CS 188
Spring 2019

Midterm Review Logic Solutions

Q1. Propositional logic

(a) Consider a vocabulary with only four symbols, A, B, C, and D. For each of the following sentences, how many possible worlds make it true?

1. $(A \wedge B) \vee (C \wedge D)$ 7 (4 for $A \wedge B$, 4 for $C \wedge D$, minus 1 for the model that satisfies both).

2. $\neg(A \wedge B \wedge C \wedge D)$ 15 — it's the negation of a sentence with 1 model.

3. $B \Rightarrow (A \wedge B)$ 12 — it's true when B is false (8) and when B is true and A is true (4).

(b) A certain procedure to convert a sentence to CNF contains four steps (1-4 below); each step is based on a logical equivalence. Circle ALL of the valid equivalences for each step.

1. Step 1: drop biconditionals

   [a)] $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$
   b) $(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \vee (\beta \Rightarrow \alpha))$
   c) $(\alpha \Leftrightarrow \beta) \equiv (\alpha \wedge \beta)$

2. Step 2: drop implications

   a) $(\alpha \Rightarrow \beta) \equiv (\alpha \vee \neg\beta)$
   [b)] $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$
   c) $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \wedge \beta)$

3. Step 3: move "not" inwards

   [a)] $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$
   b) $\neg(\alpha \vee \beta) \equiv (\neg\alpha \vee \neg\beta)$
   [c)] $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$

4. Step 4: move "or" inwards and "and" outwards

   a) $(\alpha \vee (\beta \wedge \gamma)) \equiv (\alpha \vee \beta \vee \gamma)$
   [b)] $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$
   c) $(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$

(c) Convert the sentence $A \Leftrightarrow (C \vee D)$ to CNF form.

$A \Leftrightarrow (C \vee D)$
$(A \Rightarrow (C \vee D)) \wedge ((C \vee D) \Rightarrow A)$
$(\neg A \vee (C \vee D)) \wedge (\neg(C \vee D) \vee A)$
$(\neg A \vee C \vee D) \wedge ((\neg C \wedge \neg D) \vee A)$
$(\neg A \vee C \vee D) \wedge ((\neg C \vee A) \wedge (\neg D \vee A))$
$(\neg A \vee C \vee D) \wedge (\neg C \vee A) \wedge (\neg D \vee A)$