CS 188 Spring 2019 | Introduction to Artificial Intelligence

Written HW 4

Due: Monday 2/25/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

Self assessment due: Monday 3/4/2019 at 11:59pm (submit via Gradescope)

For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope). For each subpart where your original answer was correct, write "correct." Otherwise, write and explain the correct answer.

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

Q1. Reinforcement Learning

Imagine an unknown game which has only two states $\{A, B\}$ and in each state the agent has two actions to choose from: {Up, Down}. Suppose a game agent chooses actions according to some policy π and generates the following sequence of actions and rewards in the unknown game:

t	s_t	a_t	s_{t+1}	r_t
0	A	Down	B	2
1	B	Down	B	-4
2	B	Up	B	0
3	B	Up	A	3
4	A	Up	A	-1

Unless specified otherwise, assume a discount factor $\gamma = 0.5$ *and a learning rate* $\alpha = 0.5$

(a) Recall the update function of Q-learning is:

$$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha(r_t + \gamma \max_{a'} Q(s_{t+1}, a'))$$

Assume that all Q-values initialized as 0. What are the following Q-values learned by running Q-learning with the above experience sequence?

$$Q(\text{A}, \text{Down}) = \_\_\_\_\_\_\_\_, \qquad Q(\text{B}, \text{Up}) = \_\_\_\_\_\_\_\_$$

Self assessment

If your answer was correct, write "**correct**" above. Otherwise, **write and explain** the correct answer.

(b) In model-based reinforcement learning, we first estimate the transition function $T(s, a, s')$ and the reward function $R(s, a, s')$. Fill in the following estimates of T and R, estimated from the experience above. Write "n/a" if not applicable or undefined.

$\hat{T}(\text{A}, \text{Up}, \text{A}) = \_\_\_\_\_\_\_\_$, $\hat{T}(\text{A}, \text{Up}, \text{B}) = \_\_\_\_\_\_\_\_$, $\hat{T}(\text{B}, \text{Up}, \text{A}) = \_\_\_\_\_\_\_\_$, $\hat{T}(\text{B}, \text{Up}, \text{B}) = \_\_\_\_\_\_\_\_$

$\hat{R}(\text{A}, \text{Up}, \text{A}) = \_\_\_\_\_\_\_\_$, $\hat{R}(\text{A}, \text{Up}, \text{B}) = \_\_\_\_\_\_\_\_$, $\hat{R}(\text{B}, \text{Up}, \text{A}) = \_\_\_\_\_\_\_\_$, $\hat{R}(\text{B}, \text{Up}, \text{B}) = \_\_\_\_\_\_\_\_$

Self assessment
If your answer was correct, write "**correct**" above. Otherwise, **write and explain** the correct answer.

(c) To decouple this question from the previous one, assume we had **a different experience** and ended up with the following estimates of the transition and reward functions:

s	a	s'	$\hat{T}(s,a,s')$	$\hat{R}(s,a,s')$
A	Up	A	1	10
A	Down	A	0.5	2
A	Down	B	0.5	2
B	Up	A	1	-5
B	Down	B	1	8

(i) Give the optimal policy $\hat{\pi}^*(s)$ and $\hat{V}^*(s)$ for the MDP with transition function $\hat{T}$ and reward function $\hat{R}$. *Hint: for any* $x \in \mathbb{R}$, $|x| < 1$, *we have* $1 + x + x^2 + x^3 + x^4 + \cdots = 1/(1-x)$.

$\hat{\pi}^*(A) =$ ________, $\hat{\pi}^*(B) =$ ________, $\hat{V}^*(A) =$ ________, $\hat{V}^*(B) =$ ________.

(ii) If we repeatedly feed this new experience sequence through our Q-learning algorithm, what values will it converge to? Assume the learning rate α_t is properly chosen so that convergence is guaranteed.

- ○ the values found above, $\hat{V}^*$
- ○ the optimal values, V^*
- ○ neither $\hat{V}^*$ nor V^*
- ○ not enough information to determine

Self assessment
If your answer was correct, write "**correct**" above. Otherwise, **write and explain** the correct answer.

Q2. Policy Evaluation

In this question, you will be working in an MDP with states S, actions A, discount factor γ, transition function T, and reward function R.

We have some fixed policy $\pi : S \to A$, which returns an action $a = \pi(s)$ for each state $s \in S$. We want to learn the Q function $Q^\pi(s,a)$ for this policy: the expected discounted reward from taking action a in state s and then continuing to act according to π: $Q^\pi(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma Q^\pi(s', \pi(s'))]$. The policy π will not change while running any of the algorithms below.

(a) Can we guarantee anything about how the values Q^π compare to the values Q^* for an optimal policy π^*?

- ◯ $Q^\pi(s,a) \le Q^*(s,a)$ for all s, a
- ◯ $Q^\pi(s,a) = Q^*(s,a)$ for all s, a
- ◯ $Q^\pi(s,a) \ge Q^*(s,a)$ for all s, a
- ◯ None of the above are guaranteed

Self assessment
If your answer was correct, write "**correct**" above. Otherwise, **write and explain** the correct answer.

(b) Suppose T and R are *unknown*. You will develop sample-based methods to estimate Q^π. You obtain a series of *samples* $(s_1, a_1, r_1), (s_2, a_2, r_2), \ldots (s_T, a_T, r_T)$ from acting according to this policy (where $a_t = \pi(s_t)$, for all t).

(i) Recall the update equation for the Temporal Difference algorithm, performed on each sample in sequence:

$$V(s_t) \leftarrow (1-\alpha)V(s_t) + \alpha(r_t + \gamma V(s_{t+1}))$$

which approximates the expected discounted reward $V^\pi(s)$ for following policy π from each state s, for a learning rate α.

Fill in the blank below to create a similar update equation which will approximate Q^π using the samples. You can use any of the terms $Q, s_t, s_{t+1}, a_t, a_{t+1}, r_t, r_{t+1}, \gamma, \alpha, \pi$ in your equation, as well as $\sum$ and max with any index variables (i.e. you could write $\max_a$, or $\sum_a$ and then use a somewhere else), but no other terms.

$$Q(s_t, a_t) \leftarrow (1-\alpha)Q(s_t, a_t) + \alpha\,[\underline{\qquad\qquad\qquad\qquad\qquad\qquad}]$$

(ii) Now, we will approximate Q^π using a linear function: $Q(s,a) = \sum_{i=1}^{d} w_i f_i(s,a)$ for weights $w_1, \ldots, w_d$ and feature functions $f_1(s,a), \ldots, f_d(s,a)$.

To decouple this part from the previous part, use Q_{samp} for the value in the blank in part (i) (i.e. $Q(s_t, a_t) \leftarrow (1-\alpha)Q(s_t, a_t) + \alpha Q_{samp}$).

Which of the following is the correct sample-based update for each w_i?

- ◯ $w_i \leftarrow w_i + \alpha[Q(s_t, a_t) - Q_{samp}]$
- ◯ $w_i \leftarrow w_i - \alpha[Q(s_t, a_t) - Q_{samp}]$

- ◯ $w_i \leftarrow w_i + \alpha[Q(s_t, a_t) - Q_{samp}]f_i(s_t, a_t)$
- ◯ $w_i \leftarrow w_i - \alpha[Q(s_t, a_t) - Q_{samp}]f_i(s_t, a_t)$
- ◯ $w_i \leftarrow w_i + \alpha[Q(s_t, a_t) - Q_{samp}]w_i$
- ◯ $w_i \leftarrow w_i - \alpha[Q(s_t, a_t) - Q_{samp}]w_i$

(iii) The algorithms in the previous parts (part i and ii) are:

☐ model-based ☐ model-free

Self assessment
If your answer was correct, write "**correct**" above. Otherwise, **write and explain** the correct answer.