Last name:_____ First name:_____ SID:_____ Class account login:_____

Collaborators: _____

CS188 Spring 2010 Written 3: Expectimax & Utility

Due: Thursday 2/25 in 283 Soda Drop Box by 11:59pm (no slip days)

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually.

1 [8 pts] Expectimax for cs188-Blackjack

Blackjack is the most widely played casino betting game in the world. The goal of the game is to be dealt a hand whose value is as close to 21 as possible without exceeding it. If the current value of a player's hand is less than 21, the player can "hit", or be dealt a single card, in hopes of acquiring a hand with higher value. However, the player runs the risk of "busting", or going over 21, which results in an immediate loss. In casino play, players bet independently against a dealer, who plays according to a fixed set of rules that govern when he should hit or stay.

In this problem set, we consider a simplified variant called cs188-Blackjack.

- There are only 3 cards in the deck: 5's, 10's and 11's. Each card appears with equal probability.
- The casino has invented an infinite deck. The probability of being dealt any given card is independent of the cards already dealt.
- To model the action of a dealer, we assume the casino gives fixed payoffs according to the following schedule (in dollars)

Hand Value	Payoff
0-14	0
15	3
16	3
17	3
18	3
19	3
20	9
21	12
Bust	-6

- There are two actions available: Hit, which draws a card uniformly at random and adds its value to your current score, and Stay, which ends the game and yields the above payoff. If your score goes above 21 the game ends immediately with a payoff of -6. It is not possible to hit on 21.

You are playing a hand of cs188-Blackjack. You have been dealt 1 card, and its value is 11.

[2 pts] (a) Build the expectimax tree for this game, starting from your current hand and including all chance and max nodes. Write the value of each state next to the given node. What is your optimal strategy? Be sure to specify your actions at all max nodes in the tree.

[2 pt] (b) Unfortunately, you are playing at a table with an unscrupulous dealer who is rigging the deck. Every time he deals a card, instead of dealing you a random card, he gives you the worst possible card you could get at that moment. What is the value of this game? What is your optimal strategy if you currently hold 11?

[2 pt] (c) When you complain about the cheating dealer to the pit boss, a new dealer is brought in. This dealer is extremely nice: half of the time, when his boss is watching, he deals you a random card. The other half of the time, he deals you the best possible card you could get at that moment. Draw out the game tree for this. What is the value of the game now? What is your optimal strategy if you currently hold 11?

[2 pt] (d) The casino owner, anxious about dwindling interest in cs188-Blackjack, asks you to help him rework the game. He would like to increase the payouts for a value of 21 to $x. What should $x be so that the optimal strategy for a player holding 16 changes? Assume fair dealers.

2 [10 pts] Whales

A whale is slang for a high-roller who bets large amounts of money in casino betting games. At this casino, there are whales and normal betters playing cs188-Blackjack. Whales start with \$24 while normal betters start with \$12. They all share the following utility function for money

$$U(x) = \frac{-1080}{x}$$

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Throughout this problem, we assume the cs188-Blackjack rules from problem 1a.

[2 pt] (a) A normal better is playing a game of cs188-Blackjack and currently holds 11. What is the expected utility gained by playing the game (i.e. difference between his final and starting utilities)? Make sure to include the normal better's starting money in your utility calculations.

[2 pts] (b) What is the minimum offer (in cash) a normal better would be willing to accept to give up his position to someone else? The normal better must be better off after receiving the offer than he would be by playing out the hand.

[2 pts] (c) A whale is playing a game of cs188-Blackjack and holds a hand of 11. What is the expected utility gained by playing the game? Make sure to include the whale's starting money in your utility calculations. Round your answer to 2 decimal places.

[2 pts] (d) Given the whale's expected utility gain from part (c), what is the dollar equivalent value of the game? Equivalently, how much would you need to pay the whale to NOT play this game? Round your answer to 2 decimal places.

[2 pts] (e) What is the maximum amount x a whale would be willing to pay to take over for a normal better holding a single 11? Taking over means paying the better and then playing out the hand, earning whatever payout results. Round your answer to 2 decimal places. (You may want to solve numerically).