

## Q1. Propositional logic

(a) Consider a vocabulary with only four symbols,  $A$ ,  $B$ ,  $C$ , and  $D$ . For each of the following sentences, how many possible worlds make it true?

1.  $(A \wedge B) \vee (C \wedge D)$

2.  $\neg(A \wedge B \wedge C \wedge D)$

3.  $B \Rightarrow (A \wedge B)$

(b) A certain procedure to convert a sentence to CNF contains four steps (1-4 below); each step is based on a logical equivalence. Circle ALL of the valid equivalences for each step.

1. Step 1: drop biconditionals

a)  $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$

b)  $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \vee (\beta \Rightarrow \alpha))$

c)  $(\alpha \Leftrightarrow \beta) \equiv (\alpha \wedge \beta)$

2. Step 2: drop implications

a)  $(\alpha \Rightarrow \beta) \equiv (\alpha \vee \neg\beta)$

b)  $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$

c)  $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \wedge \beta)$

3. Step 3: move “not” inwards

a)  $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$

b)  $\neg(\alpha \vee \beta) \equiv (\neg\alpha \vee \neg\beta)$

c)  $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$

4. Step 4: move “or” inwards and “and” outwards

a)  $(\alpha \vee (\beta \wedge \gamma)) \equiv (\alpha \vee \beta \vee \gamma)$

b)  $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$

c)  $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$

(c) Convert the sentence  $A \Leftrightarrow (C \vee D)$  to CNF form.