CS188

We've seen how AI methods can solve problems in:

- Search

CS188

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- Search
- Constraint Satisfaction Problems

CS188

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CS188

We’ve seen how AI methods can solve problems in:

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CS188

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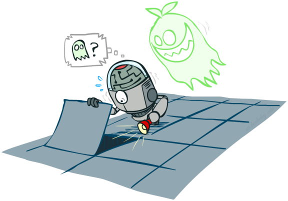
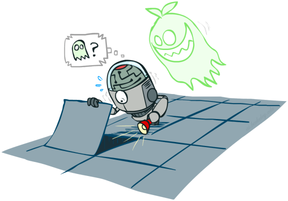
Next up: Part II: Uncertainty and Learning!

Our Status in CS188



Our Status in CS188



We're done with Part I Search and Planning!

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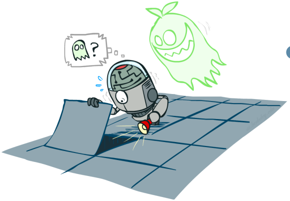

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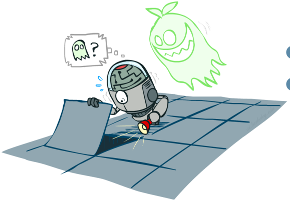
Part II: Probabilistic Reasoning

- Diagnosis

Our Status in CS188



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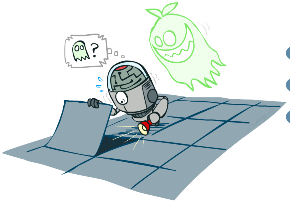
Part II: Probabilistic Reasoning
- Diagnosis
- Speech recognition

Our Status in CS188



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Part II: Probabilistic Reasoning

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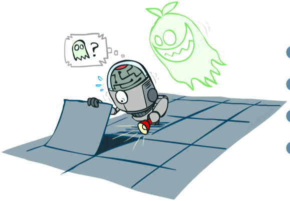
- Speech recognition
- Tracking objects

Our Status in CS188



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- Tracking objects
- Robot mapping

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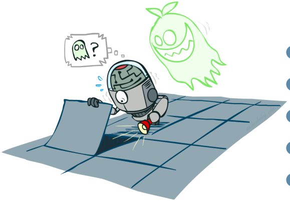

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Part II: Probabilistic Reasoning

- Diagnosis
- Speech recognition
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- Genetics

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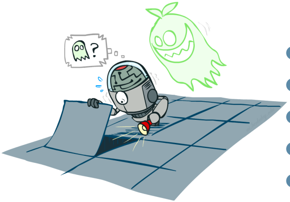

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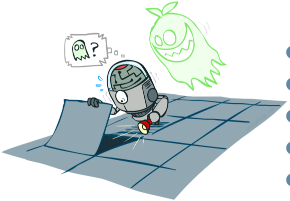

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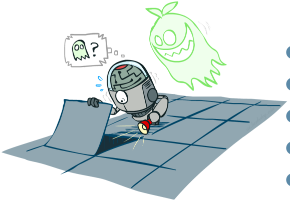

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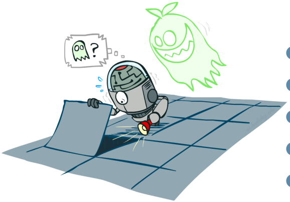

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Part II: Probabilistic Reasoning

- Diagnosis
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- Genetics
- Error correcting codes
- . . . lots more!

Part III: Machine Learning

CS 188: Artificial Intelligence

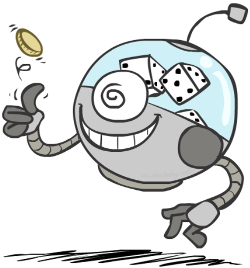
CS 188: Artificial Intelligence

Probability

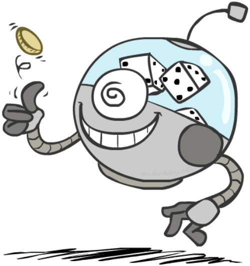
CS 188: Artificial Intelligence

Probability

Today

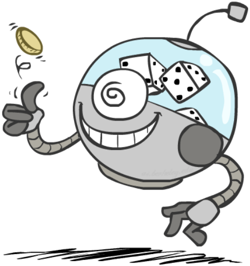


Today



- Probability
  - Random Variables

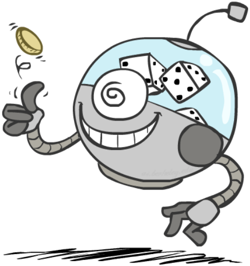
Today



Probability

- Random Variables
- Joint and Marginal Distributions i

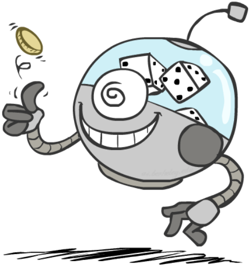
Today



Probability

- Random Variables
- Joint and Marginal Distributions i
- Conditional Distribution

Today



Probability

- Random Variables
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- Product Rule, Chain Rule, Bayes' Rule

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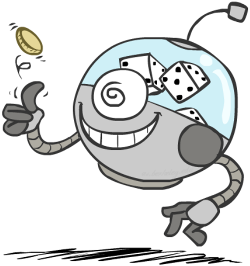

Probability

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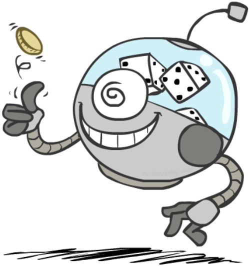

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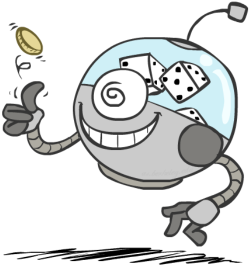

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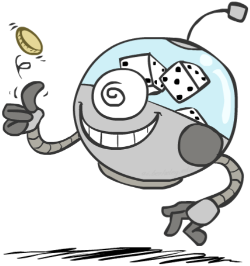

Probability

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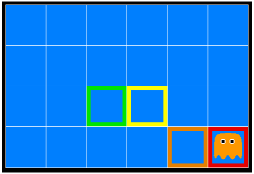
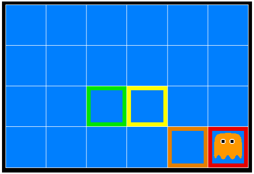
- Independence

You'll need all this stuff A LOT for the next few weeks, so make sure you get this!

Inference in Ghostbusters

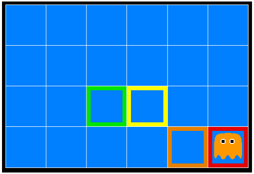


Inference in Ghostbusters



A ghost is in the grid somewhere

Inference in Ghostbusters

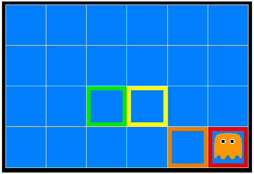


A ghost is in the grid somewhere

Sensor readings tell how close a square is to the ghost

- On the ghost: red

Inference in Ghostbusters



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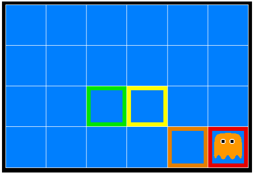
Sensor readings tell how close a square is to the ghost

- On the ghost: red
- 1 or 2 away: orange

Inference in Ghostbusters



A ghost is in the grid somewhere

Sensor readings tell how close a square is to the ghost

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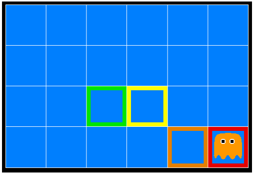
- 1 or 2 away: orange
- 3 or 4 away: yellow

Inference in Ghostbusters



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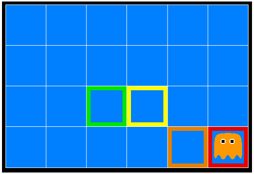
Inference in Ghostbusters



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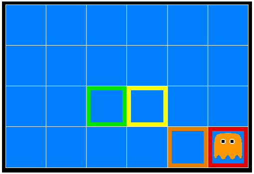

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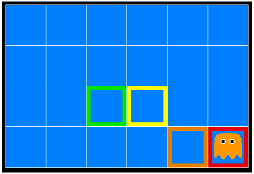
Sensors are noisy, but we know $P(Color|Distance)$

Inference in Ghostbusters



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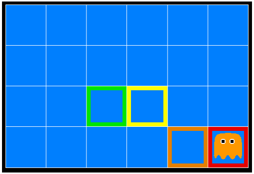
$P(red|3)$	$P(orange|3)$	$P(yellow|3)$	$P(green|3)$
0.05	0.15	0.5	0.3

Inference in Ghostbusters



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[Demo: Ghostbuster – no probability (L12D1)]

Video of Demo Ghostbuster – No probability

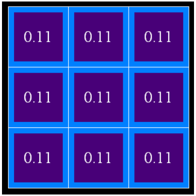
Uncertainty

0.11	0.11	0.11
0.11	0.11	0.11
0.11	0.11	0.11

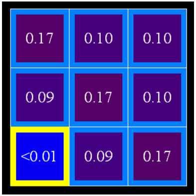
0.17	0.10	0.10
0.09	0.17	0.10
<0.01	0.09	0.17

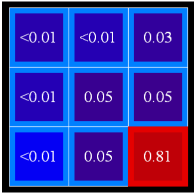
<0.01	<0.01	0.03
<0.01	0.05	0.05
<0.01	0.05	0.81

Uncertainty

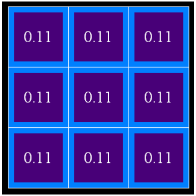


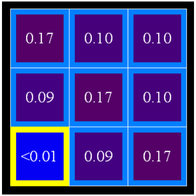
General situation:

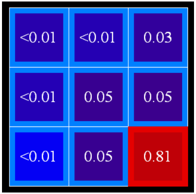




Uncertainty



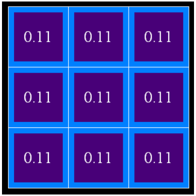


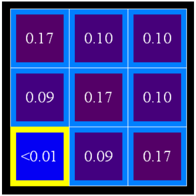


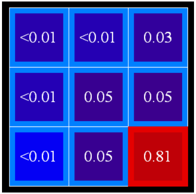
General situation:

- **Observed variables (evidence):** Agent knows certain things about the state of the world (e.g., sensor readings or symptoms)

Uncertainty







General situation:

- **Observed variables (evidence):** Agent knows certain things about the state of the world (e.g., sensor readings or symptoms)
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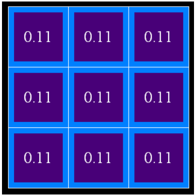

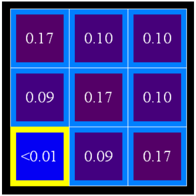

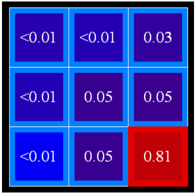

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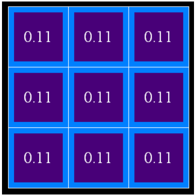

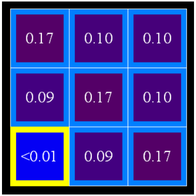

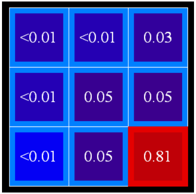

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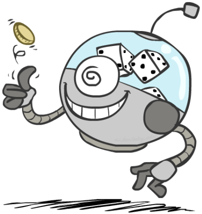
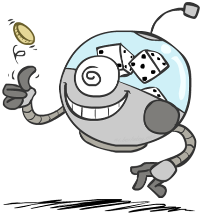
Probabilistic reasoning gives us a framework for managing our beliefs and knowledge

Random Variables



Random Variables

A random variable is some aspect of the world about which we (may) have uncertainty



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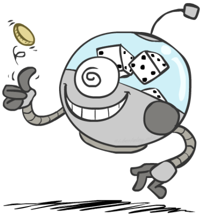

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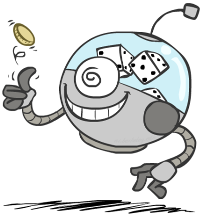
- $R =$ Is it raining?

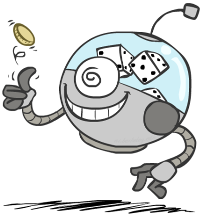
Random Variables



A random variable is some aspect of the world about which we (may) have uncertainty

- $R =$ Is it raining?
- $T =$ Is it hot or cold?

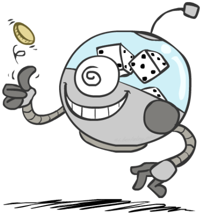
Random Variables



A random variable is some aspect of the world about which we (may) have uncertainty

- $R =$ Is it raining?
- $T =$ Is it hot or cold?
- $D =$ How long will it take to drive to work?

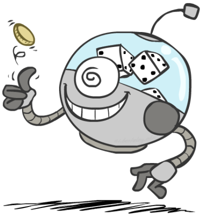
Random Variables



A random variable is some aspect of the world about which we (may) have uncertainty

- $R =$ Is it raining?
- $T =$ Is it hot or cold?
- $D =$ How long will it take to drive to work?
- $L =$ Where is the ghost?

Random Variables



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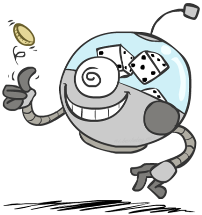

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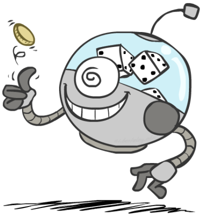
We denote random variables with capital letters

Random Variables



A random variable is some aspect of the world about which we (may) have uncertainty

- $R =$ Is it raining?
- $T =$ Is it hot or cold?
- $D =$ How long will it take to drive to work?
- $L =$ Where is the ghost?

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Like CSP, variables (random) have domains

- $R \in \{true, false\}$ (often write as $\{+r, -r\}$)

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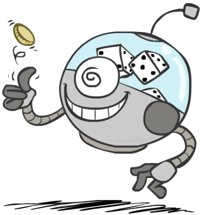

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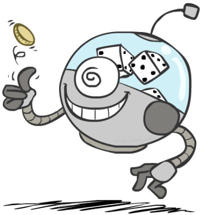

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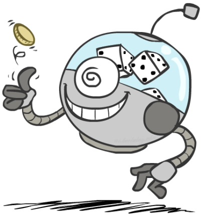

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- $T \in \{hot, cold\}$
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- L in possible locations, maybe $\{(0,0), (0,1), \ldots\}$

Probability Distributions

Associate a probability with each value

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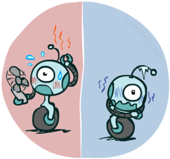

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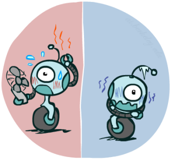

$$P(T)$$

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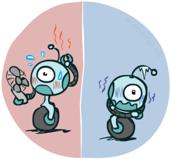

$P(T)$

T	P
hot	0.5
cold	0.5

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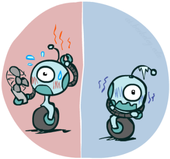

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- Weather:

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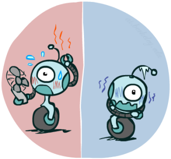

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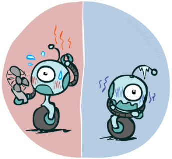

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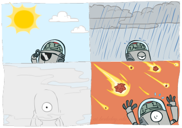

$P(W)$

Probability Distributions

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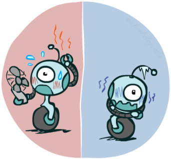

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hot	0.5
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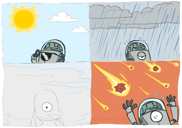

$P(W)$

W	P
sun	0.6
rain	0.1
fog	0.3
meteor	0.0

Probability Distributions

Unobserved random variables have distributions:

T	P
hot	0.5
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Shorthand notation:

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Shorthand notation:

$$P(hot) = P(T = hot)$$

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Same table:

W$\times$T	hot	cold
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Constraint satisfaction problems:
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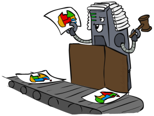
Constraints over T,W

T	W	P
hot	sun	T
hot	rain	F
cold	sun	F
cold	rain	T



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From a joint distribution, we can calculate the probability of any event

- Probability that it's hot AND sunny?

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cold	rain	0.3

From a joint distribution, we can calculate the probability of any event

- Probability that it's hot AND sunny?
- Probability that it's hot?
- Probability that it's hot OR sunny?

Events

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

An event is a set E of outcomes:

$$P(E) = \sum_{(x_1,\ldots,x_n)\in E} P(x_1,\ldots,x_n).$$

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Typically, the events we care about are partial assignments:
examples: $P(T = hot)$. $P(W = sun)$.

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$P(hot)$

Events

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$P(hot) = P(hot, sun) + P(hot, rain)$

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$P(hot) = P(hot, sun) + P(hot, rain) = .5$

Quiz: Events

http://bit.ly/cs188prob

$$P(X, Y)$$

Quiz: Events

http://bit.ly/cs188prob

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$?

Quiz: Events

http://bit.ly/cs188prob

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

Quiz: Events

http://bit.ly/cs188prob

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

$P(+x)$?

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-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

$P(+x)$? 0.2 + 0.3

Quiz: Events

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$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

$P(+x)$? 0.2 + 0.3 = 0.5

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+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

$P(+x)$? 0.2 + 0.3 = 0.5

$P(-y\ OR\ +x's)$?

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+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

$P(+x)$? 0.2 + 0.3 = 0.5

$P(-y\ OR\ +x's)$?

P(+x,-y) + P(+x,+y) + P(+x,-y)

Quiz: Events

http://bit.ly/cs188prob

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x, +y)$? .2

$P(+x)$? 0.2 + 0.3 = 0.5

$P(-y \ OR \ +x's)$?

P(+x,-y) + P(+x,+y) + P(+x,-y) = 0.6

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

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$P(T, W)$

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Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

Marginal for Temparature.

$P(T, W)$

T	W	P
hot	sun	0.4
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cold	sun	0.2
cold	rain	0.3

Marginal Distributions

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$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal for Weather.

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal for Weather.

W	P
rain	

Marginal Distributions

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$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal for Weather.

W	P
rain	0.4
sun	

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal for Weather.

W	P
rain	0.4
sun	0.6

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal for Weather.

W	P
rain	0.4
sun	0.6

Marginalization (summing out): Combine collapsed rows by adding.

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
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Marginal for Temparature.

T	P
hot	0.5
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Marginal for Weather.

W	P
rain	0.4
sun	0.6

Marginalization (summing out): Combine collapsed rows by adding.

Same idea

W $\times$ T	hot	cold	M(W)
sun	0.4	0.2	0.6
rain	0.1	0.3	0.4
M(T)	0.5	0.5	

Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
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Marginal for Temparature.

T	P
hot	0.5
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Marginal for Weather.

W	P
rain	0.4
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W $\times$ T	hot	cold	M(W)
sun	0.4	0.2	0.6
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Marginal Distributions

Marginal distributions are sub-tables which eliminate variables

$P(T,W)$

T	W	P
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Marginal for Temparature.

T	P
hot	0.5
cold	0.5

Marginal for Weather.

W	P
rain	0.4
sun	0.6

Marginalization (summing out): Combine collapsed rows by adding.

Same idea

W $\times$ T	hot	cold	M(W)
sun	0.4	0.2	0.6
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M(T)	0.5	0.5	

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

Quiz: Marginal Distributions

http://bit.ly/cs188prob

P(X)

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

P(Y)

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

P(Y)

Y	P
+y	

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

P(Y)

Y	P
+y	0.6

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

P(Y)

Y	P
+y	0.6
-y	

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

P(Y)

Y	P
+y	0.6
-y	0.4

Quiz: Marginal Distributions

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

P(X)

X	P
+x	0.5
-x	0.5

P(Y)

Y	P
+y	0.6
-y	0.4

Conditional Probabilities

A simple relation between joint and conditional probabilities

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- In fact, this is taken as the definition of a conditional probability

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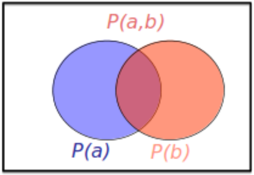
The probability of event *a* given event *b*.



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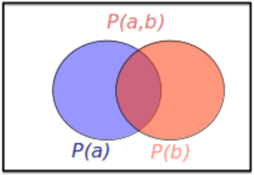

The probability of event *a* given event *b*.

$$P(a|b)$$

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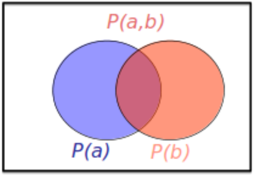

The probability of event *a* given event *b*.

$$P(a|b) = \frac{P(a,b)}{P(b)}$$

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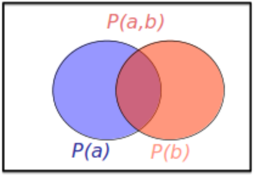

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Probability of a given b.

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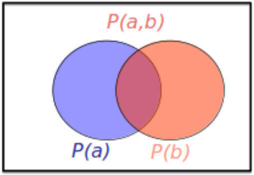

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Natural?

Conditional Probabilities

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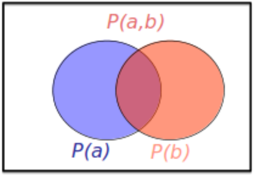

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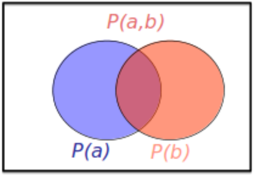

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Natural? Yes!

T	W	P
hot	sun	0.4
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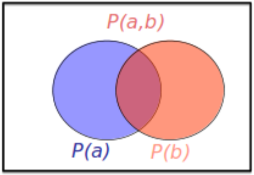

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$$P(W = s|T = c) = \frac{P(w=s, T=c)}{P(T=c)}.$$

Conditional Probabilities

A simple relation between joint and conditional probabilities

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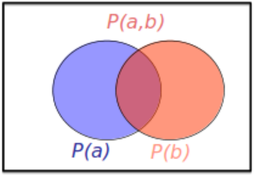

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Conditional Probabilities

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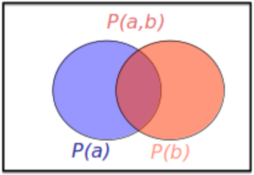

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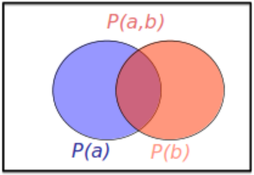

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$$= 0.2 + 0.3 = 0.5$$

Conditional Probabilities

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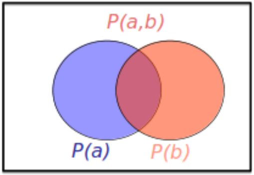

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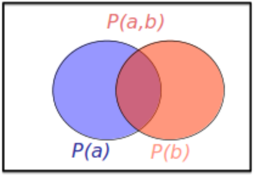

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$$= 0.2 + 0.3 = 0.5$$

$$P(W=s|T=c) = \frac{P(w=s,T=c)}{P(T=c)} = \frac{.2}{.5} = 2/5.$$

Quiz: Conditional Probabilities

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

Quiz: Conditional Probabilities

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X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

Quiz: Conditional Probabilities

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x \mid +y)$?

Quiz: Conditional Probabilities

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X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x|+y) \ ? \ \frac{P(+x,+y)}{P(+y)}$

Quiz: Conditional Probabilities

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x|+y)$? $\frac{P(+x,+y)}{P(+y)} = \frac{.2}{.6} = 1/3$

Quiz: Conditional Probabilities

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Quiz: Conditional Probabilities

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$P(+x|+y)$? $\frac{P(+x,+y)}{P(+y)} = \frac{.2}{.6} = 1/3$

$P(-x|+y)$? $= 1 - P(+x|+y)$

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Quiz: Conditional Probabilities

http://bit.ly/cs188prob

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$P(+x|+y)$? $\frac{P(+x,+y)}{P(+y)} = \frac{.2}{.6} = 1/3$

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$P(-y|+x)$?

Quiz: Conditional Probabilities

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x|+y)$? $\frac{P(+x,+y)}{P(+y)} = \frac{.2}{.6} = 1/3$

$P(-x|+y)$? $= 1 - P(+x|+y) = \frac{2}{3}$.

$P(-y|+x)$? $= \frac{P(-y,+x)}{P(+x)}$

Quiz: Conditional Probabilities

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
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-x	-y	0.1

$P(+x|+y)$? $\frac{P(+x,+y)}{P(+y)} = \frac{.2}{.6} = 1/3$

$P(-x|+y)$? $= 1 - P(+x|+y) = \frac{2}{3}$.

$P(-y|+x)$? $= \frac{P(-y,+x)}{P(+x)} = \frac{.3}{.5}$

Quiz: Conditional Probabilities

http://bit.ly/cs188prob

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$P(+x|+y)$? $\frac{P(+x,+y)}{P(+y)} = \frac{.2}{.6} = 1/3$

$P(-x|+y)$? $= 1 - P(+x|+y) = \frac{2}{3}$.

$P(-y|+x)$? $= \frac{P(-y,+x)}{P(+x)} = \frac{.3}{.5} = 3/5$

Conditional Distributions

Conditional distributions are probability distributions over some variables given fixed values of others

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Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Conditional Distributions

Conditional distributions are probability distributions over some variables given fixed values of others

Conditional Distributions

Joint Distribution

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Conditional distributions are probability distributions over some variables given fixed values of others

Conditional Distributions

$$P(W|T = hot)$$

Joint Distribution

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Conditional Distributions

Conditional distributions are probability distributions over some variables given fixed values of others

Conditional Distributions

$$P(W|T = hot)$$

W	P
sun	0.8
cold	0.2

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Conditional Distributions

Conditional distributions are probability distributions over some variables given fixed values of others

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Conditional Distributions

$P(W|T = hot)$

W	P
sun	0.8
cold	0.2

$P(W|T = cold)$

Conditional Distributions

Conditional distributions are probability distributions over some variables given fixed values of others

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Conditional Distributions

$P(W|T = hot)$

W	P
sun	0.8
cold	0.2

$P(W|T = cold)$

W	P
sun	0.4
cold	0.6

Normalization Trick

$$P(W = s | T = c) = \frac{P(w{=}s, T{=}c)}{P(T{=}c)}$$
$$= \frac{.2}{P(T{=}c)}.$$

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Normalization Trick

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(W = s|T = c) = \frac{P(w=s,T=c)}{P(T=c)}$$
$$= \frac{.2}{P(T=c)}.$$

$$P(T = c) = P(W = s, T = c) + P(W = r, T = c)$$
$$= 0.2 + 0.3$$

Normalization Trick

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
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$$= 0.2 + 0.3 = 0.5$$

Normalization Trick

Joint Distribution

T	W	P
hot	sun	0.4
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cold	sun	0.2
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$$P(T = c) = P(W = s, T = c) + P(W = r, T = c)$$
$$= 0.2 + 0.3 = 0.5$$

$$P(W = r | T = h) = \frac{P(w{=}r, T{=}h)}{P(T{=}c)}$$
$$= \frac{.3}{P(T{=}c)}.$$

Normalization Trick

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(W=s|T=c) = \frac{P(w=s,T=c)}{P(T=c)}$$
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$$P(T=c) = P(W=s,T=c) + P(W=r,T=c)$$
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$$= \frac{.3}{P(T=c)}.$$

$$P(T=c) = P(W=s,T=c) + P(W=r,T=c)$$

$$= 0.2 + 0.3$$

Normalization Trick

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(W=s|T=c)=\frac{P(w=s,T=c)}{P(T=c)}$$
$$=\frac{.2}{P(T=c)}.$$

$$P(T=c)=P(W=s,T=c)+P(W=r,T=c)$$
$$=0.2+0.3=0.5$$

$$P(W=r|T=h)=\frac{P(w=r,T=h)}{P(T=c)}$$
$$=\frac{.3}{P(T=c)}.$$

$$P(T=c)=P(W=s,T=c)+P(W=r,T=c)$$

$$=0.2+0.3=0.5$$

$$P(W|T=cold)$$

Normalization Trick

Joint Distribution

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$P(W=s|T=c) = \frac{P(w=s,T=c)}{P(T=c)}$$
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$$= \frac{.3}{P(T=c)}.$$

$$P(T=c) = P(W=s,T=c) + P(W=r,T=c)$$

$$= 0.2 + 0.3 = 0.5$$

$P(W|T=cold)$

W	P
sun	0.4
cold	0.6

Why does normalization work?

Why does normalization work?

Answer: Work it out!

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Will discuss on Monday,

Why does normalization work?

Answer: Work it out!

Will discuss on Monday,

Have a nice weekend!