CS 188: Artificial Intelligence
Spring 2011

Lecture 8: Games, MDPs

2/14/2010

Pieter Abbeel – UC Berkeley

Many slides adapted from Dan Klein

Outline

- Zero-sum deterministic two player games
  - Minimax
  - Evaluation functions for non-terminal states
  - Alpha-Beta pruning
- Stochastic games
  - Single player: expectimax
  - Two player: expectiminimax
- Non-zero sum
- Markov decision processes (MDPs)

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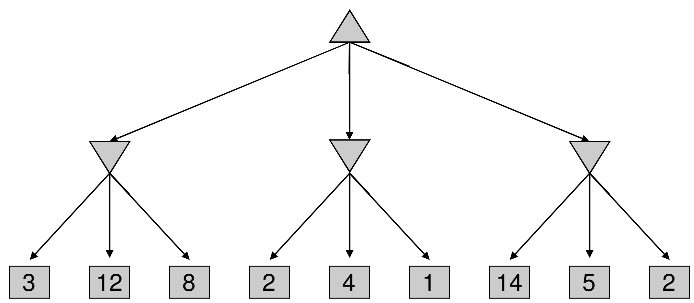
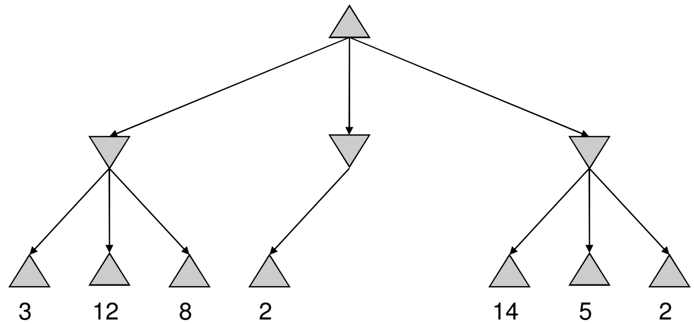
Minimax Example



Speeding Up Game Tree Search

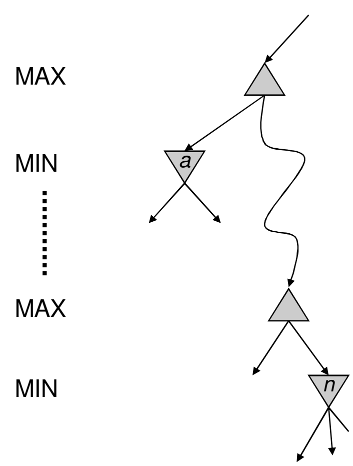
- Evaluation functions for non-terminal states
- Pruning: not search parts of the tree
  - Alpha-Beta pruning does so without losing accuracy, $O(b^d) \rightarrow O(b^{d/2})$

Pruning

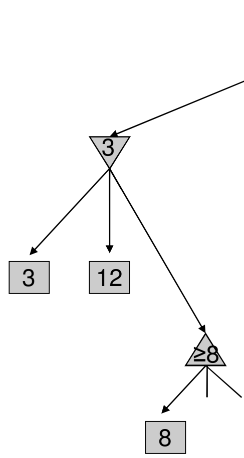


Alpha-Beta Pruning

- General configuration
  - We're computing the MIN-VALUE at *n*
  - We're looping over *n*'s children
  - *n*'s value estimate is dropping
  - *a* is the best value that MAX can get at any choice point along the current path
  - If *n* becomes worse than *a*, MAX will avoid it, so can stop considering *n*'s other children
  - Define *b* similarly for MIN

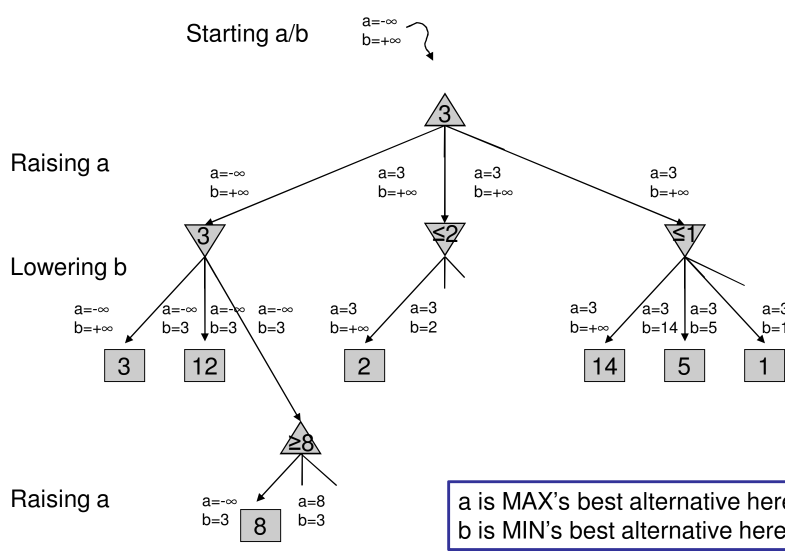


Alpha-Beta Pruning Example



a is MAX's best alternative here or above
b is MIN's best alternative here or above

Alpha-Beta Pruning Example



a is MAX's best alternative here or above
b is MIN's best alternative here or above

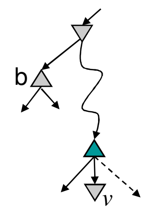
Alpha-Beta Pseudocode

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do** $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MAX-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v



Alpha-Beta Pruning Properties

- This pruning has no effect on final result at the root
- Values of intermediate nodes might be wrong!
- Good child ordering improves effectiveness of pruning
  - Heuristic: order by evaluation function or based on previous search
- With “perfect ordering”:
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…
- This is a simple example of metareasoning (computing about what to compute)

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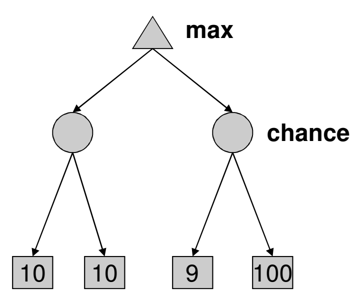
Expectimax Search Trees

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, next card is unknown
  - In minesweeper, mine locations
  - In pacman, the ghosts act randomly
- Can do **expectimax search** to maximize average score
  - Chance nodes, like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Max nodes as in minimax search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize the underlying problem as a **Markov Decision Process**



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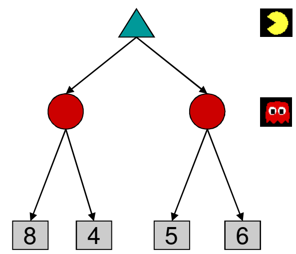
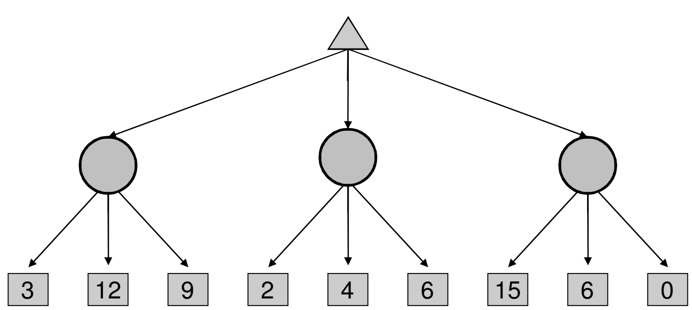
Expectimax Pseudocode

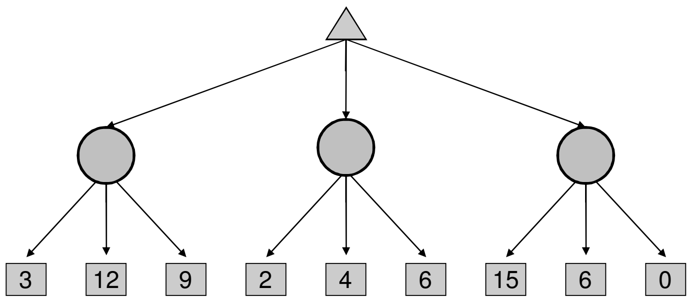
```
def value(s)
    if s is a max node return maxValue(s)
    if s is an exp node return expValue(s)
    if s is a terminal node return evaluation(s)

def maxValue(s)
    values = [value(s’) for s’ in successors(s)]
    return max(values)

def expValue(s)
    values = [value(s’) for s’ in successors(s)]
    weights = [probability(s, s’) for s’ in successors(s)]
    return expectation(values, weights)
```



Expectimax Quantities



Expectimax Pruning?



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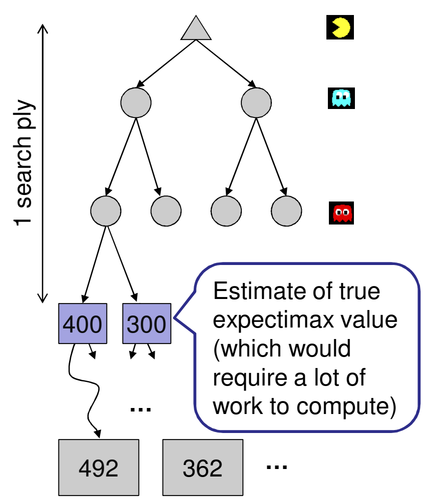
Expectimax Search

- Chance nodes
  - Chance nodes are like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Chance nodes average successor values (weighted)
- Each chance node has a probability distribution over its outcomes (called a model)
  - For now, assume we're given the model
- Utilities for terminal states
  - Static evaluation functions give us limited-depth search



Expectimax for Pacman

- Notice that we've gotten away from thinking that the ghosts are trying to minimize pacman's score
- Instead, they are now a part of the environment
- Pacman has a belief (distribution) over how they will act
- Quiz: Can we see minimax as a special case of expectimax?
- Quiz: what would pacman's computation look like if we assumed that the ghosts were doing 1-ply minimax and taking the result 80% of the time, otherwise moving randomly?
- If you take this further, you end up calculating belief distributions over your opponents' belief distributions over your belief distributions, etc…
  - Can get unmanageable very quickly!

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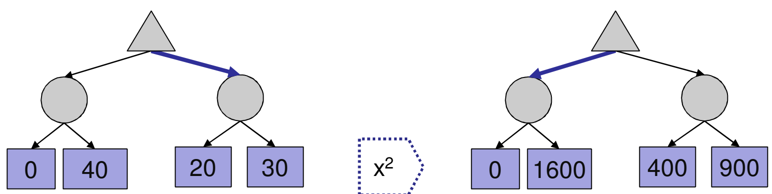
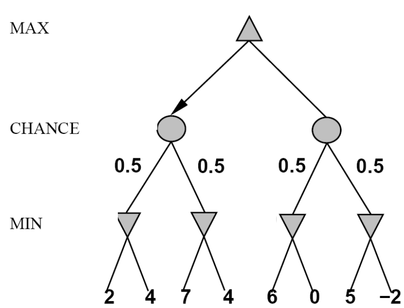
Expectimax Utilities

- For minimax, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For expectimax, we need *magnitudes* to be meaningful



Stochastic Two-Player

- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



if *state* is a MAX node **then**
  return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
  return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
  return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

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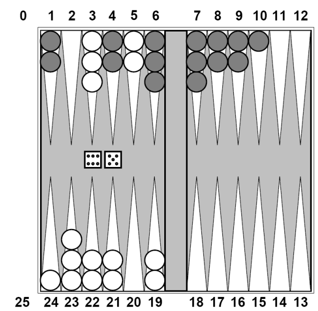
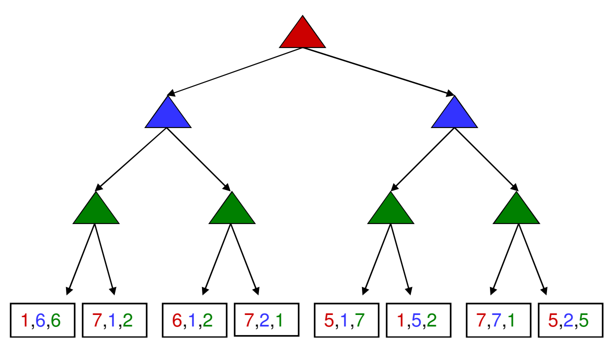
Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = 20 x (21 x 20)^3 = 1.2 x 10^9
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1st AI world champion in any game!



Non-Zero-Sum Utilities

- Similar to minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own utility and propagate (or back up) nodes from children
  - Can give rise to cooperation and competition dynamically…



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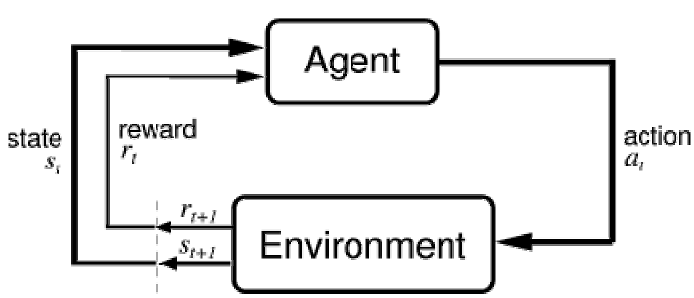
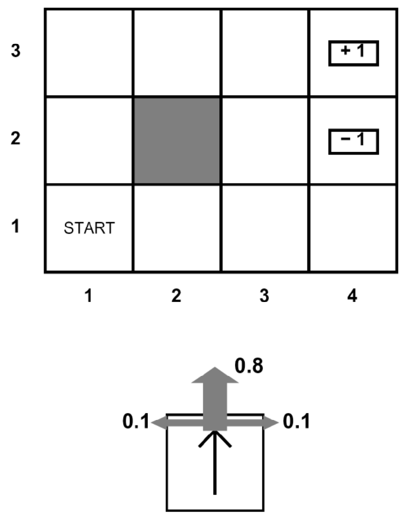
Reinforcement Learning

- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must learn to act so as to maximize expected rewards

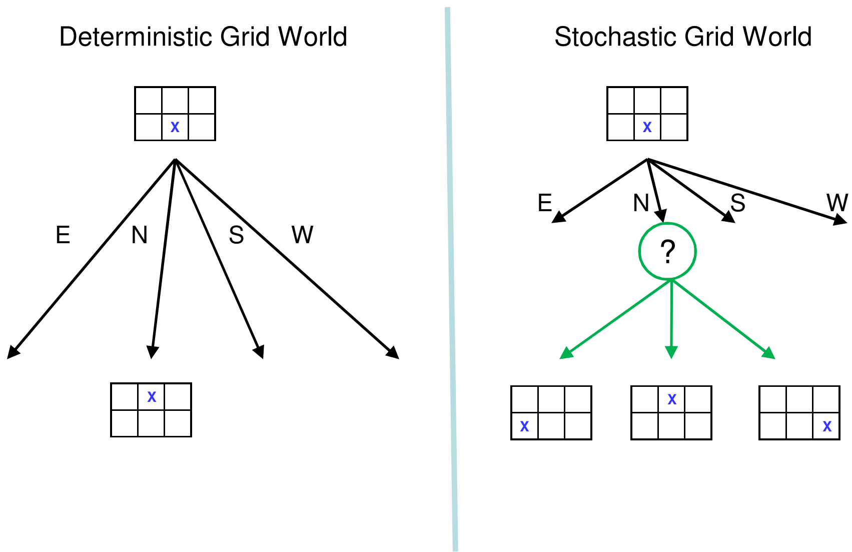


Grid World

- The agent lives in a grid
- Walls block the agent's path
- The agent's actions do not always go as planned:
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- Small "living" reward each step
- Big rewards come at the end
- Goal: maximize sum of rewards



Grid Futures



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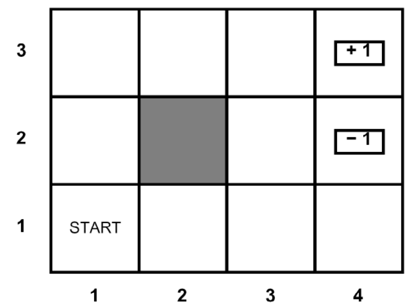
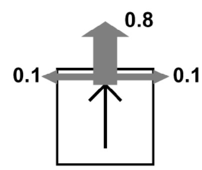
Markov Decision Processes

- An MDP is defined by:
  - A set of states $s \in S$
  - A set of actions $a \in A$
  - A transition function T(s,a,s')
    - Prob that a from s leads to s'
    - i.e., P(s' | s,a)
    - Also called the model
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state (or distribution)
  - Maybe a terminal state
- MDPs are a family of non-deterministic search problems
  - Reinforcement learning: MDPs where we don't know the transition or reward functions





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What is Markov about MDPs?

- Andrey Markov (1856-1922)
- "Markov" generally means that given the present state, the future and the past are independent
- For Markov decision processes, "Markov" means:

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

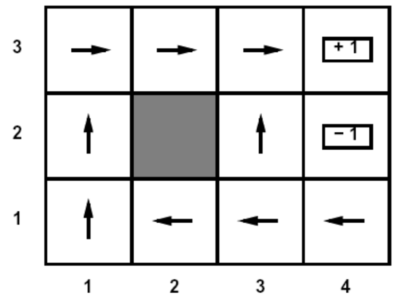
$$=$$

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

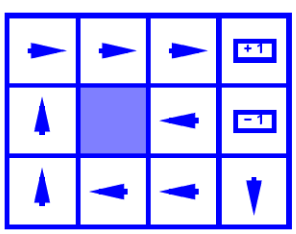
Solving MDPs

- In deterministic single-agent search problems, want an optimal plan, or sequence of actions, from start to a goal
- In an MDP, we want an optimal policy $\pi^*: S \rightarrow A$
  - A policy π gives an action for each state
  - An optimal policy maximizes expected utility if followed
  - Defines a reflex agent

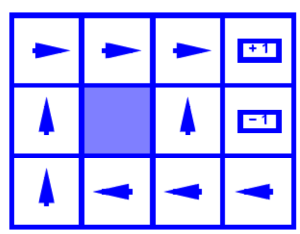
Optimal policy when R(s, a, s') = -0.03 for all non-terminals s



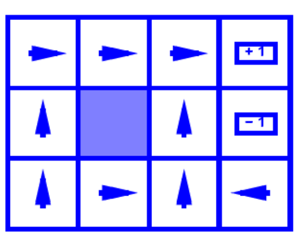
Example Optimal Policies



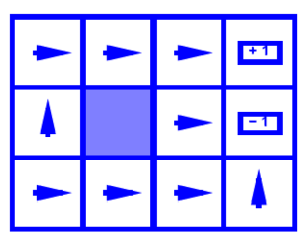
R(s) = -0.01



R(s) = -0.03



R(s) = -0.4



R(s) = -2.0

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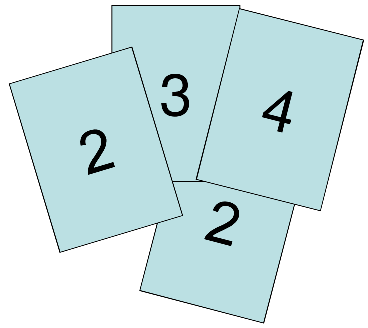
Example: High-Low

- Three card types: 2, 3, 4
- Infinite deck, twice as many 2's
- Start with 3 showing
- After each card, you say "high" or "low"
- New card is flipped
- If you're right, you win the points shown on the new card
- Ties are no-ops
- If you're wrong, game ends



- Differences from expectimax:
  - #1: get rewards as you go
  - #2: you might play forever!

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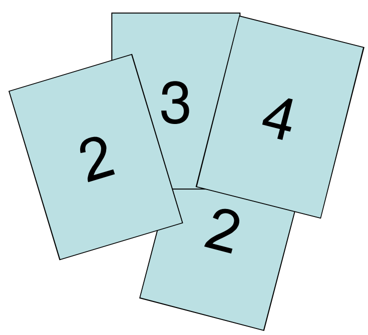
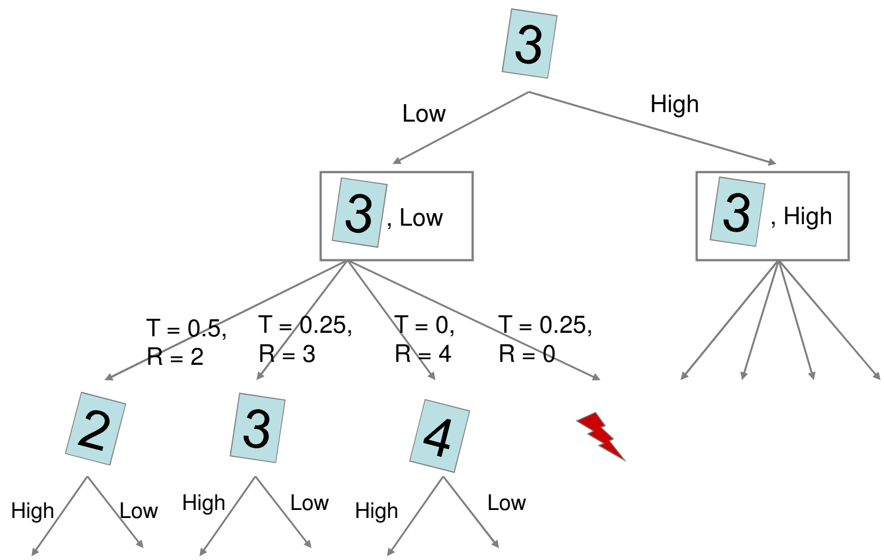
High-Low as an MDP

- States: 2, 3, 4, done
- Actions: High, Low
- Model: T(s, a, s'):
  - P(s'=4 | 4, Low) = 1/4
  - P(s'=3 | 4, Low) = 1/4
  - P(s'=2 | 4, Low) = 1/2
  - P(s'=done | 4, Low) = 0
  - P(s'=4 | 4, High) = 1/4
  - P(s'=3 | 4, High) = 0
  - P(s'=2 | 4, High) = 0
  - P(s'=done | 4, High) = 3/4
  - …
- Rewards: R(s, a, s'):
  - Number shown on s' if s ≠ s' and a is "correct"
  - 0 otherwise
- Start: 3



Example: High-Low



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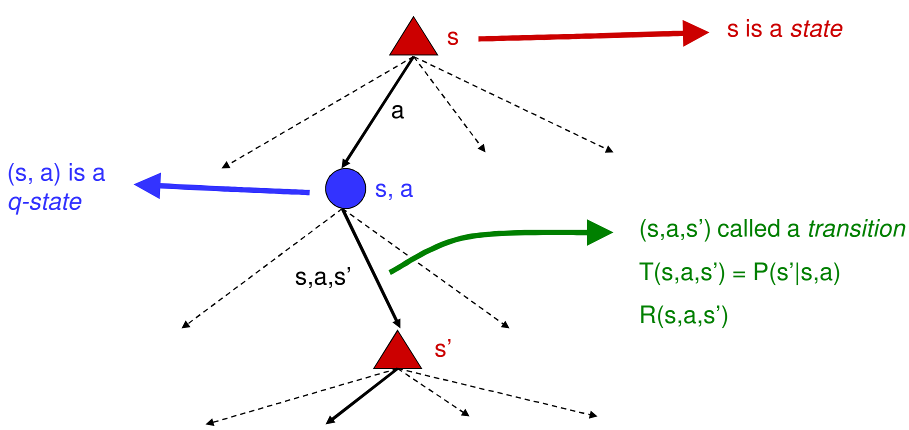
MDP Search Trees

- Each MDP state gives an expectimax-like search tree



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Utilities of Sequences

- In order to formalize optimality of a policy, need to understand utilities of sequences of rewards
- Typically consider stationary preferences:

$$[r, r_0, r_1, r_2, \ldots] \succ [r, r'_0, r'_1, r'_2, \ldots]$$
$$\Leftrightarrow$$
$$[r_0, r_1, r_2, \ldots] \succ [r'_0, r'_1, r'_2, \ldots]$$

- Theorem: only two ways to define stationary utilities
  - Additive utility:

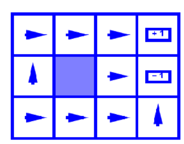
$$U([r_0, r_1, r_2, \ldots]) = r_0 + r_1 + r_2 + \cdots$$

  - Discounted utility:

$$U([r_0, r_1, r_2, \ldots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \cdots$$

Infinite Utilities?!

- Problem: infinite state sequences have infinite rewards
- Solutions:



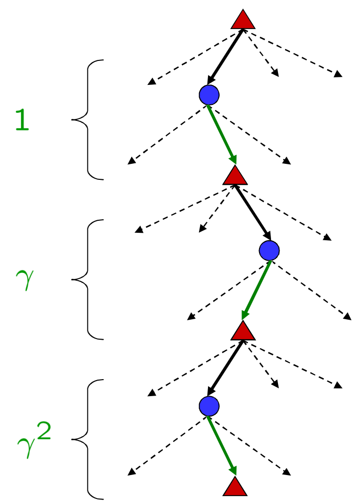
  - Finite horizon:
    - Terminate episodes after a fixed T steps (e.g. life)
    - Gives nonstationary policies (π depends on time left)
  - Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "done" for High-Low)
  - Discounting: for $0 < \gamma < 1$

$$U([r_0, \ldots r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\max}/(1-\gamma)$$

    - Smaller γ means smaller "horizon" – shorter term focus

Discounting

- Typically discount rewards by $\gamma < 1$ each time step
  - Sooner rewards have higher utility than later rewards
  - Also helps the algorithms converge



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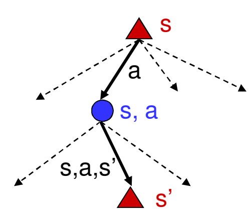
Recap: Defining MDPs

- Markov decision processes:
  - States S
  - Start state s_0
  - Actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility (or return) = sum of discounted rewards

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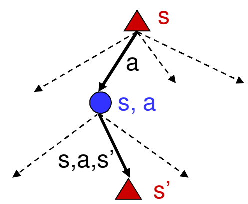
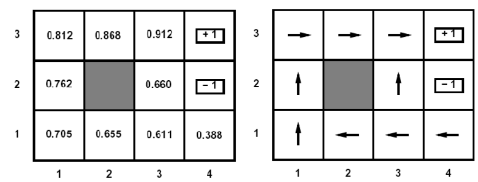
Optimal Utilities

- Fundamental operation: compute the values (optimal expectimax utilities) of states s
- Why? Optimal values define optimal policies!
- Define the value of a state s:
  $V^*(s)$ = expected utility starting in s and acting optimally
- Define the value of a q-state (s,a):
  $Q^*(s,a)$ = expected utility starting in s, taking action a and thereafter acting optimally
- Define the optimal policy:
  $\pi^*(s)$ = optimal action from state s





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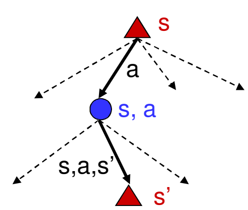
The Bellman Equations

- Definition of "optimal utility" leads to a simple one-step lookahead relationship amongst optimal utility values:

  Optimal rewards = maximize over first action and then follow optimal policy



- Formally:

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

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Value Estimates

- Calculate estimates $V_k^*(s)$
  - Not the optimal value of s!
  - The optimal value considering only next k time steps (k rewards)
  - As k $\rightarrow \infty$, it approaches the optimal value
- Almost solution: recursion (i.e. expectimax)
- Correct solution: dynamic programming