CS 188: Artificial Intelligence
Spring 2011

Lecture 5: CSPs II

2/2/2011

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Many slides from Dan Klein

A*: Robotics Examples

- Urban Challenge
  - Successor function?
  - Heuristic?
- Door Opening
  - Successor function?
  - Heuristic?

Other A* Applications

- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

Announcements

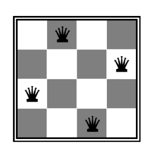
- Project 1 due Friday 4:59pm
  - See course website for this week’s office hours, held by P1 GSI
- Lecture videos online
- Written Assignment Policy

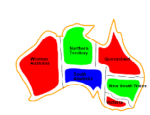
Today

- CSPs
- Efficient Solution of CSPs
  - Search
  - Constraint propagation
- Local Search

Constraint Satisfaction Problems

- Standard search problems:
  - State is a “black box”: arbitrary data structure
  - Goal test: any function over states
  - Successor function can be anything
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Simple example of a *formal representation language*
- Allows useful general-purpose algorithms with more power than standard search algorithms



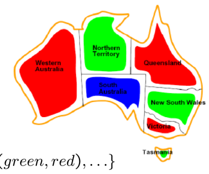


Example CSP: Map-Coloring

- Variables: $WA,\ NT,\ Q,\ NSW,\ V,\ SA,\ T$
- Domain: $D = \{red, green, blue\}$
- Constraints: adjacent regions must have different colors

  $WA \neq NT$

  $(WA, NT) \in \{(red, green), (red, blue), (green, red), \ldots\}$

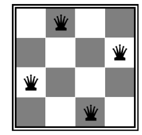


- Solutions are assignments satisfying all constraints, e.g.:

  $\{WA = red, NT = green, Q = red,$
  $NSW = green, V = red, SA = blue, T = green\}$

Example CSP: N-Queens

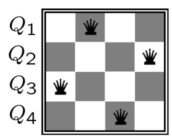
- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0, 1\}$
  - Constraints



$$\forall i, j, k \ \ (X_{ij}, X_{ik}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \ \ (X_{ij}, X_{kj}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \ \ (X_{ij}, X_{i+k,j+k}) \in \{(0,0), (0,1), (1,0)\}$$
$$\forall i, j, k \ \ (X_{ij}, X_{i+k,j-k}) \in \{(0,0), (0,1), (1,0)\}$$
$$\sum_{i,j} X_{ij} = N$$

Example CSP: N-Queens

- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \ldots N\}$



  - Constraints:

Implicit: $\forall i, j$ non-threatening(Q_i, Q_j)

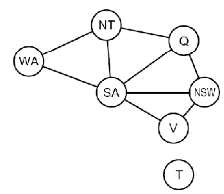
-or-

Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

$\cdots$

Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



Example CSP: Cryptarithmetic

- Variables (circles):

  $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$
- Domains:

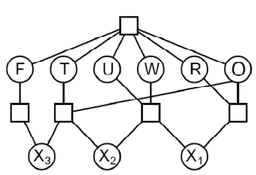
  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Constraints (boxes):

  alldiff(F, T, U, W, R, O)

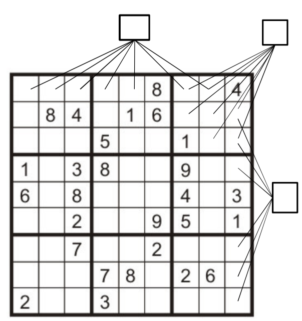
  $O + O = R + 10 \cdot X_1$

  $\cdots$

```
  T W O
+ T W O
F O U R
```



Example CSP: Sudoku



- Variables:
  - Each (open) square
- Domains:
  - {1,2,…,9}
- Constraints:

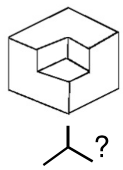
9-way alldiff for each column

9-way alldiff for each row

9-way alldiff for each region

Example CSP: The Waltz Algorithm

- The Waltz algorithm is for interpreting line drawings of solid polyhedra
- An early example of a computation posed as a CSP



- Look at all intersections
- Adjacent intersections impose constraints on each other

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Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable
- Continuous variables
  - E.g., start-end state of a robot
  - Linear constraints solvable in polynomial time by LP methods (see cs170 for a bit of this theory)

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Varieties of Constraints

- Varieties of Constraints
  - Unary constraints involve a single variable (equiv. to shrinking domains):

    $$SA \neq green$$

  - Binary constraints involve pairs of variables:

    $$SA \neq WA$$

  - Higher-order constraints involve 3 or more variables: e.g., cryptarithmetic column constraints
- Preferences (soft constraints):
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

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Real-World CSPs

- Assignment problems: e.g., who teaches what class
- Timetabling problems: e.g., which class is offered when and where?
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Floorplanning
- Fault diagnosis
- … lots more!
- Many real-world problems involve real-valued variables…

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What is Search For?

- Models of the world: single agents, deterministic actions, fully observed state, discrete state space
- Planning: sequences of actions
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics to guide, fringe to keep backups
- Identification: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulations)
  - CSPs are specialized for identification problems

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Standard Search Formulation

- Standard search formulation of CSPs (incremental)
- Let's start with the straightforward, dumb approach, then fix it
- States are defined by the values assigned so far
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints
- Simplest CSP ever: two bits, constrained to be equal

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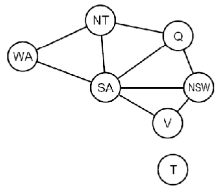
Search Methods

- What does BFS do?
- What does DFS do?
  - [demo]
- What's the obvious problem here?
- What's the slightly-less-obvious problem?



Backtracking Search

- Idea 1: Only consider a single variable at each point
  - Variable assignments are commutative, so fix ordering
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step
  - How many leaves are there?
- Idea 2: Only allow legal assignments at each point
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to figure out whether a value is ok
  - "Incremental goal test"
- Depth-first search for CSPs with these two improvements is called *backtracking search* (useless name, really)
  - [DEMO]
- Backtracking search is the basic uninformed algorithm for CSPs
- Can solve n-queens for n ≈ 25

Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)
function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

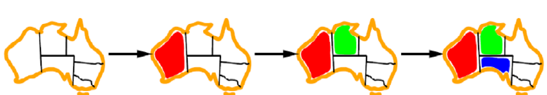
- What are the choice points?

Improving Backtracking

- General-purpose ideas give huge gains in speed
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Filtering: Can we detect inevitable failure early?
- Structure: Can we exploit the problem structure?

Minimum Remaining Values

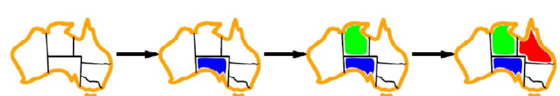
- Minimum remaining values (MRV):
  - Choose the variable with the fewest legal values



- Why min rather than max?
- Also called "most constrained variable"
- Also called "fail-fast" ordering

Degree Heuristic

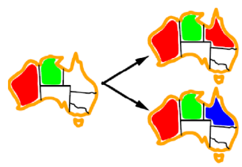
- Tie-breaker among MRV variables
- Degree heuristic:
  - Choose the variable participating in the most constraints on remaining variables



- Why most rather than fewest constraints?

Least Constraining Value

- Given a choice of variable:
  - Choose the *least constraining value*
  - The one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this!
- Why least rather than most?
- Combining these heuristics makes 1000 queens feasible

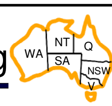


Filtering: Forward Checking

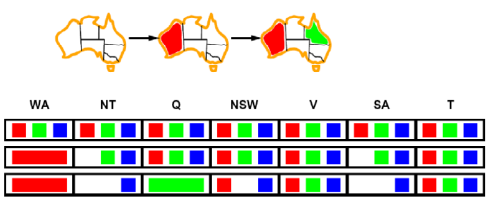
- Idea: Keep track of remaining legal values for unassigned variables (using immediate constraints)
- Idea: Terminate when any variable has no legal values

[demo: forward checking animation]

Filtering: Forward Checking

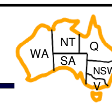


- Forward checking propagates information from assigned to adjacent unassigned variables, but doesn't detect more distant failures:

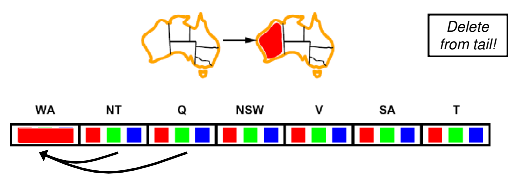


- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation* repeatedly enforces constraints (locally)

Consistency of An Arc

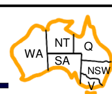


- An arc X → Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint

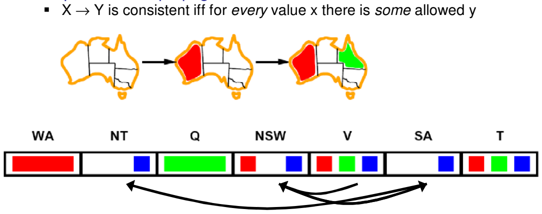


- What happens?
- Forward checking = Enforcing consistency of each arc pointing to the new assignment

Arc Consistency of a CSP



- Simplest form of propagation makes each arc *consistent*
  - X → Y is consistent iff for *every* value x there is *some* allowed y



- If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure earlier than forward checking
- What's the downside of arc consistency?
- Can be run as a preprocessor or after each assignment

Establishing Arc Consistency

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
```

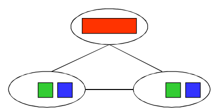
```
function REMOVE-INCONSISTENT-VALUES( Xi, Xj) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[Xi] do
      if no value y in DOMAIN[Xj] allows (x,y) to satisfy the constraint Xi ↔ Xj
         then delete x from DOMAIN[Xi]; removed ← true
   return removed
```

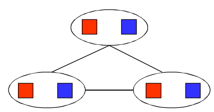
- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

[demo: arc consistency animation]

Limitations of Arc Consistency

- After running arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)





What went wrong here?

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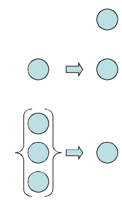
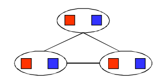
K-Consistency

- Increasing degrees of consistency
  - 1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints
  - 2-Consistency (Arc Consistency): For each pair of nodes, any consistent assignment to one can be extended to the other
  - K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the k^{th} node.



- Higher k more expensive to compute



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Strong K-Consistency

- Strong k-consistency: also k-1, k-2, … 1 consistent
- Claim: strong n-consistency means we can solve without backtracking!
- Why?
  - Choose any assignment to any variable
  - Choose a new variable
  - By 2-consistency, there is a choice consistent with the first
  - Choose a new variable
  - By 3-consistency, there is a choice consistent with the first 2
  - …
- Lots of middle ground between arc consistency and n-consistency! (e.g. path consistency)

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