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Written Homework 2 is out.
  Get started!

Artificial Intelligence: Bayes' Nets

Representation. ✓

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Conditional Independences (Next.)

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Probabilistic Inference

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Learning Bayes’ Nets from Data

Conditional Independence

X and Y are independent if

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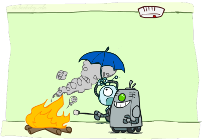
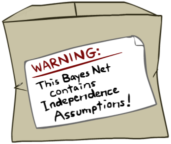
(Conditional) independence is a property of a distribution

Example: Alarm $\perp\!\!\!\perp$ Fire $|$ Smoke



Bayes Nets: Assumptions



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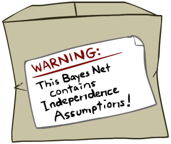

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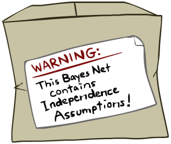

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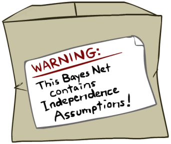

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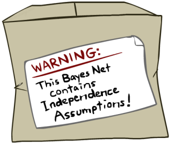

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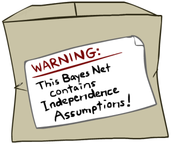

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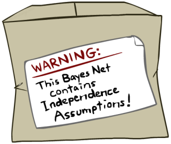

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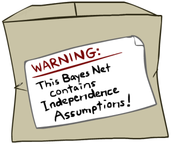

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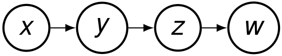
Important for modeling: understand assumptions made when choosing a Bayes net graph

Example



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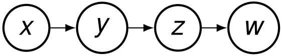

Conditional independence assumptions directly from simplifications in chain rule:

Example

$$x \rightarrow y \rightarrow z \rightarrow w$$

Conditional independence assumptions directly from simplifications in chain rule:

Additional implied conditional independence assumptions?

Independence in a BN

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Important question about a BN:

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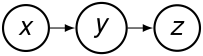

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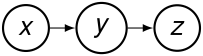

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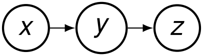

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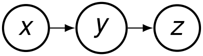

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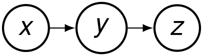

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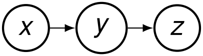

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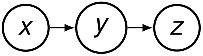

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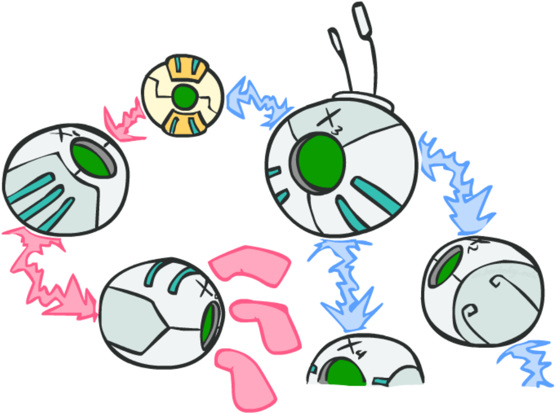
Same as Markov chain.

D-separation: Outline



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D-separation: a condition / algorithm for answering such queries

Causal Chains

This configuration is a "causal chain"

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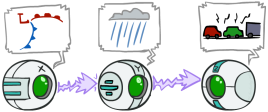

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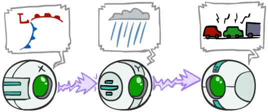

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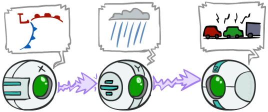

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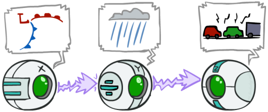

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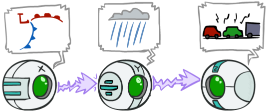

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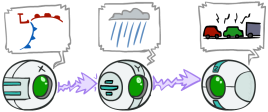

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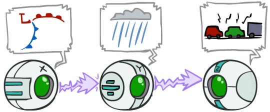

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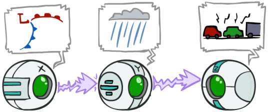

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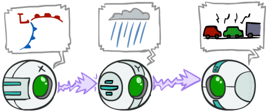

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- Evidence along the chain “blocks” the influence

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Guaranteed X independent of Z given Y?

$$\begin{aligned} P(z|y,x) &= \frac{P(x,y,z)}{P(x,y)} \\ &= \frac{P(x)P(y|x)P(z|y)}{P(x)P(y|x)} \\ &= P(z|y) \end{aligned}$$

Yes!!!!!

- Evidence along the chain “blocks” the influence

Common Cause

This configuration is a “common cause”

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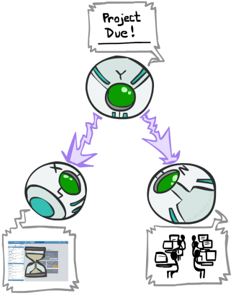
Y: Project due



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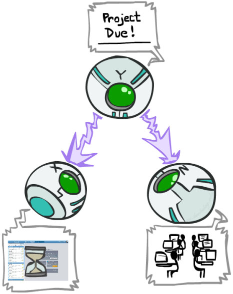

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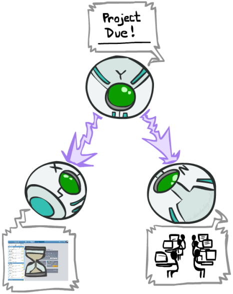

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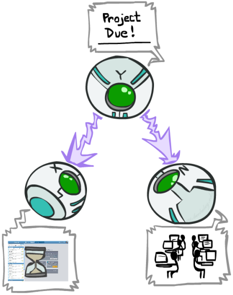

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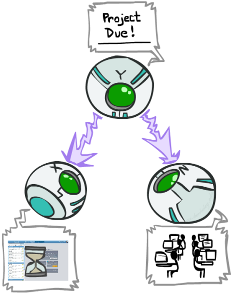

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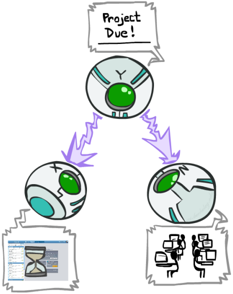

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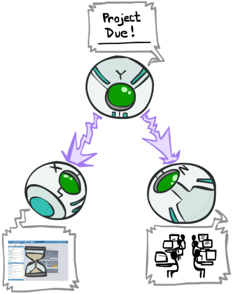

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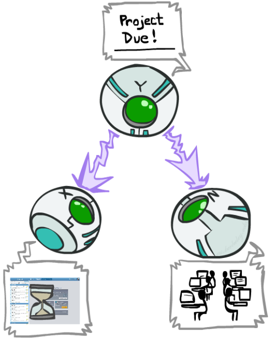

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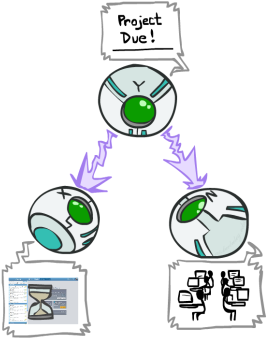

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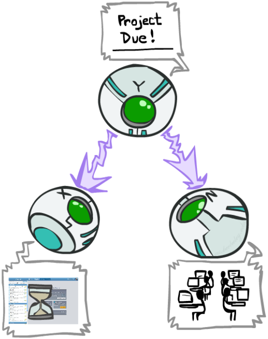

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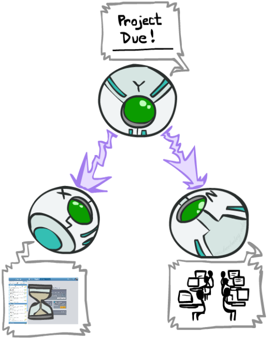

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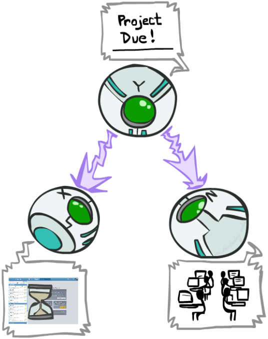

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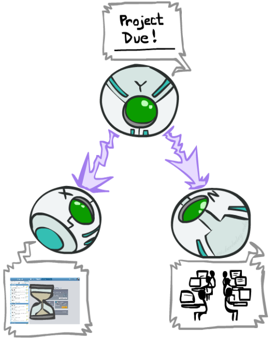

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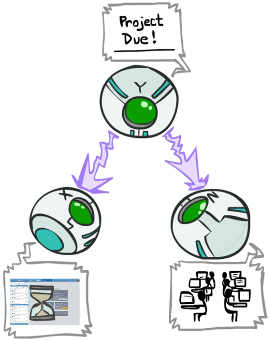

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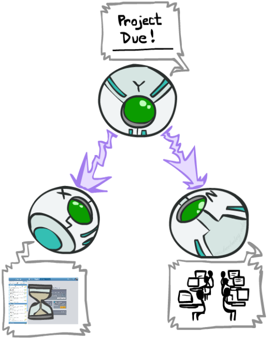

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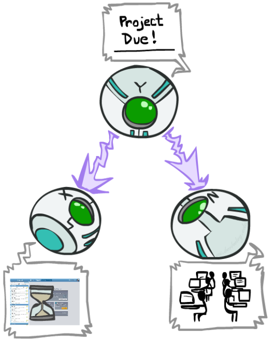

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Observing the cause blocks influence between effects.

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One last configuration: two causes of one effect (v-structures)

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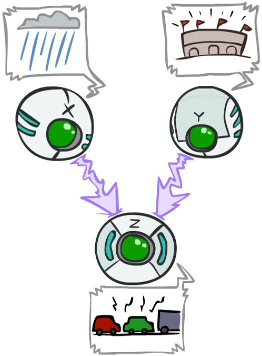

Z: Traffic

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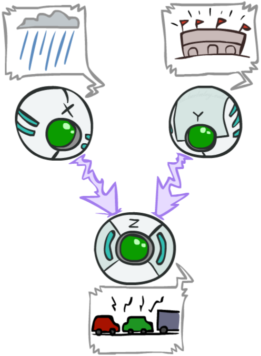

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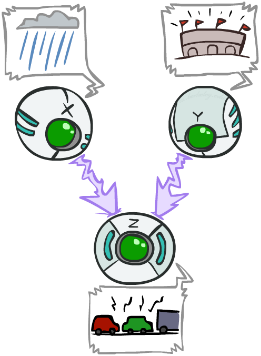

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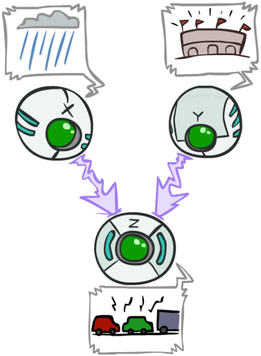

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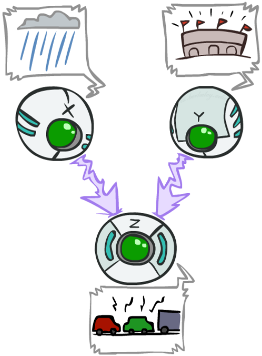

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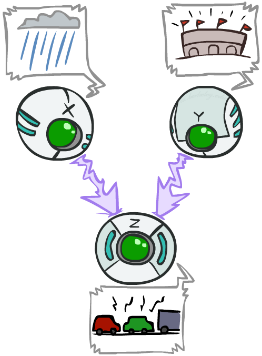

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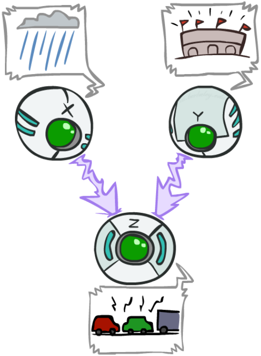

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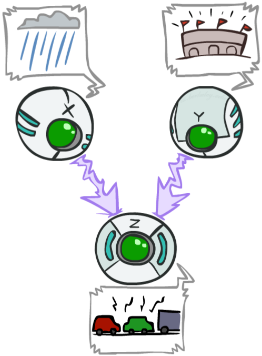

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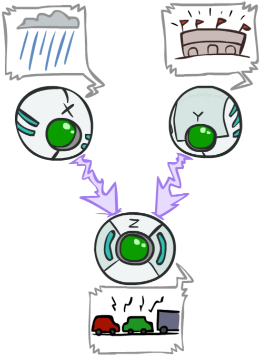

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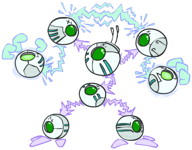

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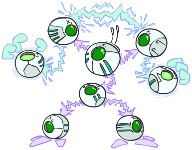

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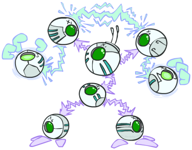

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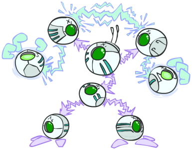

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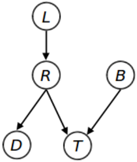
Any complex example can be broken into repetitions of the three canonical cases

Reachability



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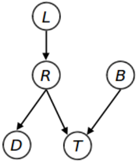

Recipe: shade evidence nodes, look for paths in the resulting graph

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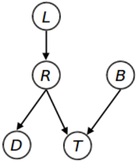

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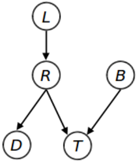

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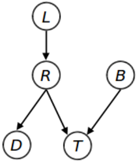

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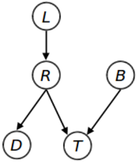

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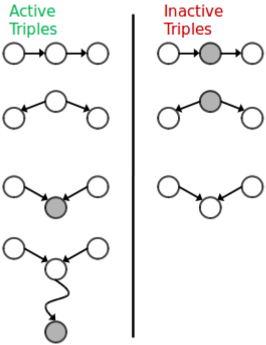

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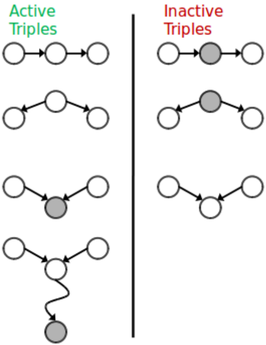

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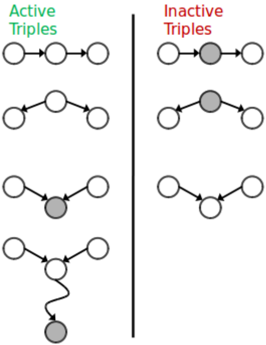

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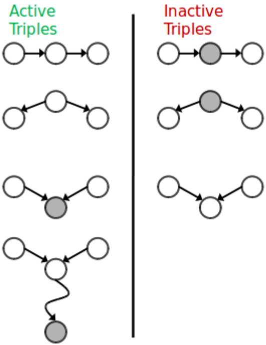

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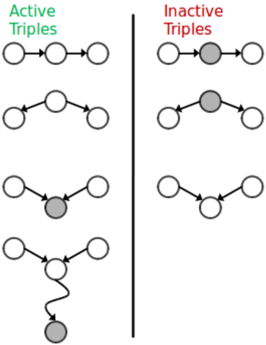

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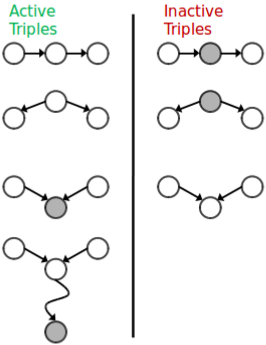

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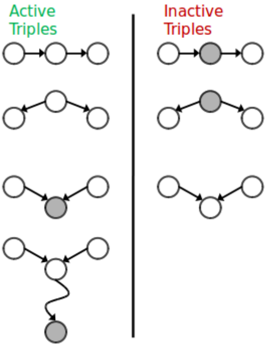

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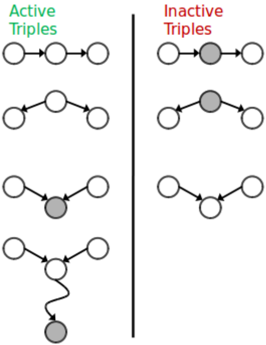

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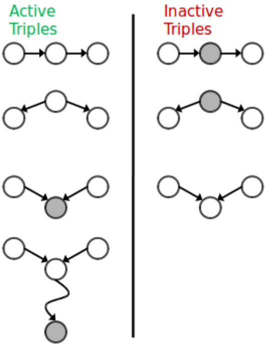

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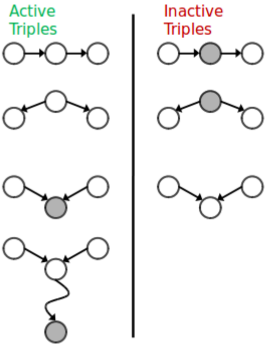
Active / Inactive Paths



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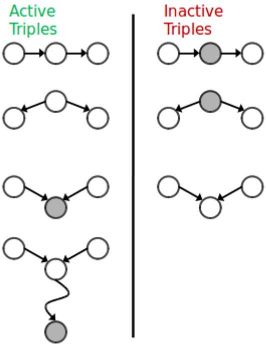

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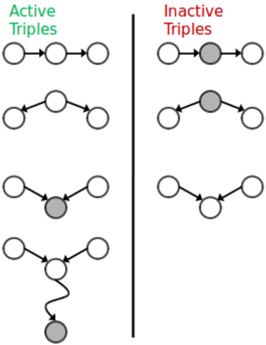

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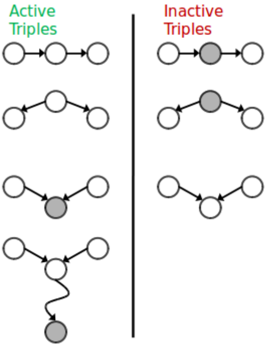

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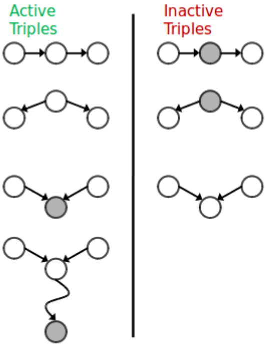

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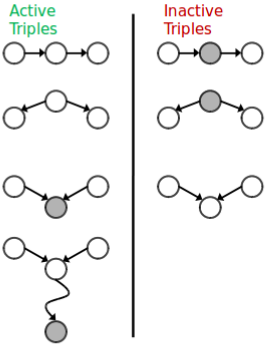

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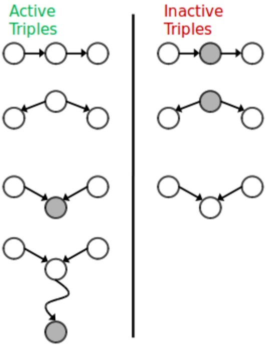

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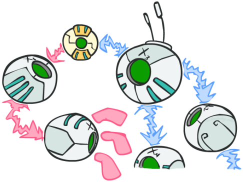
All it takes to block a path is a single inactive segment

D-separation

Query: $X_i \perp\!\!\!\perp X_j | \{X_{k_1}, \ldots, X_{k_n}\}$?



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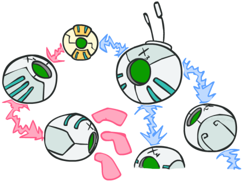

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Check all (undirected!) paths between X_i and X_j

- If one or more active, then independence not guaranteed

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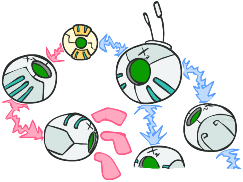

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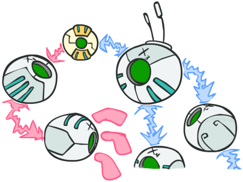

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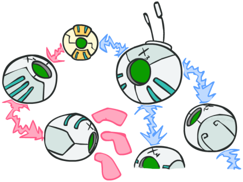

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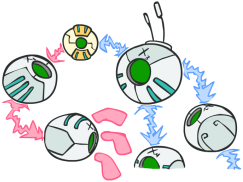

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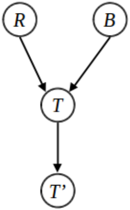
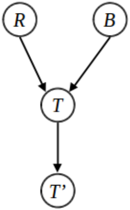
Example

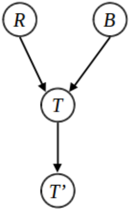


Example



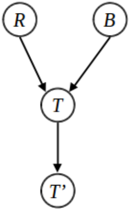
$R \perp\!\!\!\perp B$?

Example



$R \perp\!\!\!\perp B$? Yes.

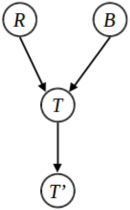
Example



$R \perp\!\!\!\perp B$? Yes.

$R \perp\!\!\!\perp B | T$?

Example



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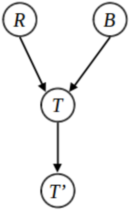

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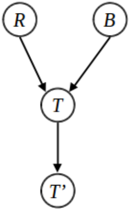

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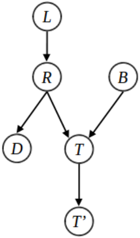
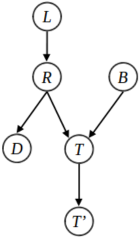


$L \perp\!\!\!\perp T' \mid T$?

Example



$L \perp\!\!\!\perp T' | T$? Yes

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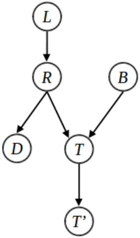

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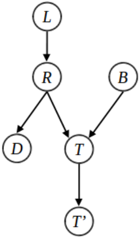

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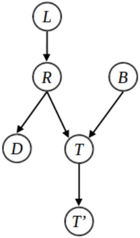

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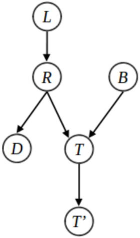

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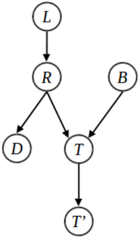

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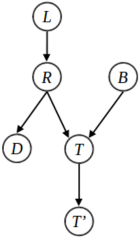

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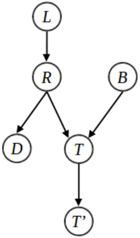

$L \perp\!\!\!\perp T' | T$? Yes

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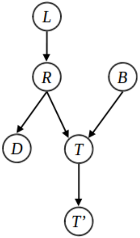
$L \perp\!\!\!\perp B | T$? No

$L \perp\!\!\!\perp B | T'$? No

$L \perp\!\!\!\perp B | T, R$?

Example



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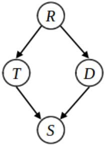

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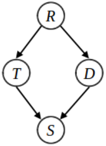
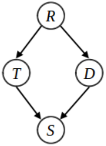


Variables:

- R: Raining

Example



Variables:

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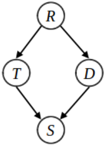
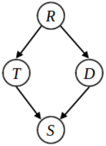
- T: Traffic

Example



Variables:
- R: Raining
- T: Traffic
- D: Roof drips

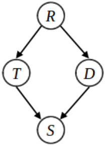
Example



Variables:

- R: Raining
- T: Traffic
- D: Roof drips
- S: I'm sad

Example



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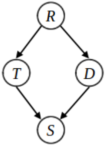

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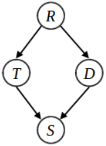
Questions:

Example



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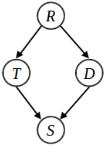
$T \perp\!\!\!\perp D$?

Example



Variables:

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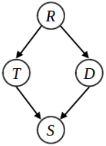
Example



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$T \perp\!\!\!\perp D | R$?

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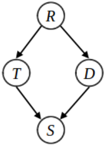

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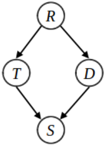

Variables:

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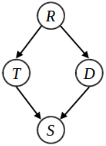
Questions:

$T \perp\!\!\!\perp D$? No..

$T \perp\!\!\!\perp D | R$? Yes.

$T \perp\!\!\!\perp D | R, S$?

Example



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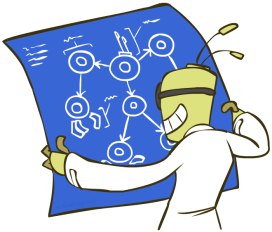
Structure Implications



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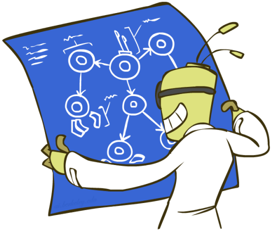

Given a Bayes net structure, can run d-separation algorithm to build a complete list of conditional independences that are necessarily true of the form

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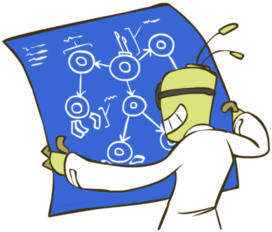

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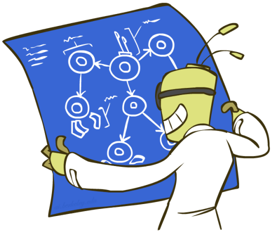

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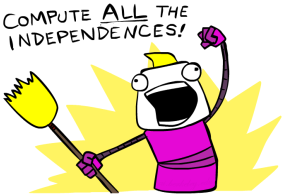
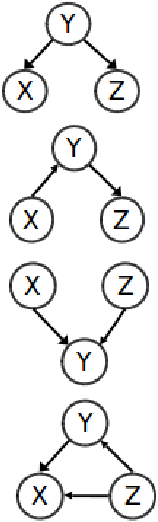
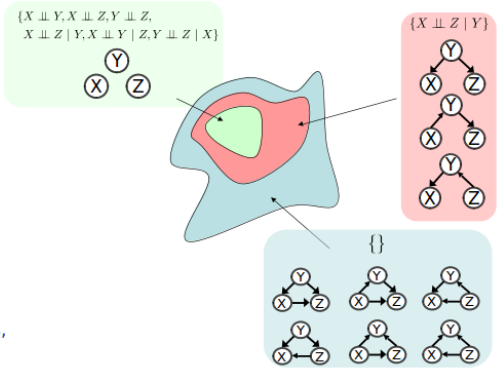
This list determines the set of probability distributions that can be represented

Computing All Independences





Topology Limits Distributions



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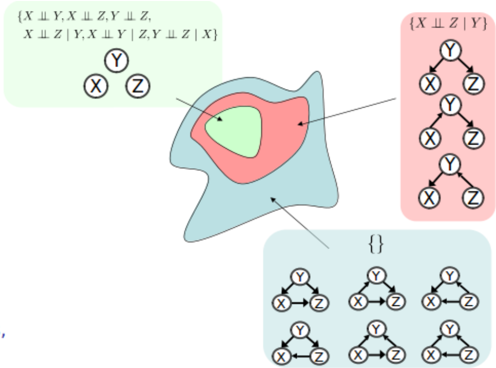
Topology Limits Distributions

Given some graph topology G, only certain joint distributions can be encoded



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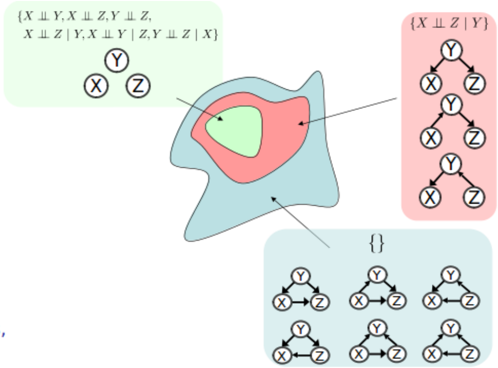

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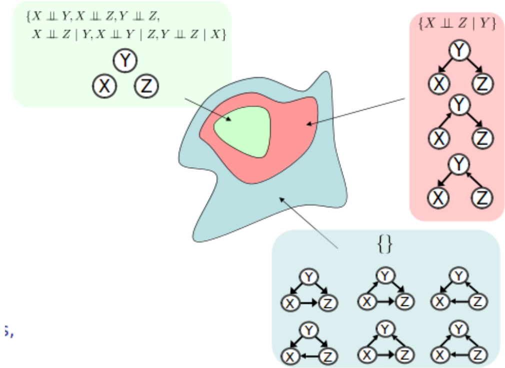
The graph structure guarantees certain (conditional) independences

Topology Limits Distributions



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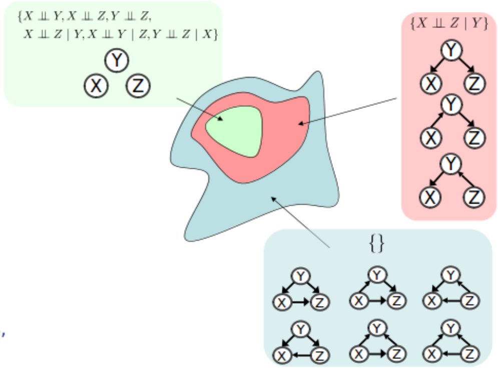
Topology Limits Distributions



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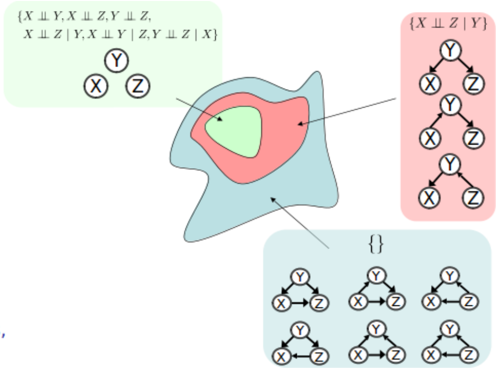

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The graph structure guarantees certain (conditional) independences

(There might be more independence)

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Full conditioning can encode any distribution

Bayes Nets Representation Summary

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Bayes nets compactly encode joint distributions

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Guaranteed independencies of distributions can be deduced from BN graph structure

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D-separation gives precise conditional independence guarantees from graph alone

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Bayes nets compactly encode joint distributions

Guaranteed independencies of distributions can be deduced from BN graph structure

D-separation gives precise conditional independence guarantees from graph alone

A Bayes' net's joint distribution may have further (conditional) independence that is not detectable until you inspect its specific distribution

Bayes' Nets

Bayes' Nets

Representation

Bayes' Nets

Representation ✓

Bayes' Nets

Representation ✓

Conditional Independences

Bayes' Nets

Representation ✓

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Probabilistic Inference

- Enumeration (exact, exponential complexity)

Bayes' Nets

Representation ✓

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- Variable elimination (exact, worst-case exponential complexity, often better)

Bayes' Nets

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Learning Bayes' Nets from Data

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Representation

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Conditional Independences

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Probabilistic Inference (Next).

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Learning Bayes' Nets from Data

Examples:

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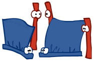
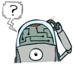

- Posterior probability
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Inference

Examples:

- Posterior probability
- Most likely explanation:

Inference

Inference: calculating some useful quantity from a joint probability distribution

Inference by Enumeration

General case:

- Evidence variables:

$E_1, \ldots, E_k = e_1, \ldots, e_k$

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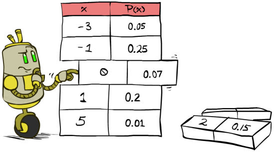

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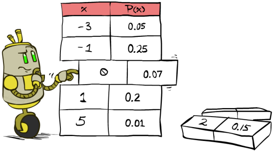
Step 2: Sum out H to get joint of Query and evidence



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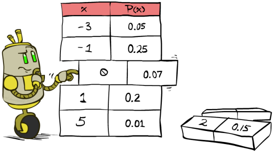

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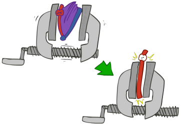

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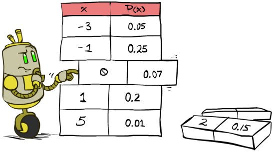

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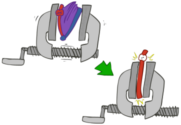

$P(Q, e_1, \ldots, e_k) =$
$\sum_{h_1, \ldots, h_r} P(Q, h_1, \ldots, h_r, e_1, \ldots, e_k)$

Step 3: Normalize.

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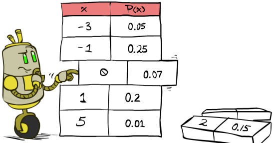

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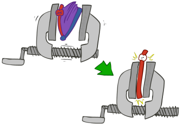

$P(Q, e_1, \ldots, e_k) = \sum_{h_1,\ldots,h_r} P(Q, h_1, \ldots, h_r, e_1, \ldots, e_k)$

Step 3: Normalize.

$Z = \sum_q P(Q = q, e_1, \ldots, e_k)$

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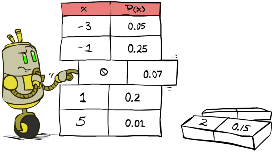

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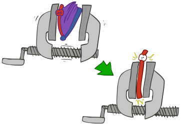

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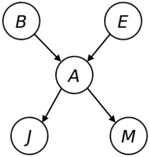
Step 3: Normalize.

$Z = \sum_q P(Q = q, e_1, \ldots, e_k)$

$P(Q|e_1, \ldots, e_k) = \frac{1}{Z} P(Q, e_1, \ldots, e_k)$

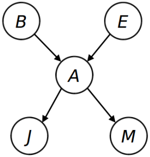
Inference by Enumeration in Bayes' Net



Given unlimited time, inference in BNs is easy.

$P(B|+j,+m) \propto P(B,+j,+m)$

Inference by Enumeration in Bayes' Net



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$$P(B \mid +j, +m) \propto P(B, +j, +m)$$

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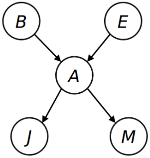

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$$P(B|+j,+m) \propto P(B,+j,+m)$$

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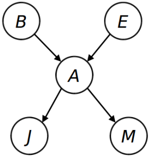
$$= \sum_{e,a} P(B)P(e)P(a|B,e)P(+j|a)P(+m|a)$$

Inference by Enumeration in Bayes' Net



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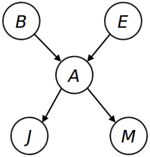

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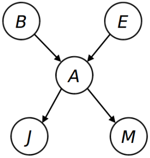
$$+P(B)P(+e)P(-a|B,+e)P(+j|-a)P(+m|-a)+$$

Inference by Enumeration in Bayes' Net

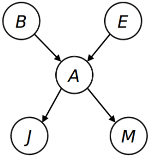


Given unlimited time, inference in BNs is easy.

$$
\begin{aligned}
P(B|+j,+m) &\propto P(B,+j,+m) \\
&= \textstyle\sum_{e,a} P(B,e,a,+j,+m) \\
&= \textstyle\sum_{e,a} P(B)P(e)P(a|B,e)P(+j|a)P(+m|a)
\end{aligned}
$$

$$
\begin{aligned}
&= P(B)P(+e)P(+a|B,+e)P(+j|+a)P(+m|+a) \\
&\quad + P(B)P(+e)P(-a|B,+e)P(+j|-a)P(+m|-a) + \\
&\quad P(B)P(-e)P(+a|B,-e)P(+j|+a)P(+m|+a)
\end{aligned}
$$

Inference by Enumeration in Bayes' Net



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$$+P(B)P(+e)P(-a|B,+e)P(+j|-a)P(+m|-a)+$$

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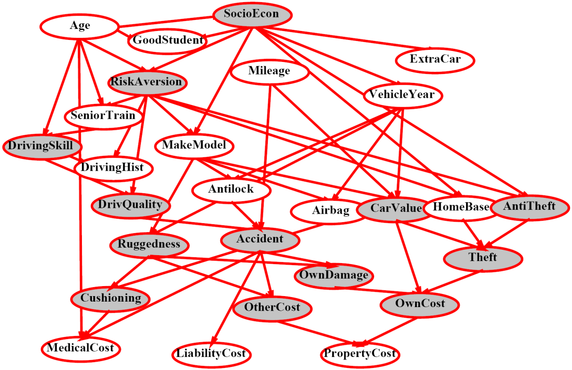
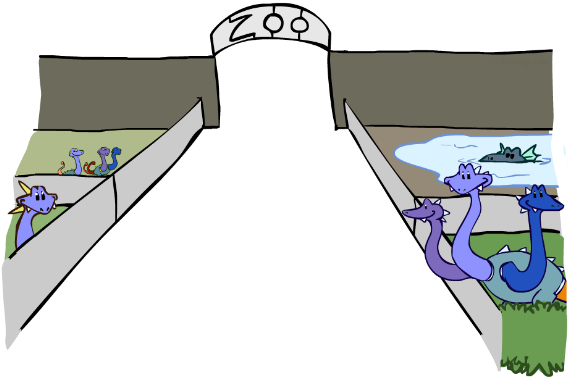
$$+P(B)P(-e)P(-a|B,-e)P(+j|-a)$$

Inference by Enumeration?

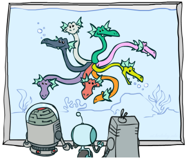


$$P(antilock|observedvariables) = ?$$

Factor Zoo



Factor Zoo I



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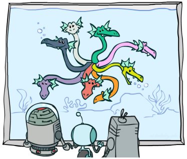
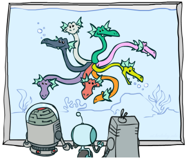


Joint distribution: P(X,Y)

- Entries P(x,y) for all x, y

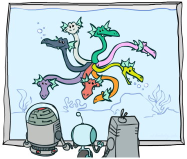
Factor Zoo I



Joint distribution: P(X,Y)

- Entries P(x,y) for all x, y
- Sums to?

Factor Zoo I



Joint distribution: P(X,Y)

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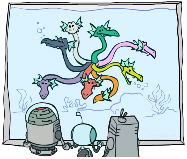
- Sums to? 1

Factor Zoo I

$P(T, W)$



Joint distribution: P(X,Y)

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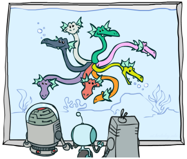
Factor Zoo I



Joint distribution: P(X,Y)

- Entries P(x,y) for all x, y
- Sums to? 1

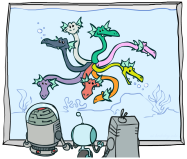
Selected joint: P(x,Y)

- Slice of joint distribution

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Factor Zoo I



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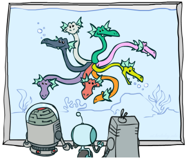
Selected joint: P(x,Y)

- Slice of joint distribution
- Entries P(x,y):

$P(T,W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Factor Zoo I



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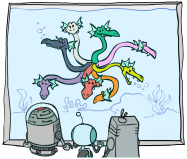

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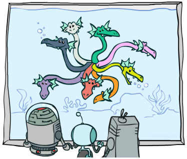

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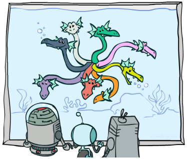

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- Sums to? P(x).

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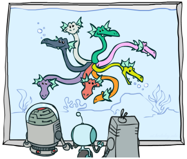
Factor Zoo I



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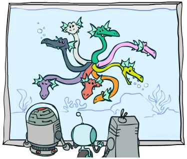
$P(cold, W)$

Factor Zoo I



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$P(cold, W)$

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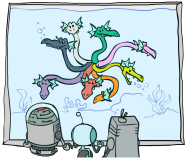
Factor Zoo I



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- Table P(cold, W)?
- Sums to? P(x).

$P(T, W)$

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hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$P(cold, W)$

T	W	P
cold	sun	0.2
cold	rain	0.3

Number of capital letters (variables)= dimensionality of the table

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y
- Sum of entries?

Factor Zoo II

Single conditional: $P(Y|x)$

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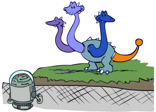
- Sum of entries? 1

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y
- Sum of entries? 1

T	W	P
cold	sun	0.4
cold	rain	0.6



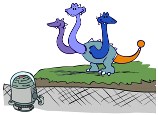
Family of conditionals:

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y
- Sum of entries? 1

T	W	P
cold	sun	0.4
cold	rain	0.6



Family of conditionals:

$P(X|Y)$

- Multiple conditionals

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y
- Sum of entries? 1

T	W	P
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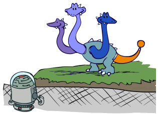

Family of conditionals:

$P(X|Y)$

- Multiple conditionals
- Entries $P(x|y)$ for all

x, y

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y
- Sum of entries? 1

T	W	P
cold	sun	0.4
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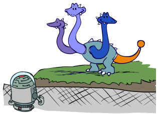

Family of conditionals:

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Factor Zoo II

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T	W	P
cold	sun	0.4
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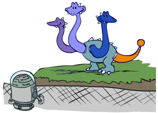


Family of conditionals:

$P(X|Y)$

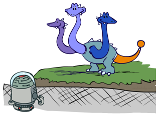
- Multiple conditionals
- Entries $P(x|y)$ for all x, y
- Sum? $|Y|$

Factor Zoo II

Single conditional: $P(Y|x)$

- Entries $P(y|x)$ for fixed x, all y
- Sum of entries? 1

T	W	P
cold	sun	0.4
cold	rain	0.6

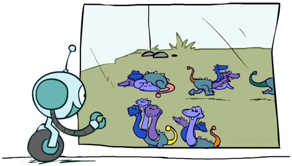


Family of conditionals:

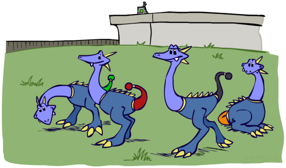
$P(X|Y)$

- Multiple conditionals
- Entries $P(x|y)$ for all x, y
- Sum? $|Y|$

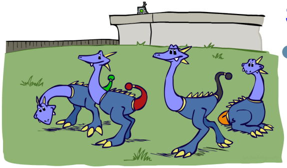
T	W	P
hot	sun	0.8
hot	rain	0.2
cold	sun	0.4
cold	rain	0.6



Factor Zoo III



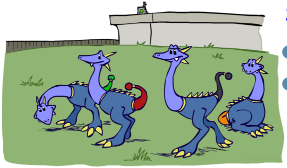
Factor Zoo III



Specified family: $P(y|X)$

- Entries $P(y|x)$ for fixed y, but for all x

Factor Zoo III



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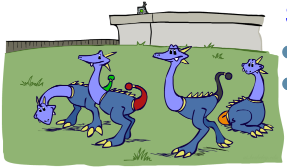
- Entries $P(y|x)$ for fixed y, but for all x
- Sums to?

Factor Zoo III



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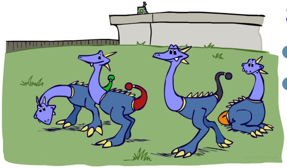
Factor Zoo III



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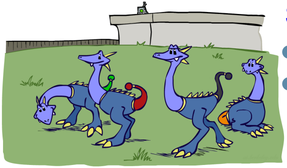


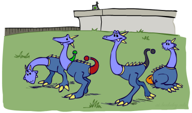
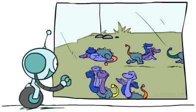
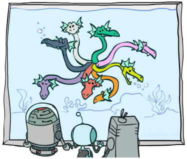
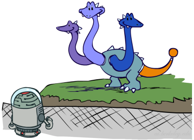
Specified family: $P(y|X)$

- Entries $P(y|x)$ for fixed y, but for all x
- Sums to? who knows!

$P(rain|T)$

T	W	P
hot	rain	0.2
cold	rain	0.6

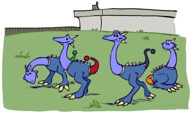
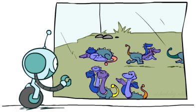
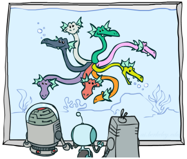
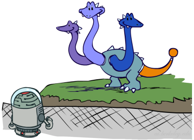
Factor Zoo Summary

In general, when we write $P(Y_1, \ldots, Y_N | X_1, \ldots, X_M)$

- It is a “factor,” a multi-dimensional array

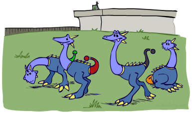
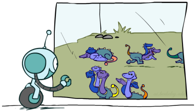
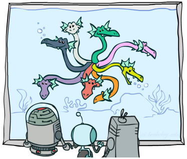
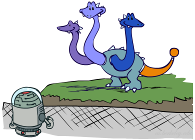
Factor Zoo Summary

In general, when we write $P(Y_1, \ldots, Y_N | X_1, \ldots, X_M)$

- It is a "factor," a multi-dimensional array
- Its values are $P(y_1, \ldots, y_N | x_1, \ldots, x_M)$

Factor Zoo Summary

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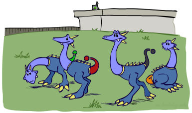
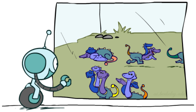
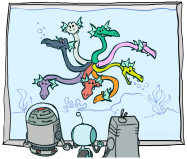
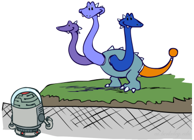
Factor Zoo Summary

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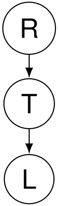
Example: Traffic Domain

$P(R)$

T	L
+r	0.1
-r	0.9



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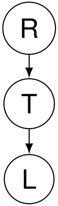


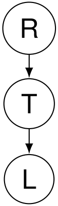
$P(R)$

T	L
+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

Example: Traffic Domain



$P(R)$

T	L
+r	0.1
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$P(T|R)$

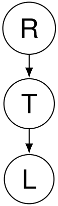
+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Random Variables

Example: Traffic Domain



$P(R)$

T	L
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$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

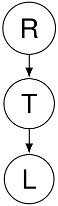
$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Random Variables

- R: Raining

Example: Traffic Domain



$P(R)$

T	L
+r	0.1
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$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

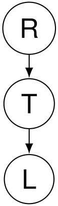
$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Random Variables

- R: Raining
- T: Traffic

Example: Traffic Domain



$P(R)$

T	L
+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

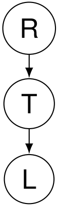
$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Random Variables

- R: Raining
- T: Traffic
- L: Late for class!

Example: Traffic Domain



$P(R)$

T	L
+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

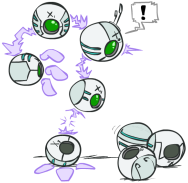
$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Random Variables

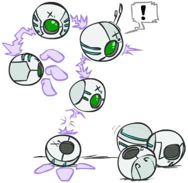
- R: Raining
- T: Traffic
- L: Late for class!

Inference by Enumeration: Procedural Outline



Inference by Enumeration: Procedural Outline

Track objects called factors.



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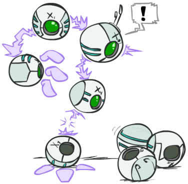

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T	L
+r	0.1
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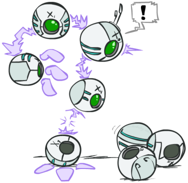

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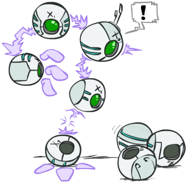

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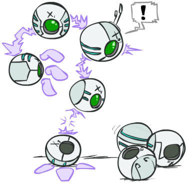

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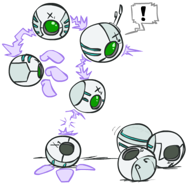

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Procedure: Join all factors, then eliminate all hidden variables

Operation 1: Join Factors

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First basic operation: joining factors

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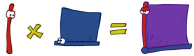

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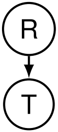
Example: Join on R



$P(R)$

T	L
+r	0.1
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$\times$

$P(T|R)$

+r	+t	0.8
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$\rightarrow$

R,T

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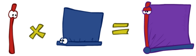

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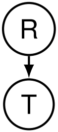

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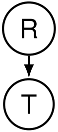

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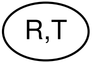

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+r	+t	0.08
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- Computation for each entry: pointwise products

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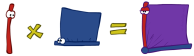

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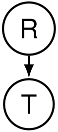

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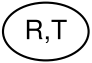

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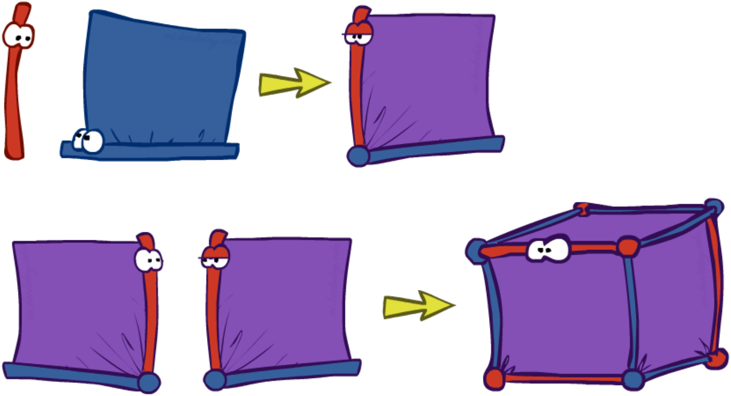

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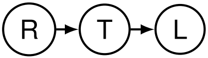

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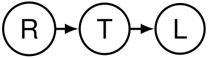

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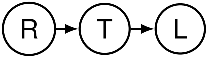

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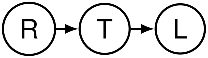

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Join R

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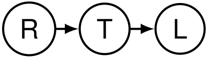

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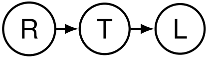

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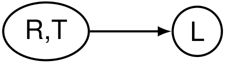

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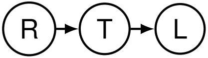

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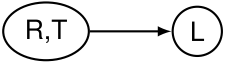

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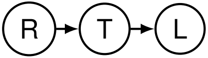

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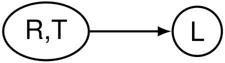

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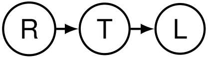

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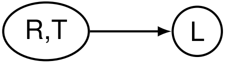

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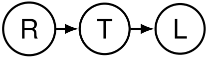

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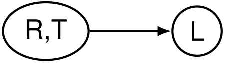

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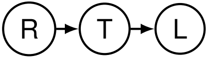

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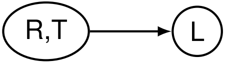

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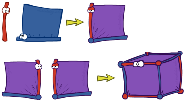
Join T.

$\rightarrow$



Example: Multiple Joins

$R \rightarrow T \rightarrow L$

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Join R $\rightarrow$

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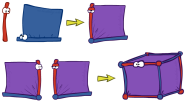
Join T. $\rightarrow$

R,T,L



$P(R, T, L)$

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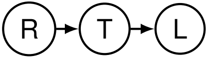

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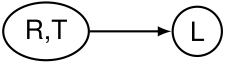
Join R

$\rightarrow$



$P(R,T)$

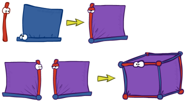
+r	+t	0.08
+r	-t	0.02
-r	+t	0.09
-r	-t	0.81

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Join T.

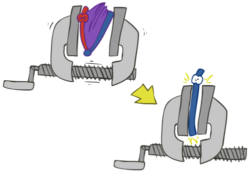
$\rightarrow$



$P(R,T,L)$

+r	+t	+l	0.024
+r	+t	-l	0.056
+r	-t	+l	0.002
+r	-t	-l	0.018
-r	+t	+l	0.027
-r	+t	-l	0.063
-r	-t	+l	0.081
-r	-t	-l	0.729

Operation 2: Eliminate



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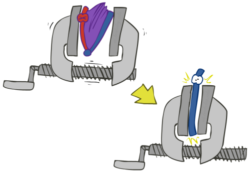
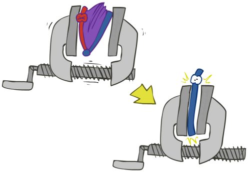


Second basic operation: marginalization

Operation 2: Eliminate

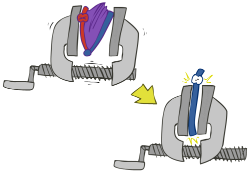


Second basic operation: marginalization

Take a factor and sum out a variable

- Shrinks a factor to a smaller one

Operation 2: Eliminate



Second basic operation: marginalization

Take a factor and sum out a variable

- Shrinks a factor to a smaller one
- A projection operation

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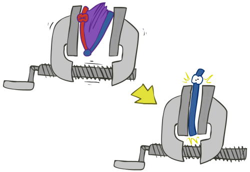

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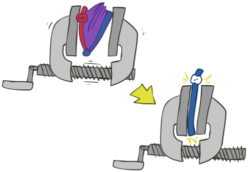

Second basic operation: marginalization

Take a factor and sum out a variable

- Shrinks a factor to a smaller one
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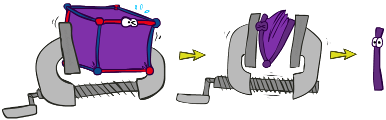
Example:

+r	+t	0.08
+r	-t	0.02
-r	+t	0.09
-r	-t	0.81

Sum R $\rightarrow$

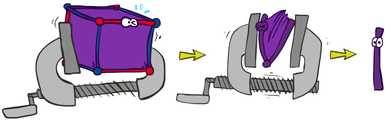
+t	0.17
-t	0.83

Multiple Elimination



R,T,L

Multiple Elimination



R,T,L

$P(R, T, L)$

+r	+t	+l	0.024
+r	+t	-l	0.056
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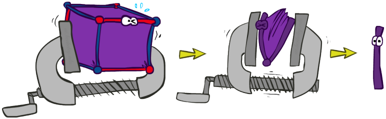
-r	-t	+l	0.081
-r	-t	-l	0.729

Sum out R $\rightarrow$

T,L

Multiple Elimination



R,T,L

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+r	+t	-l	0.056
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-r	+t	-l	0.063
-r	-t	+l	0.081
-r	-t	-l	0.729

Sum out R $\rightarrow$

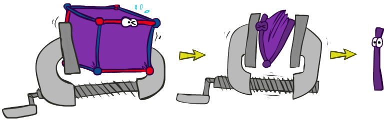
T,L

+t	+l	0.026
+t	-l	0.119
-t	+l	0.083
-t	-l	0.747

Sum out T $\rightarrow$

L

Multiple Elimination



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-r	-t	+l	0.081
-r	-t	-l	0.729

Sum out R $\rightarrow$

T,L

+t	+l	0.026
+t	-l	0.119
-t	+l	0.083
-t	-l	0.747

Sum out T $\rightarrow$

L

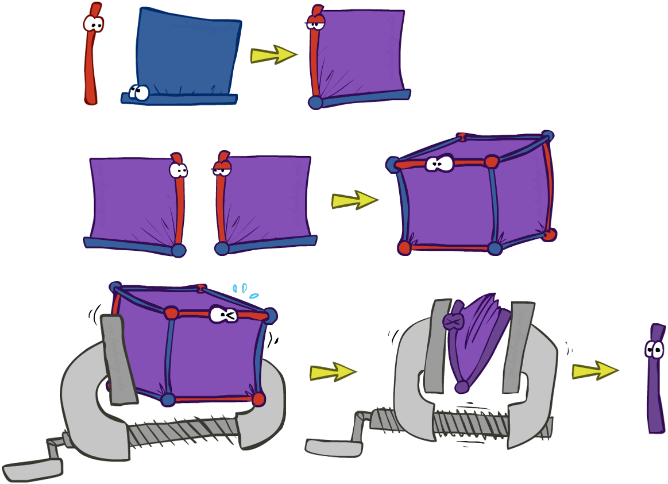
+l	0.134
-l	0.866

Thus Far: Multiple Join, Multiple Eliminate

Inference by Enumeration!

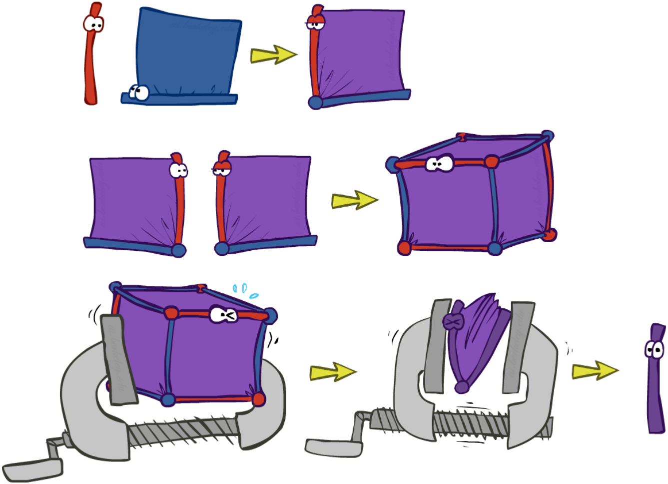
Thus Far: Multiple Join, Multiple Eliminate

Inference by Enumeration!



Thus Far: Multiple Join, Multiple Eliminate

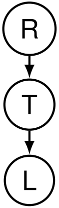
Inference by Enumeration!



Example: Traffic Domain (revisited)

$P(R)$

T	L
+r	0.1
-r	0.9



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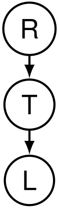

$P(R)$

T	L
+r	0.1
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$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

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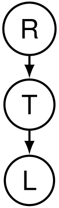

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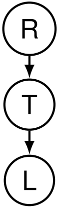
-r	+t	0.1
-r	-t	0.9

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Random Variables

Example: Traffic Domain (revisited)



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Random Variables

- R: Raining

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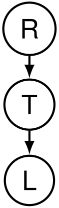

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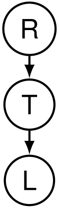
Random Variables

- R: Raining
- T: Traffic

Example: Traffic Domain (revisited)



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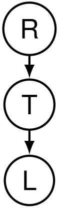
Random Variables

- R: Raining
- T: Traffic
- L: Late for class!

Example: Traffic Domain (revisited)



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Random Variables

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$P(L)$

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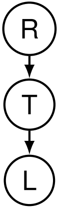

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$P(L) = \sum_{r,t} P(r,t,L)$

Random Variables

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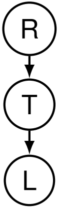

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Random Variables

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- T: Traffic
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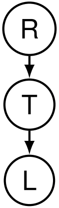
$P(L) = \sum_{r,t} P(r,t,L) = \sum_{r,t} P(r)P(t|r)P(L|t).$

Example: Traffic Domain (revisited)



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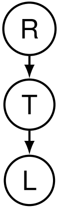
or

Example: Traffic Domain (revisited)



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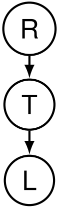
or $P(L) = \sum_t P(t)P(L|t)$ and $P(T) = \sum_r P(r)P(T|R)$.

Example: Traffic Domain (revisited)



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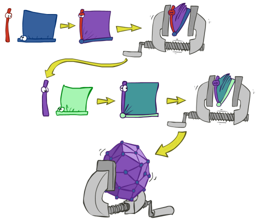
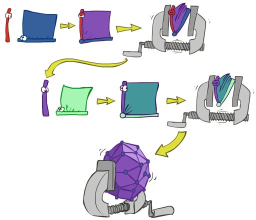
Inference by Enumeration vs. Variable Elimination

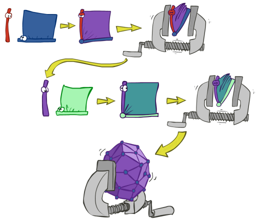


Inference by Enumeration vs. Variable Elimination



Why is inference by enumeration so slow?

Inference by Enumeration vs. Variable Elimination



Why is inference by enumeration so slow?

- Join whole joint distribution before sum out the hidden variables

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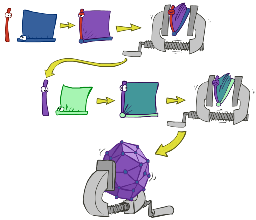

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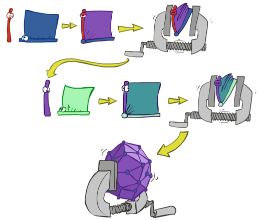

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Idea: interleave joining and marginalizing!

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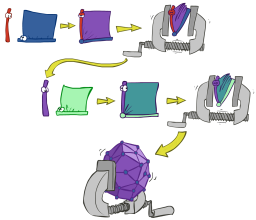

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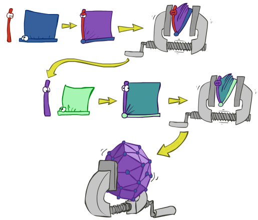

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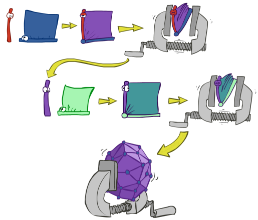
- Still NP-hard, but usually much faster than inference by enumeration

Inference by Enumeration vs. Variable Elimination



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