? = outside of scope of 188)

Next: Applications and Putting it all together!