AI (in the news)

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CS 188: Artificial Intelligence

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Constraint Satisfaction Problems II

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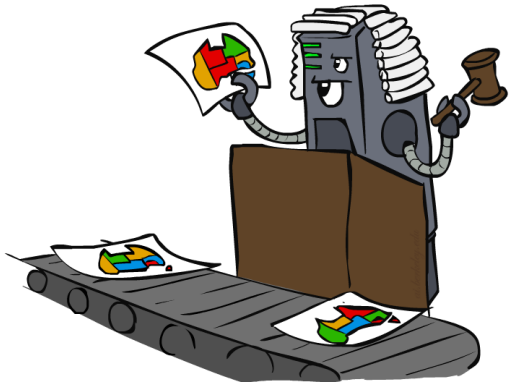
Constraint Satisfaction Problems II

Standard Search Formulation

http://bit.ly/3GEMokD
CSP: Variables, domains, and constraints.

Standard search formulation of CSPs



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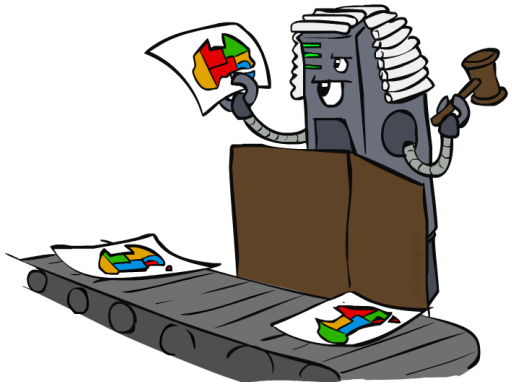

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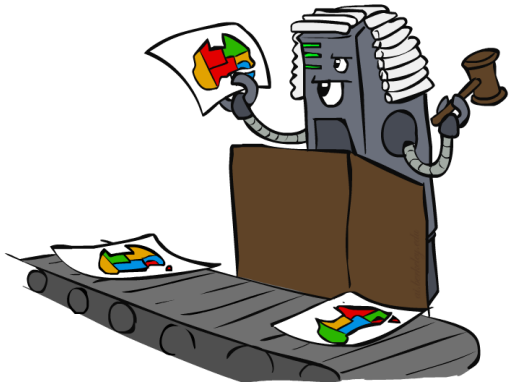

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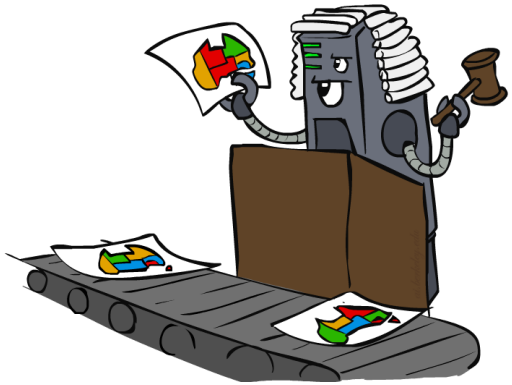

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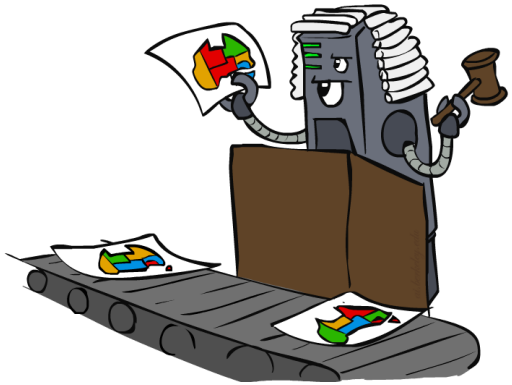

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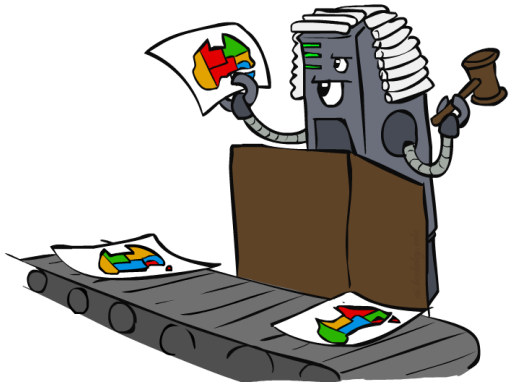

Standard search formulation of CSPs

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Goal test:
Complete satisfying assignment.

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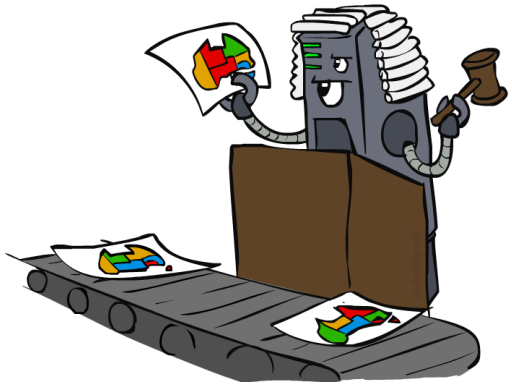

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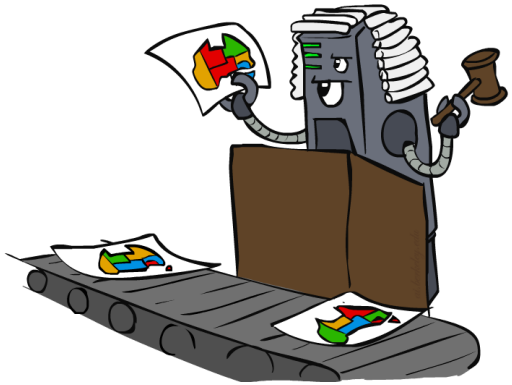

Standard search formulation of CSPs

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We'll remind ourselves of straightforward, naive approach, and then improve.

Search Methods

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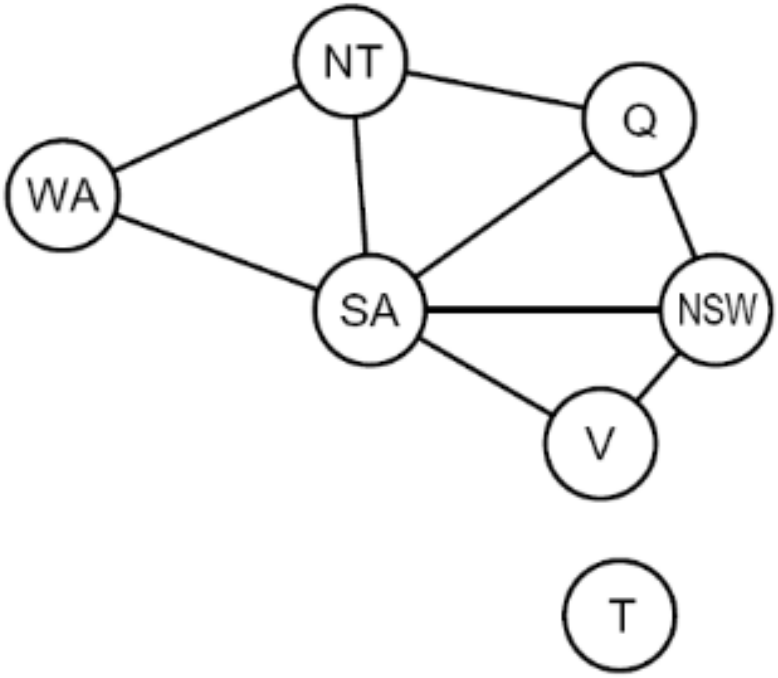

What would BFS do?

Search Methods

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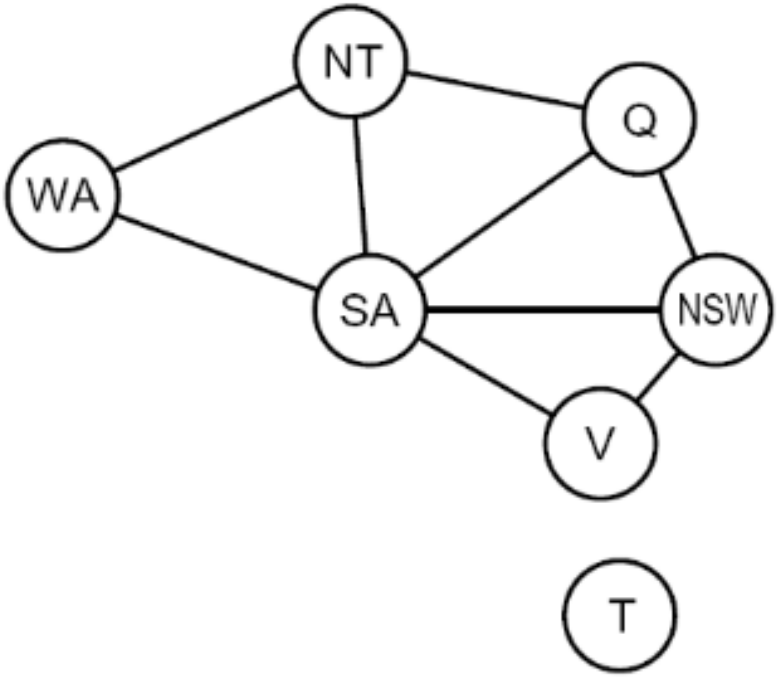

What would BFS do?

What would DFS do?

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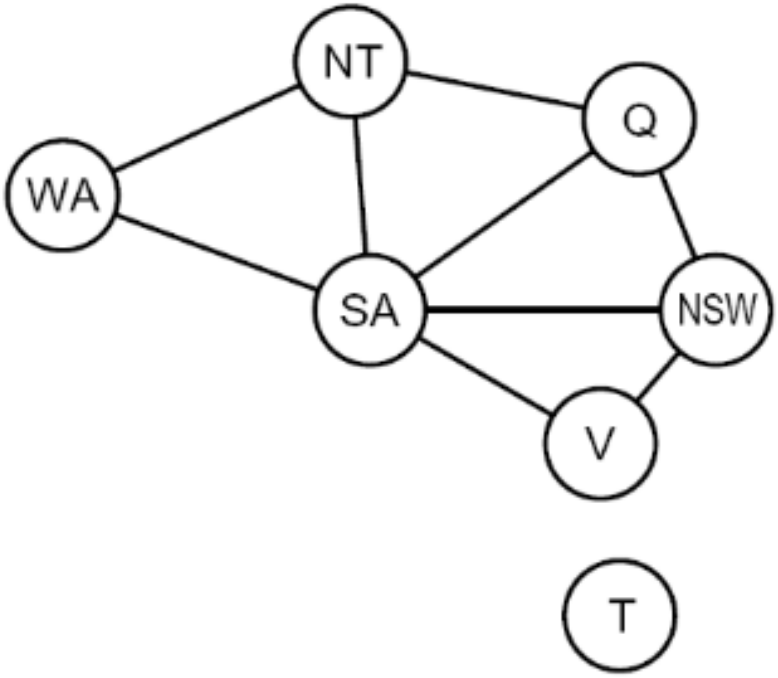
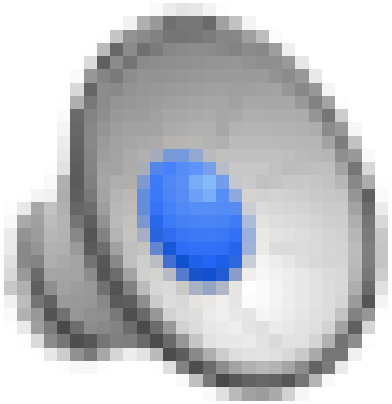


What would BFS do?

What would DFS do?

What problems does naive search have?

Video of Demo Coloring – DFS



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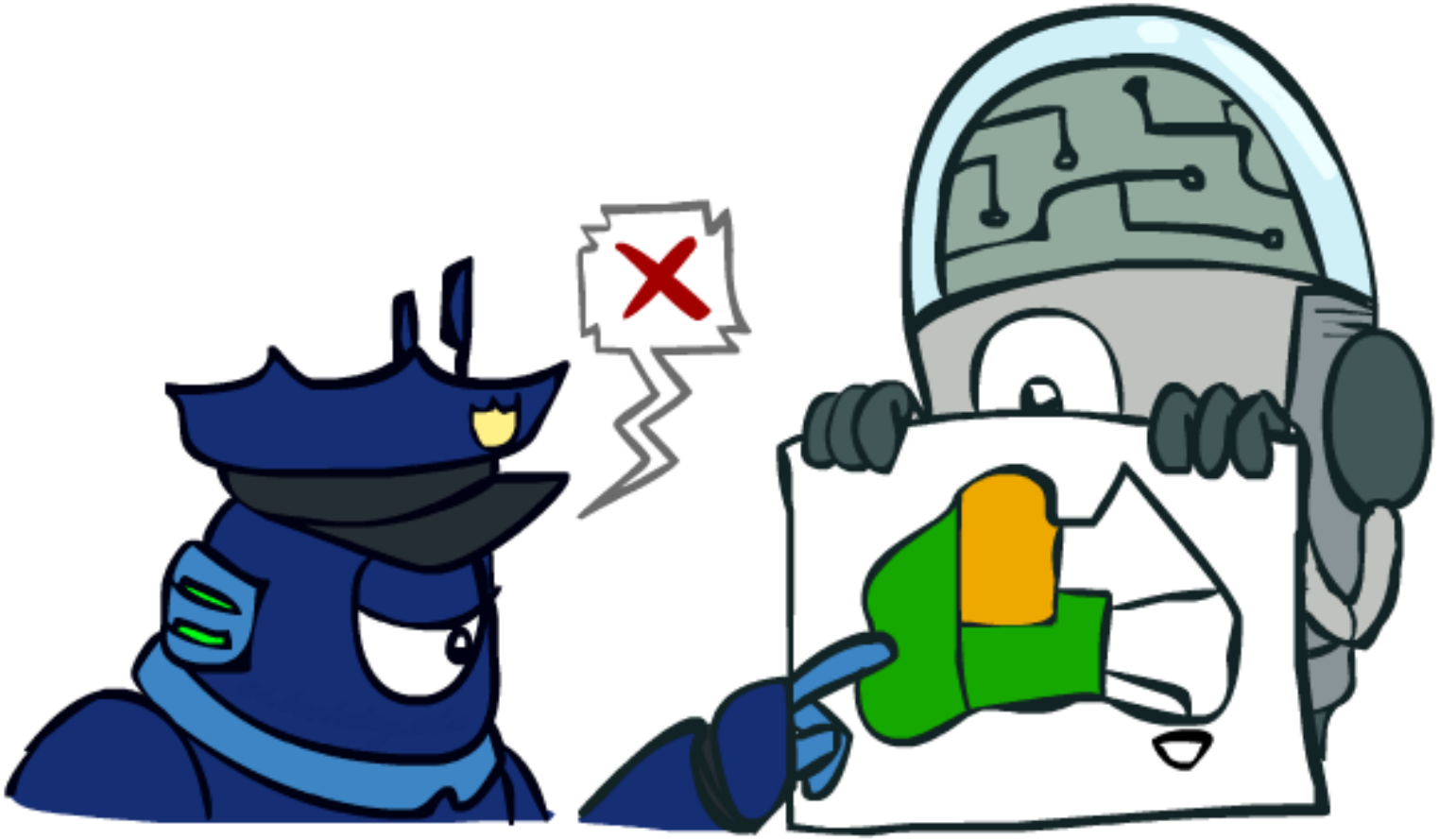

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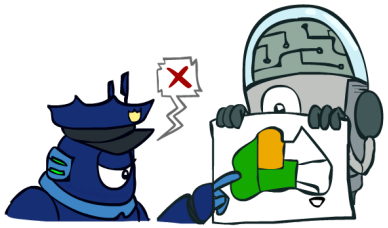
Backtracking search is the basic uninformed algorithm for solving CSPs



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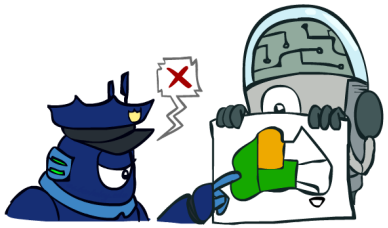

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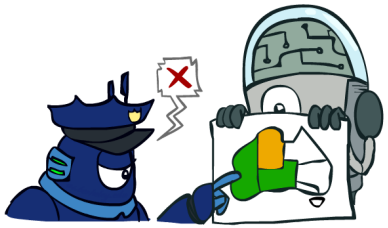

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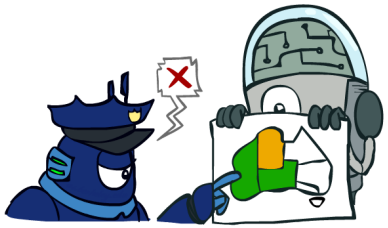

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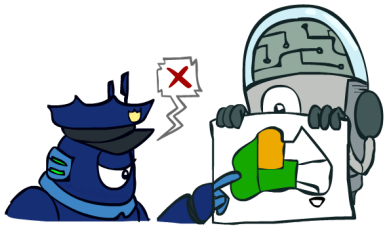

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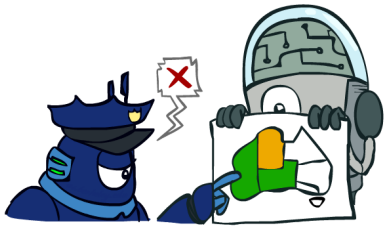

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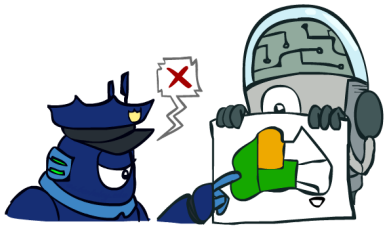

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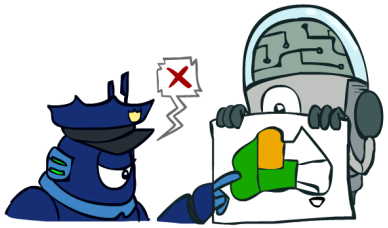

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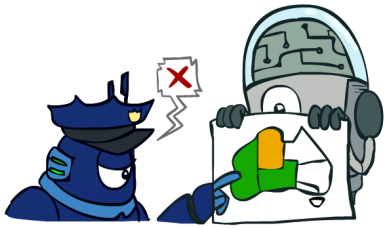

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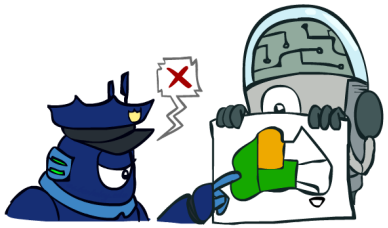

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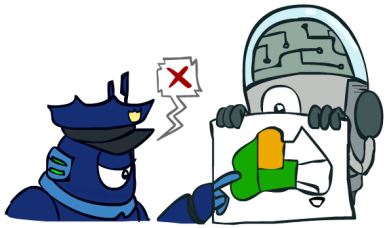

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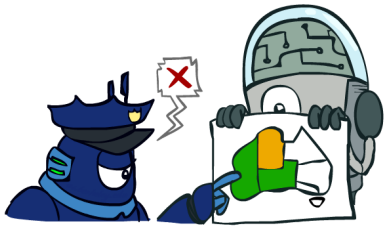

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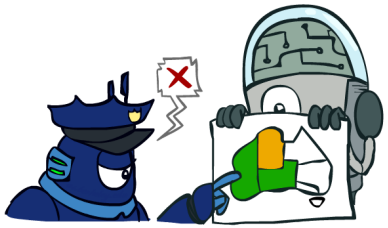

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"Incremental goal test"

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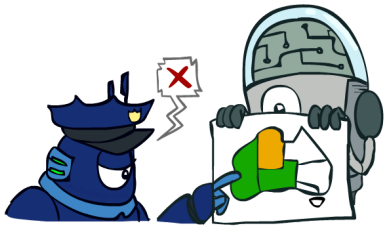

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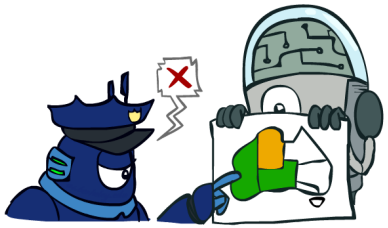

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Can solve n-queens for $n \approx 25$

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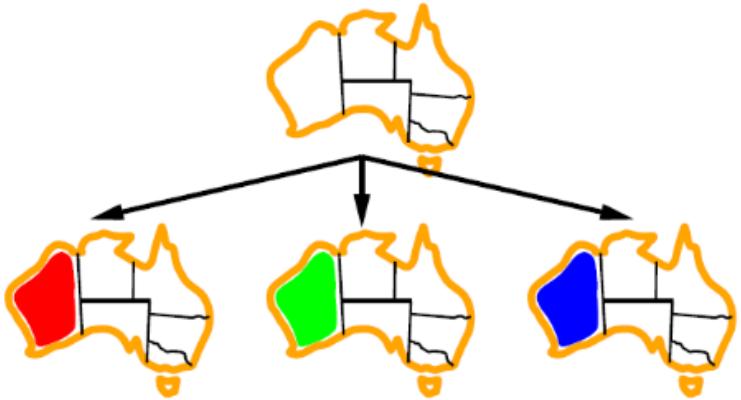

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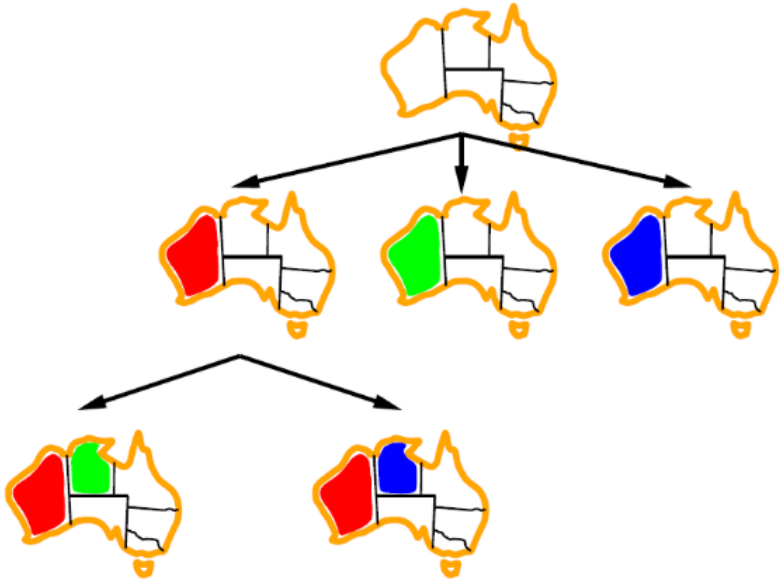
http://bit.ly/3GEMo



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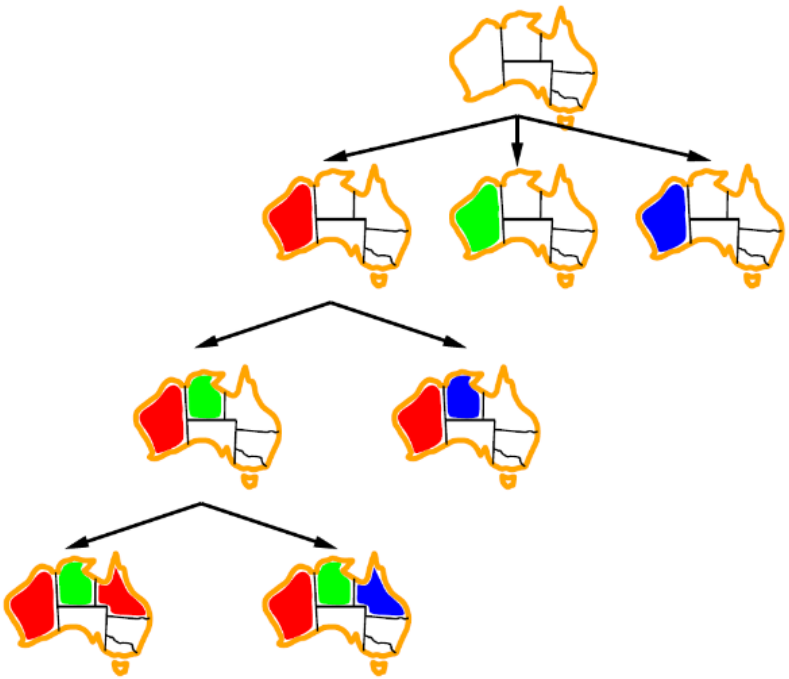

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   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
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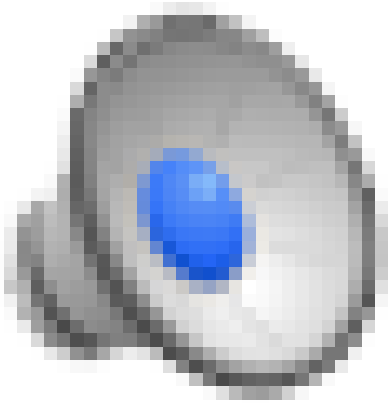
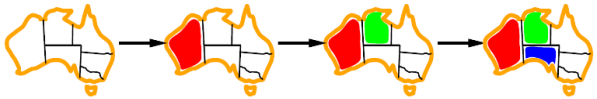
Backtracking = DFS + variable-ordering + fail-on-violation
What are the choice points?

Video of Demo Coloring – Backtracking



CSP-Backtracking Search



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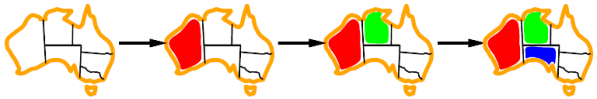

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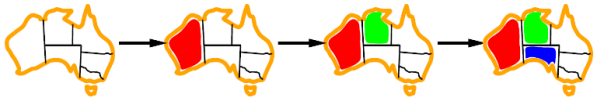

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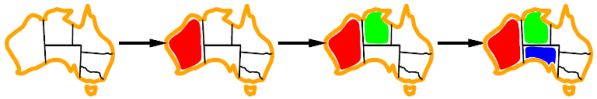
Todo:

CSP-Backtracking Search



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Todo:

Better “fail on violation”. Filtering.

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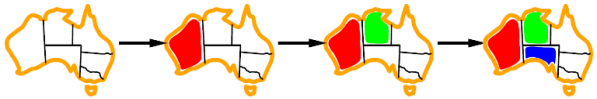

CSP-Backtracking = DFS + fail-on-violation + variable-ordering

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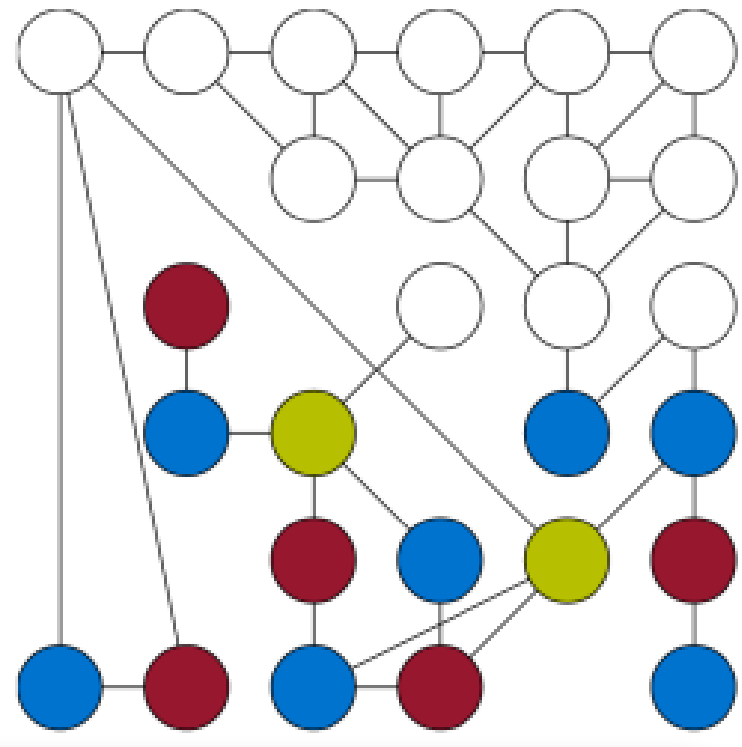
Pick “better” variable orderings and value orderings.

An issue.



Consider the partially completed CSP assignment.

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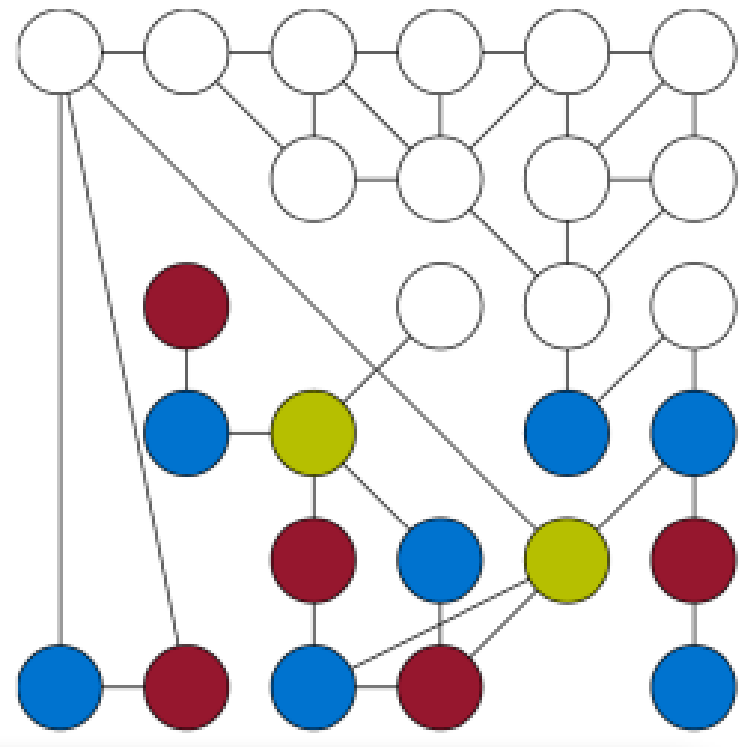

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Decisions made bottom-up, left-to-right.

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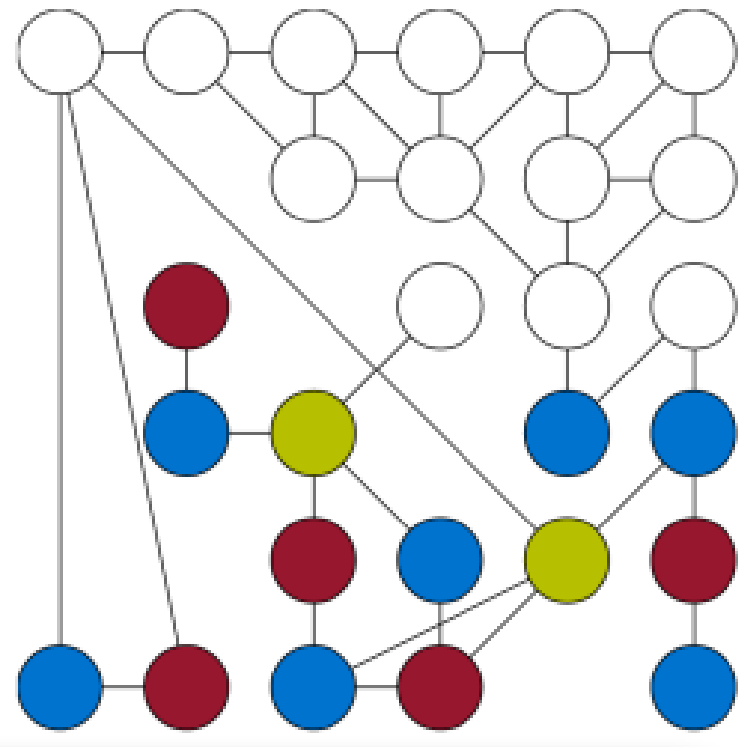

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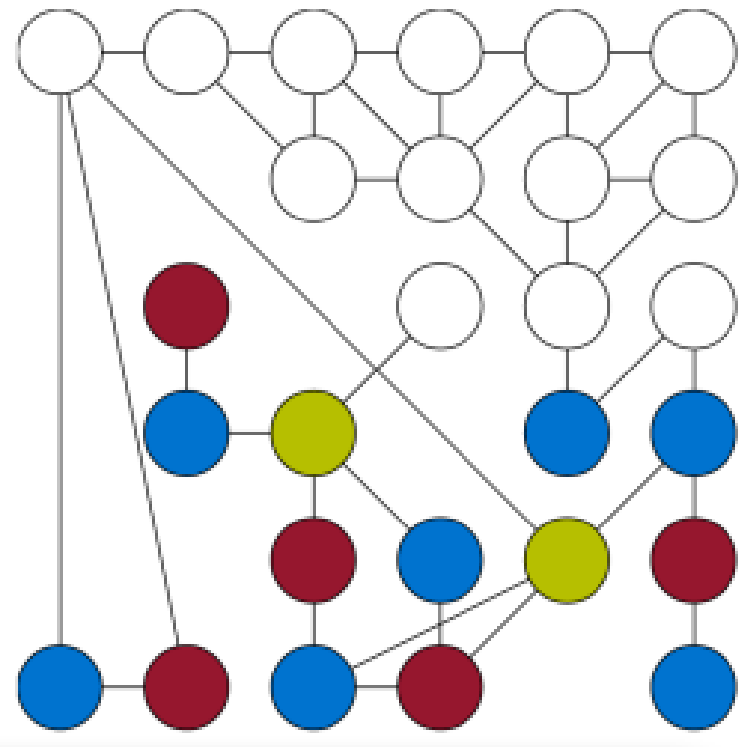

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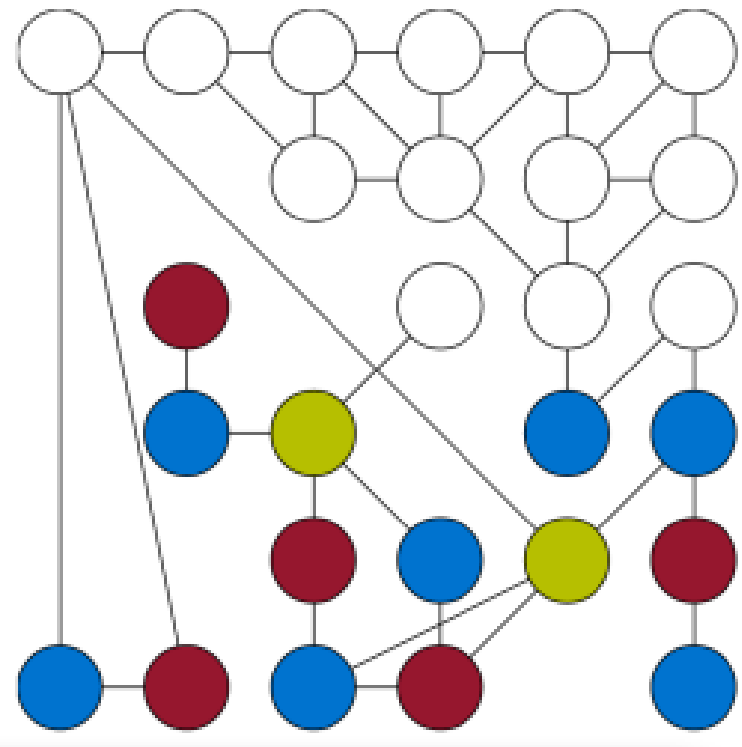

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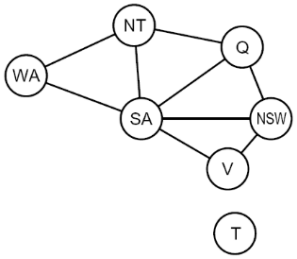
Bonus: How many decisions before CSP-Backtracking search realizes its error?

Filtering

Filtering: Forward Checking



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 Reduce domains for unassigned variables

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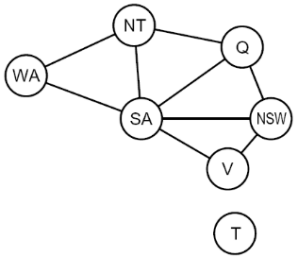

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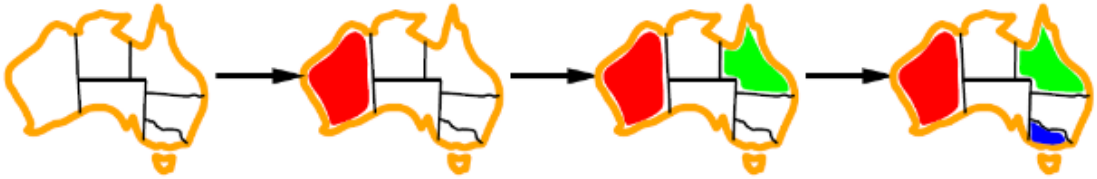
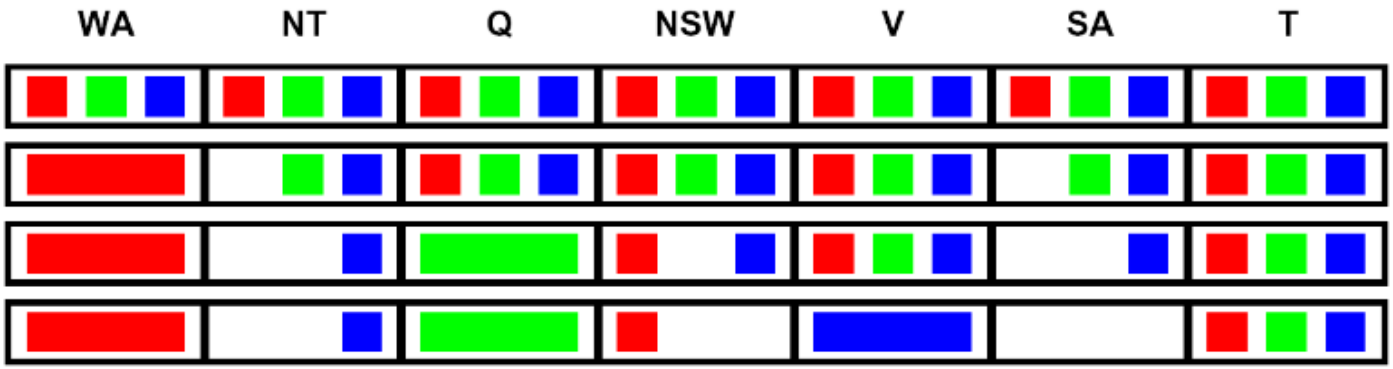
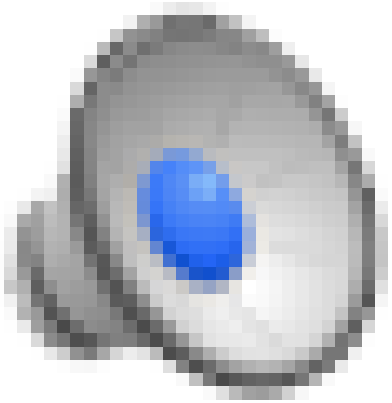
Forward checking:
Remove values that violate constraint in existing assignment





Video of Demo Coloring – Backtracking with Forward Checking



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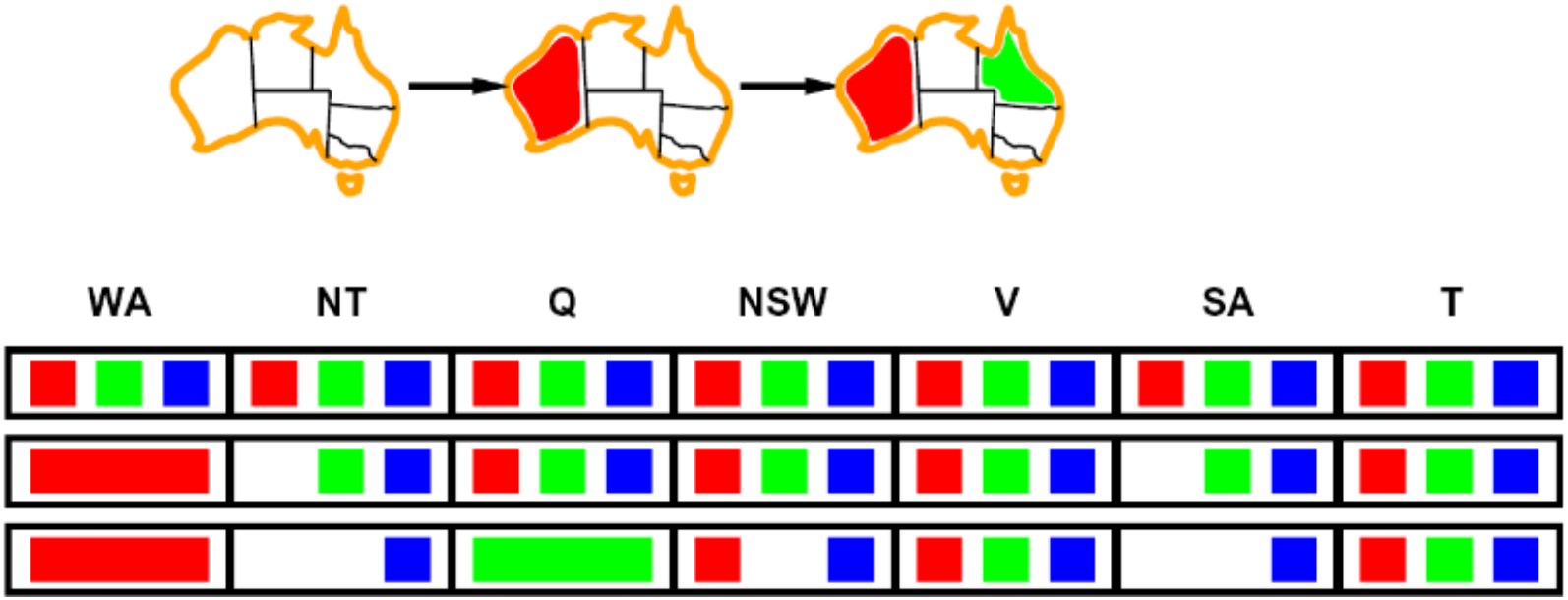

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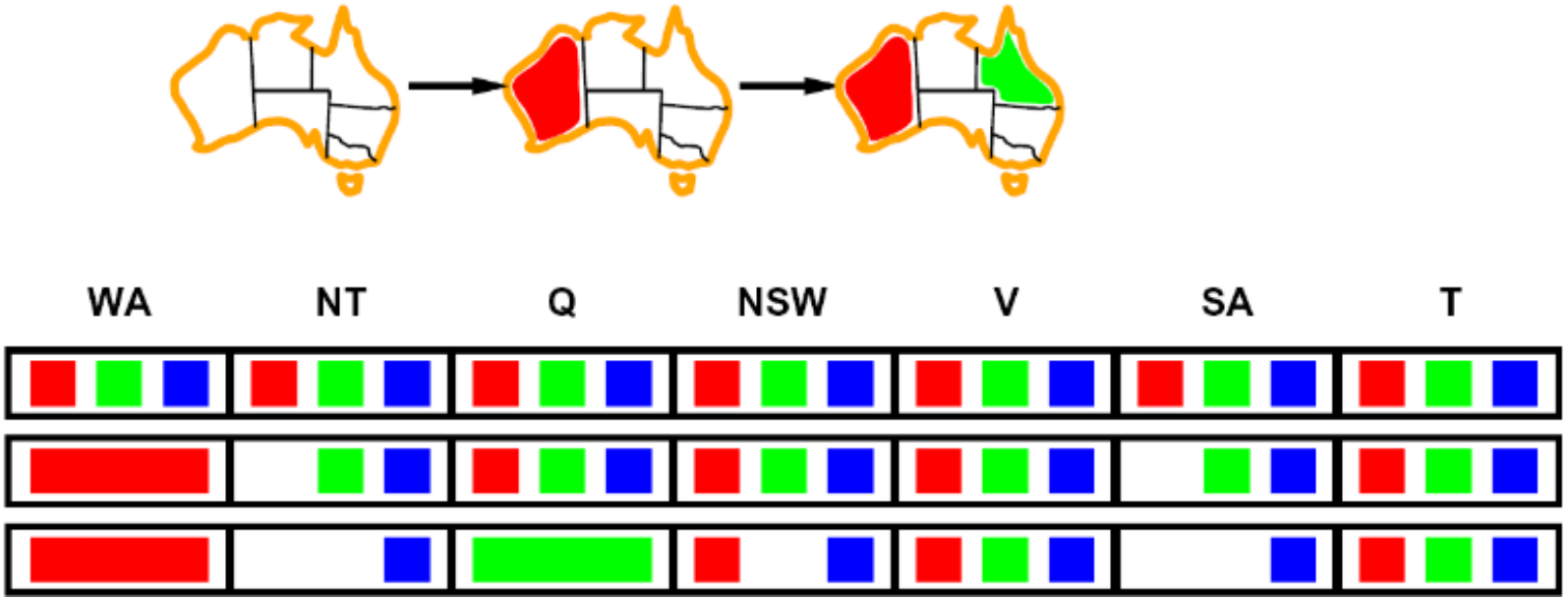

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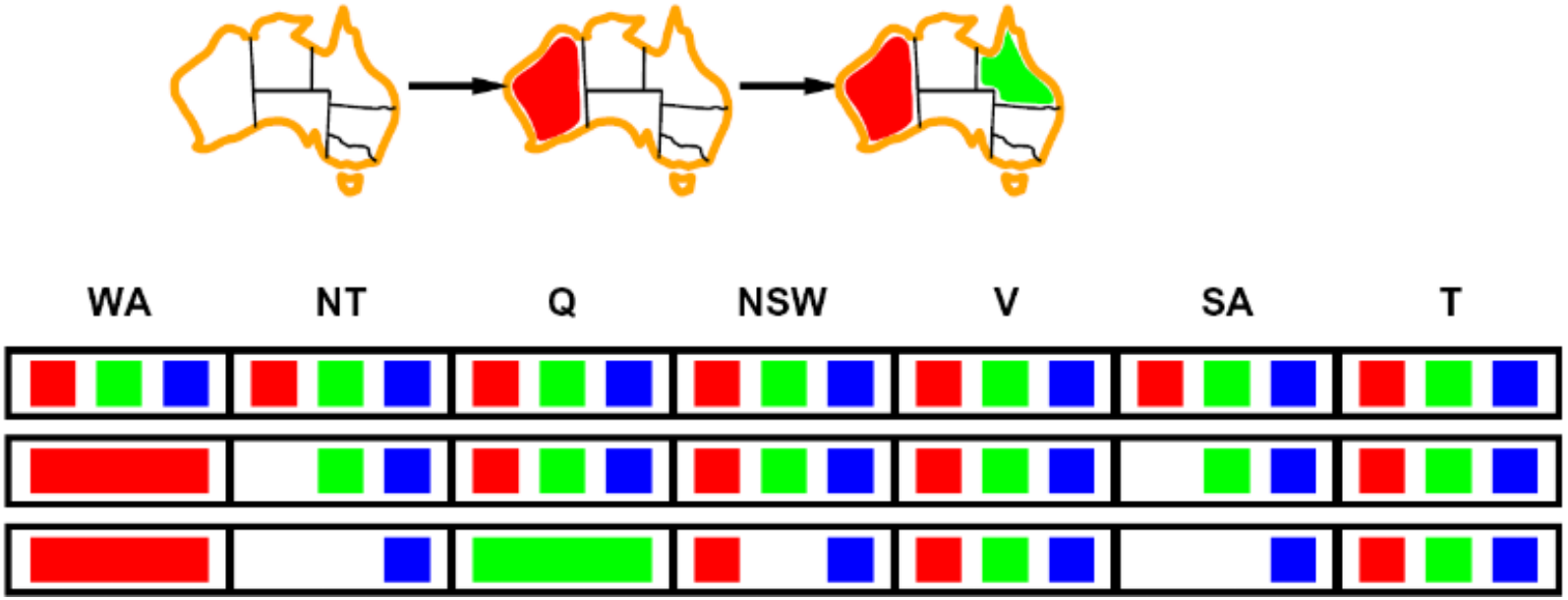

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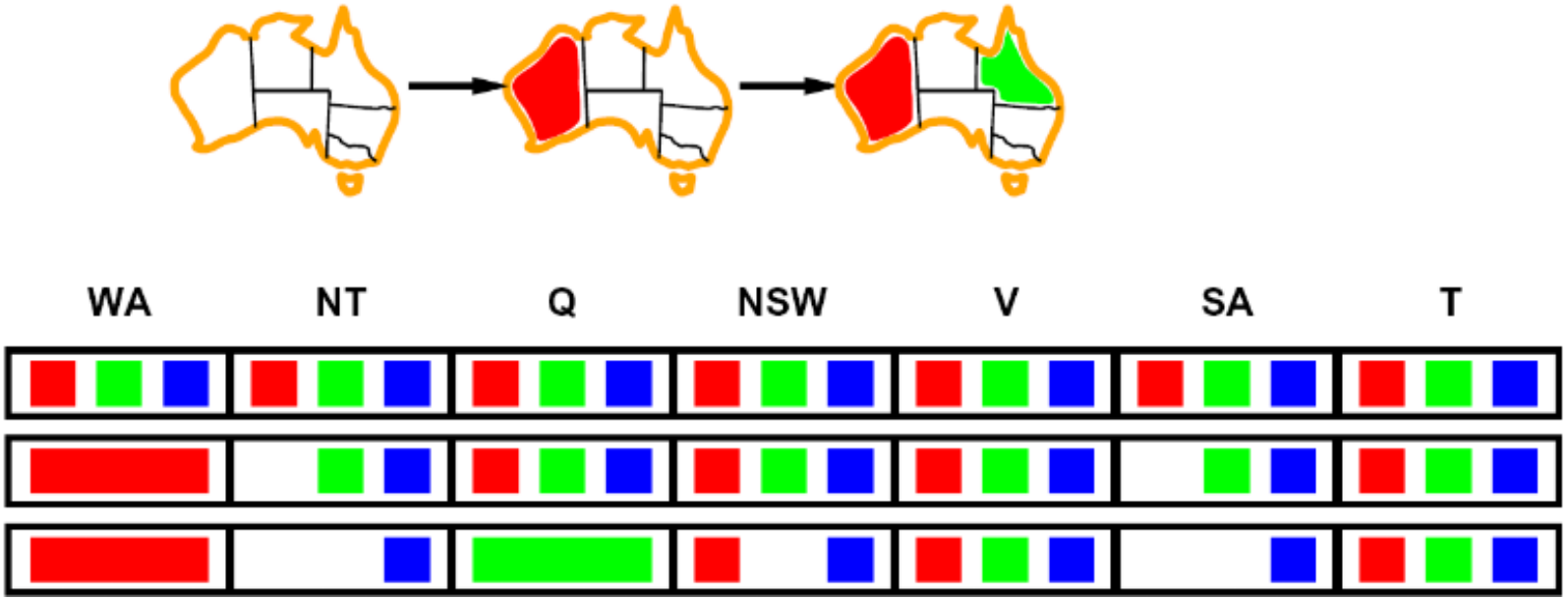

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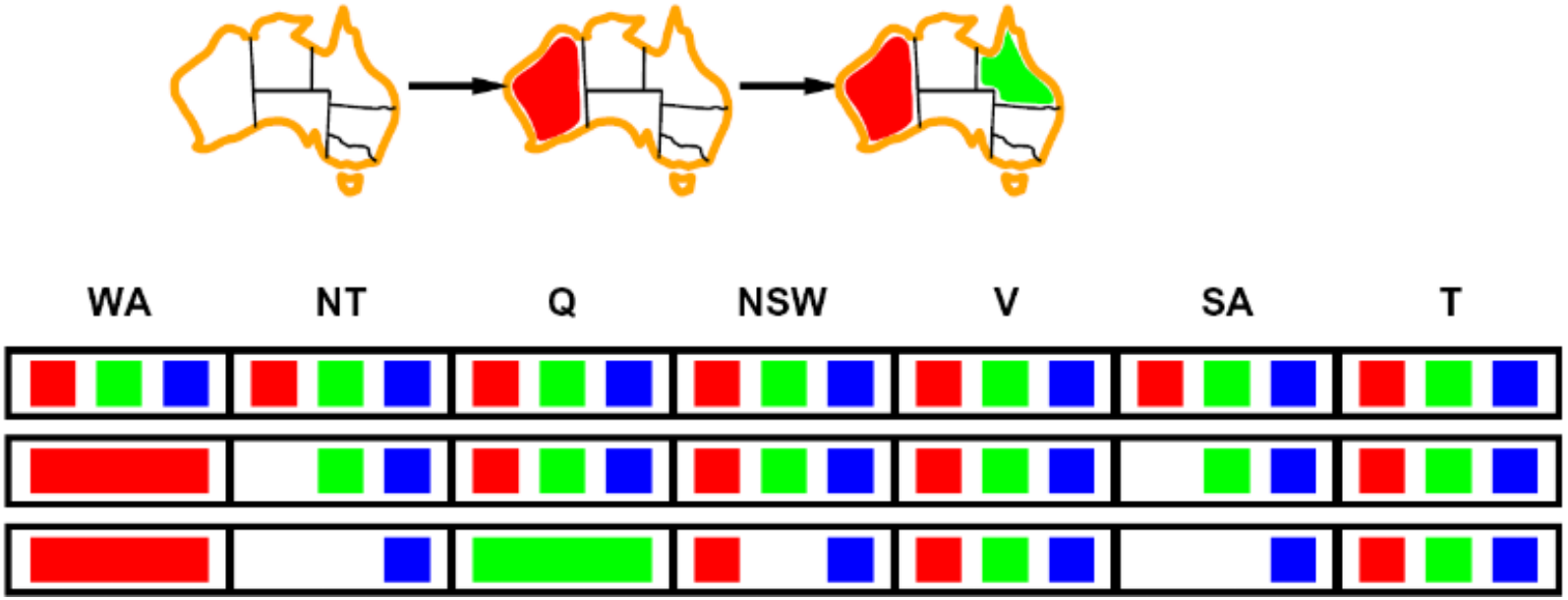

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Consistency of A Single Arc

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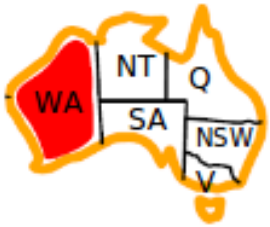

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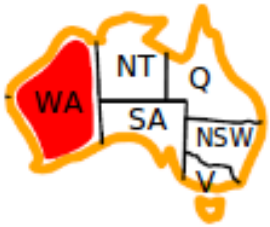

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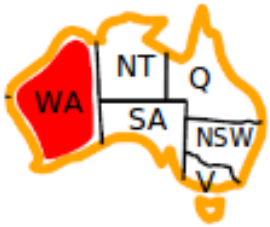

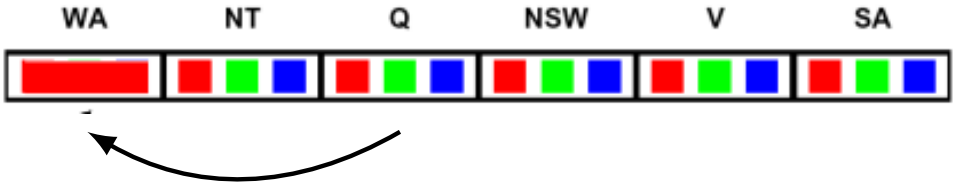

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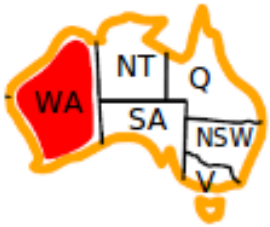

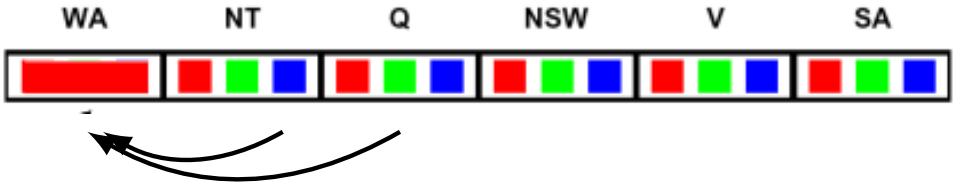

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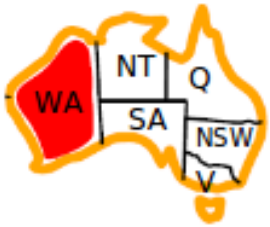

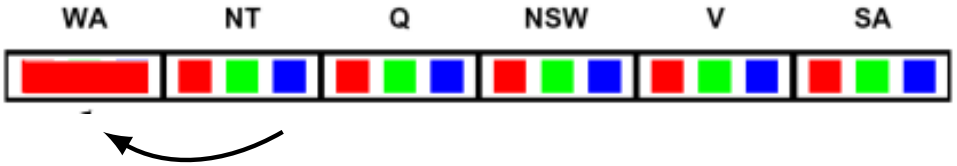

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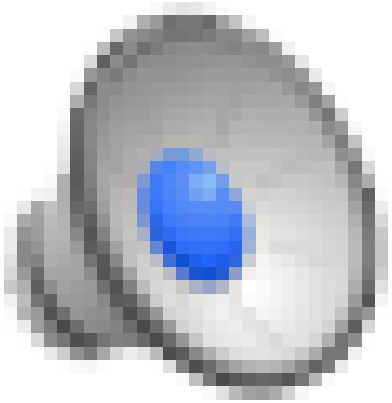
Video of Demo Coloring – Backtracking with Forward Checking – Complex Graph



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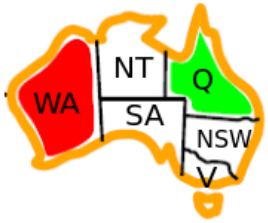

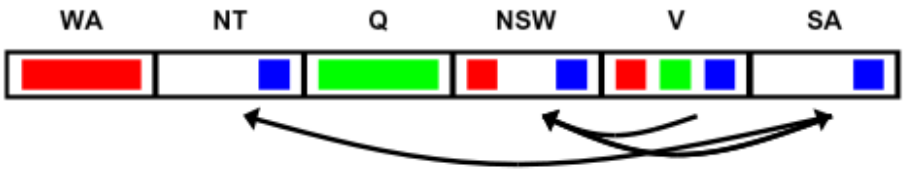

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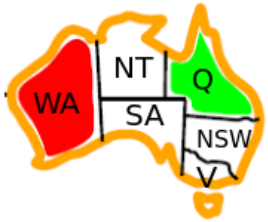

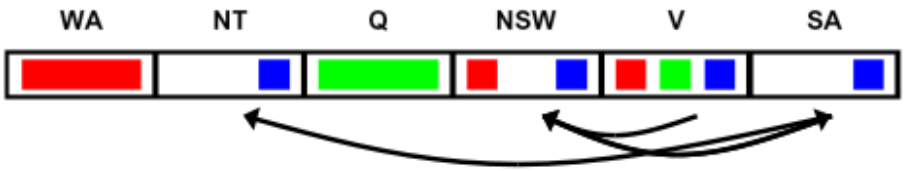

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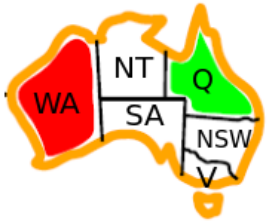

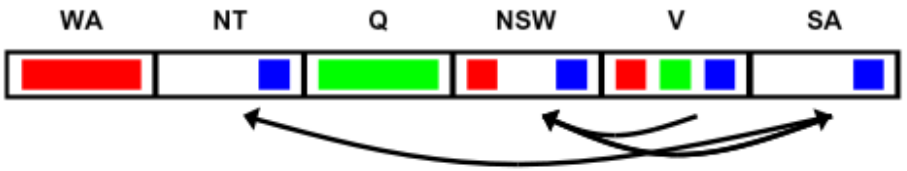

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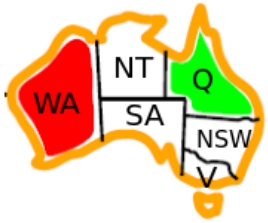

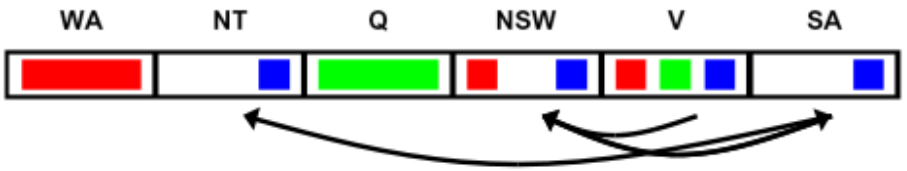

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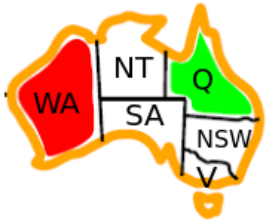

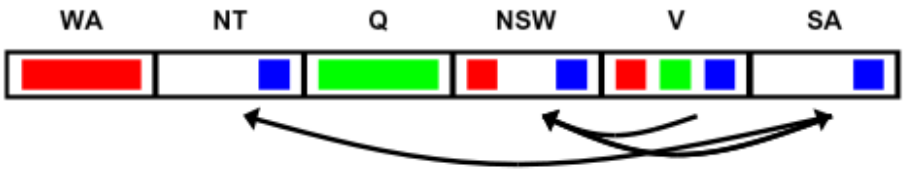

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What's the downside of enforcing arc consistency?

Enforcing Arc Consistency in a CSP

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function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
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   for each x in DOMAIN[Xi] do
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Runtime: $O(n^2 d^3)$, can be improved to $O(n^2 d^2)$.

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....but detecting all possible future problems is NP-hard – why?

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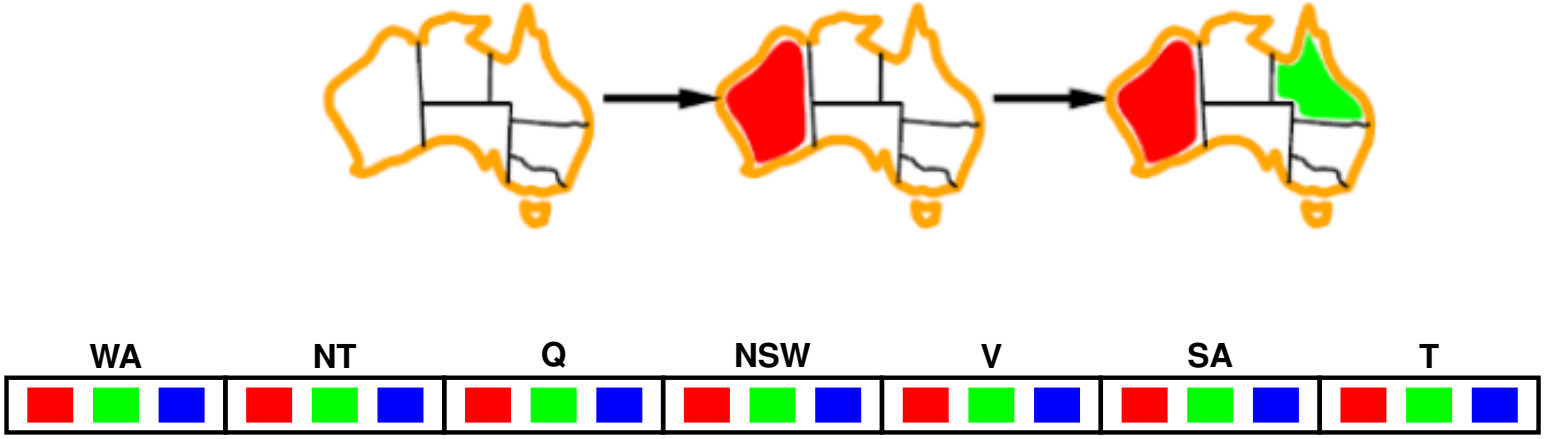

[Demo: CSP applet (made available by aispace.org) – n-queens]

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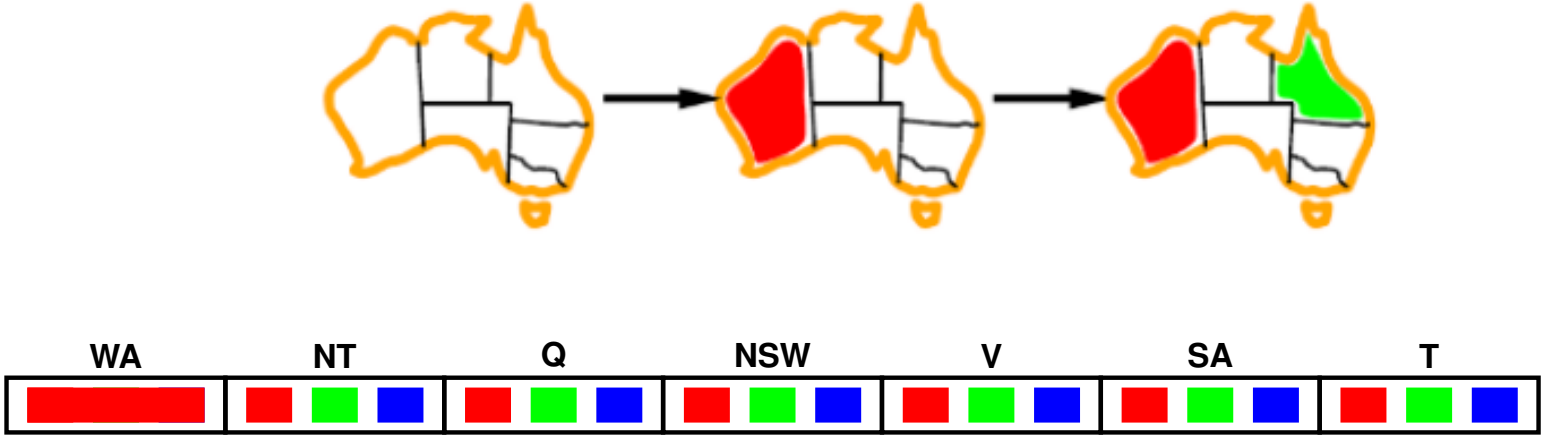

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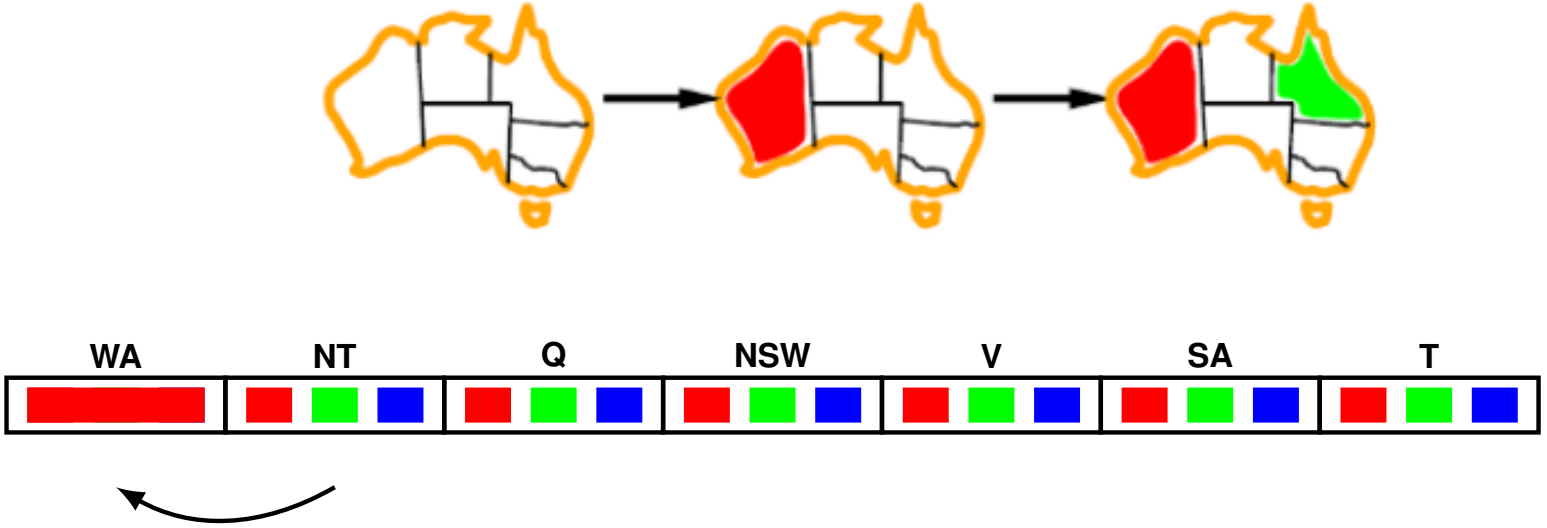

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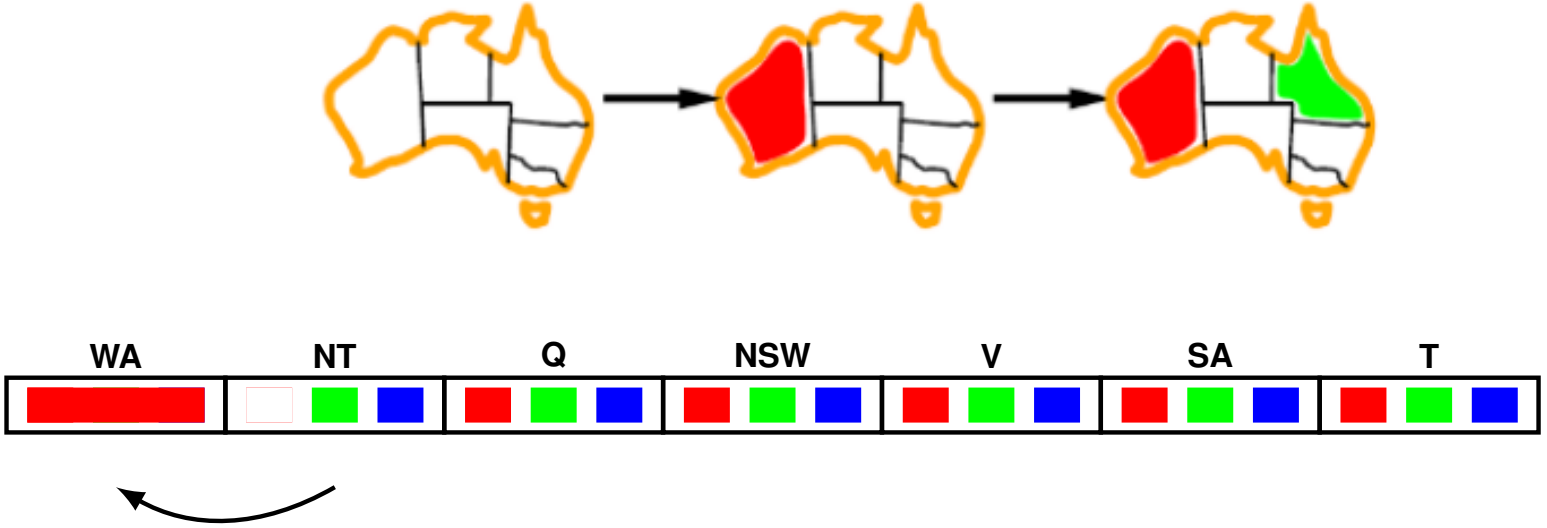

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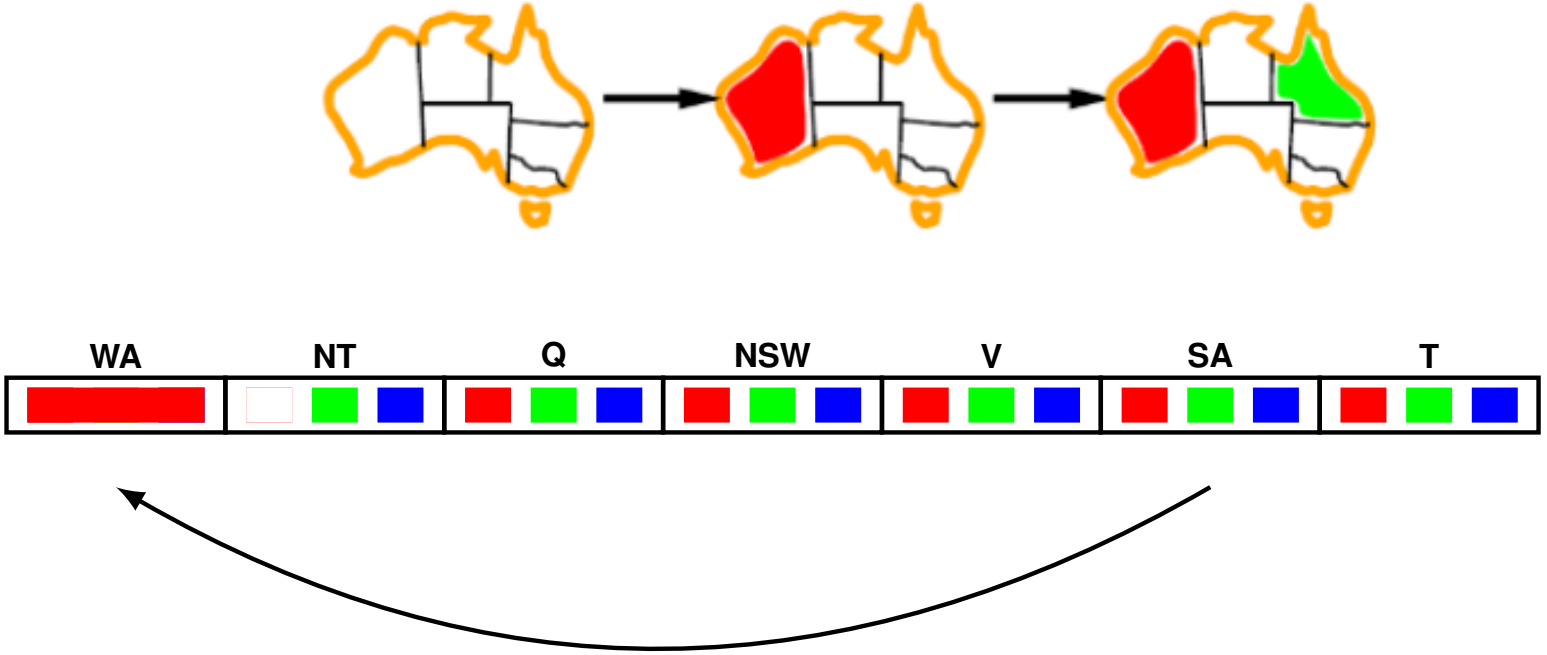

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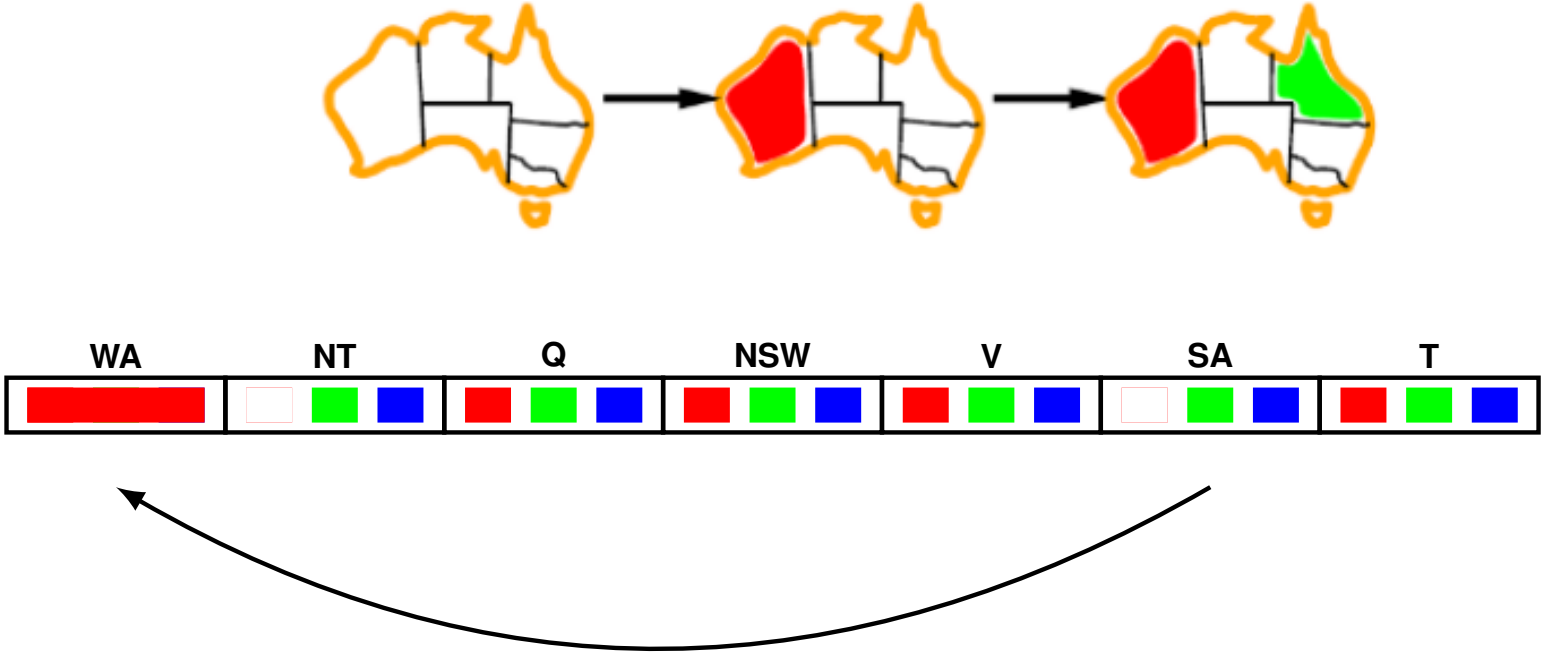

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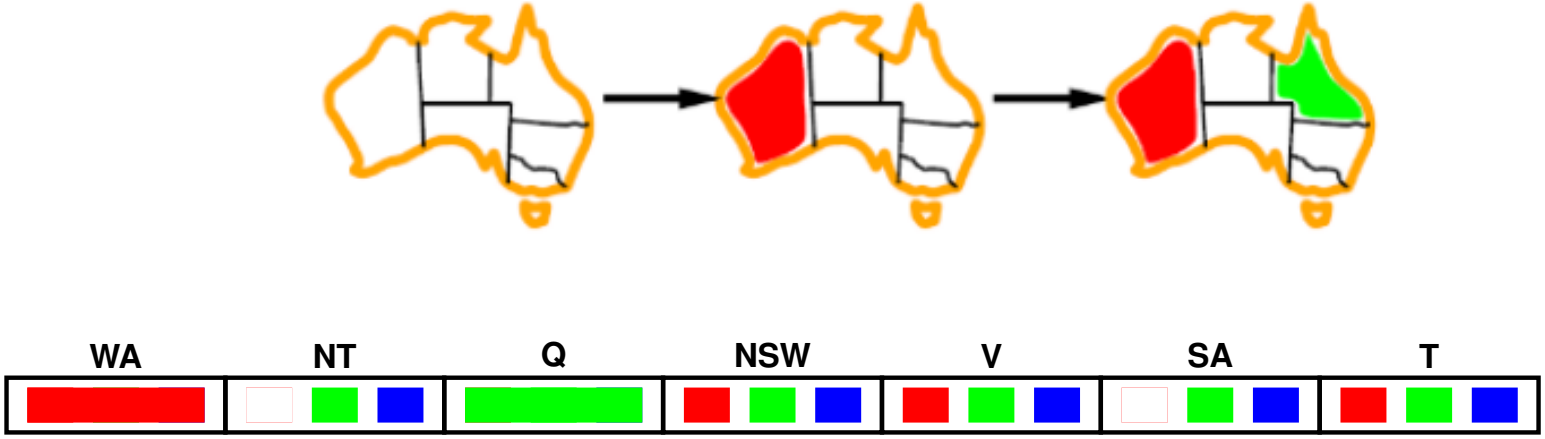

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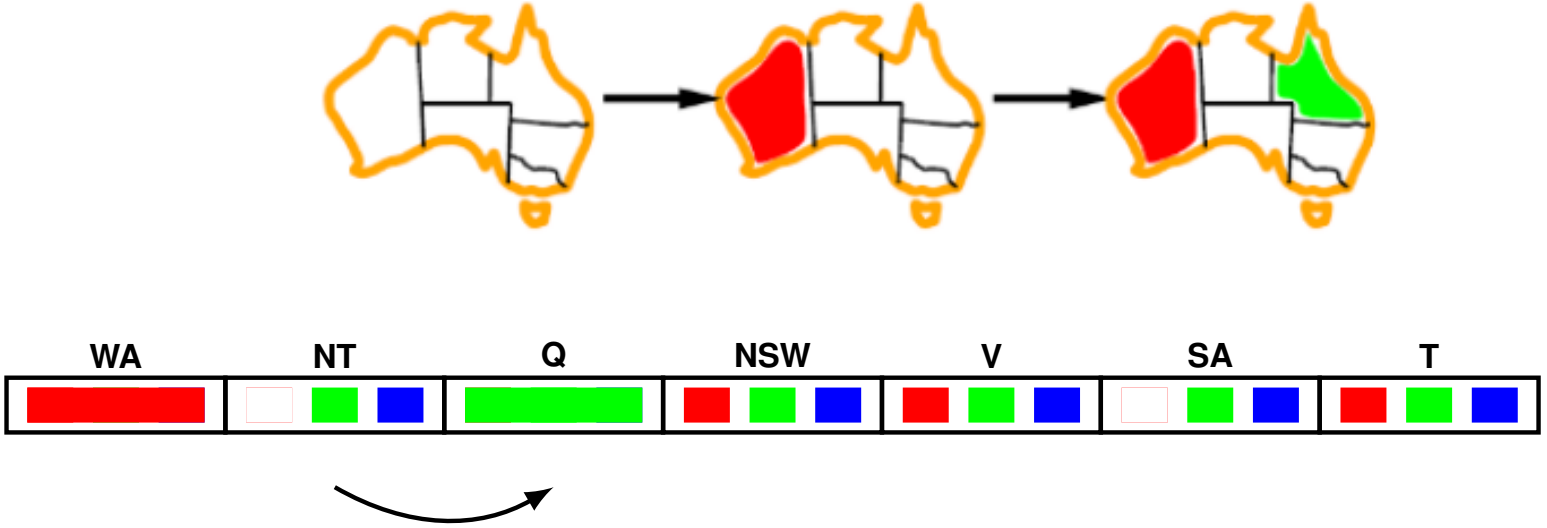

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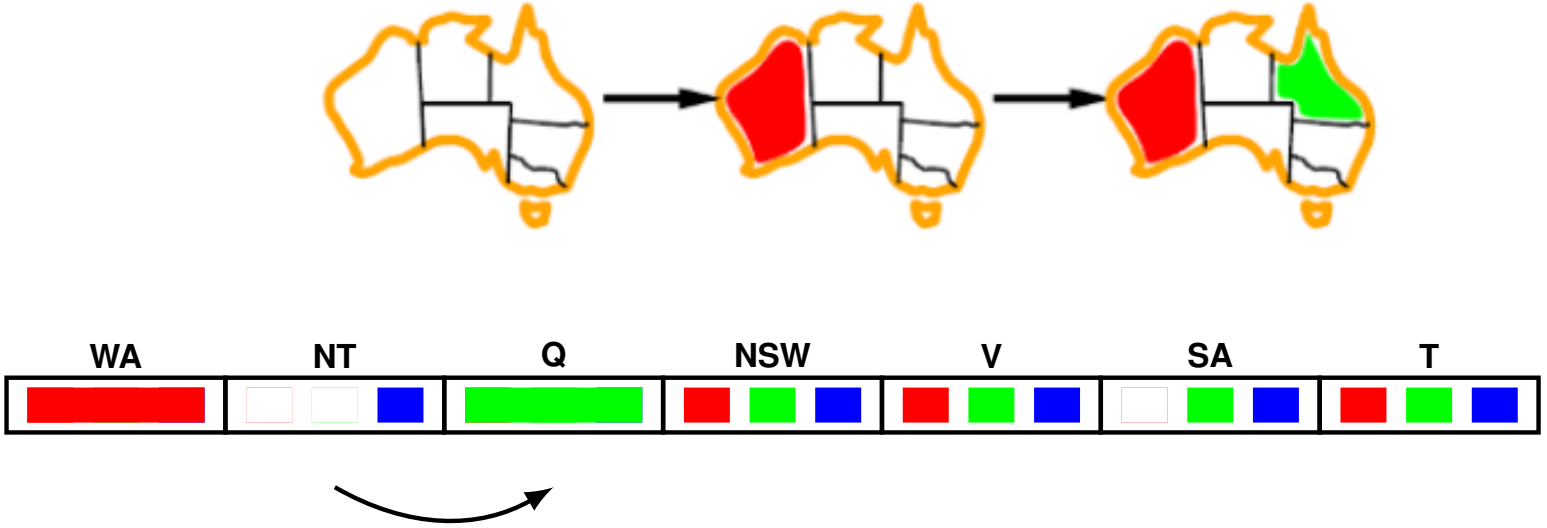

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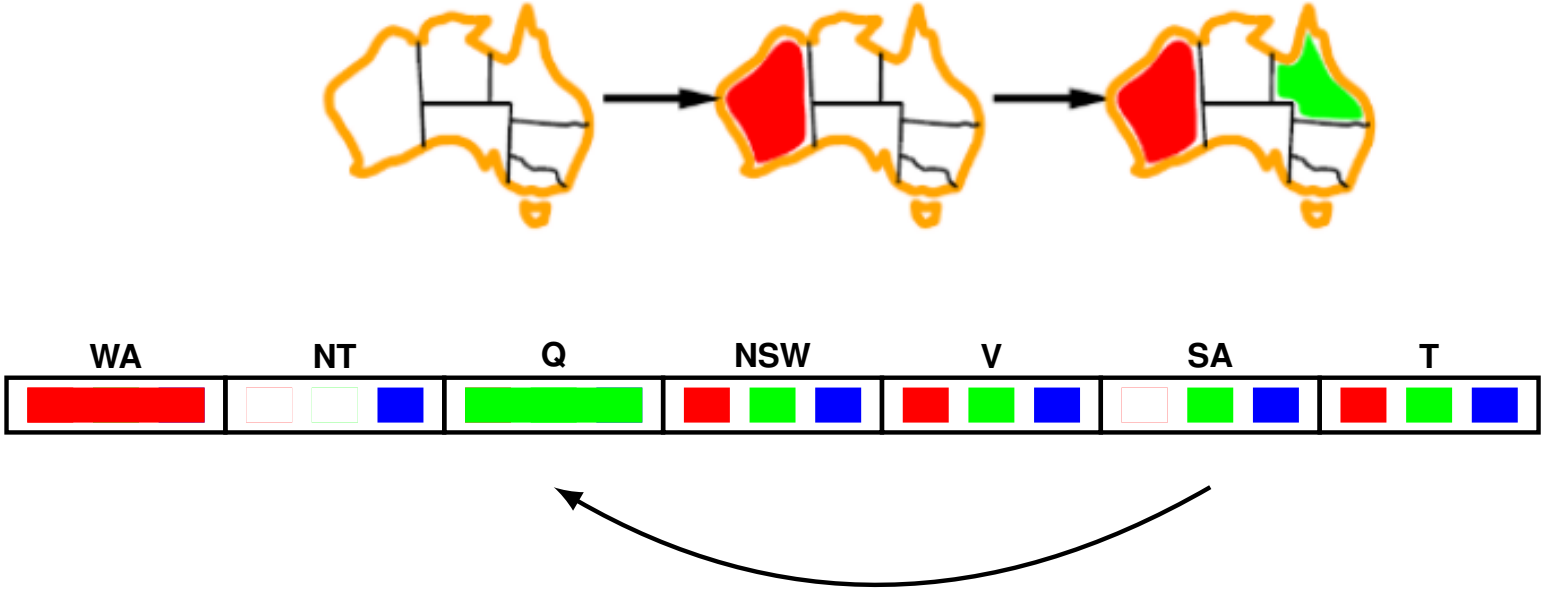

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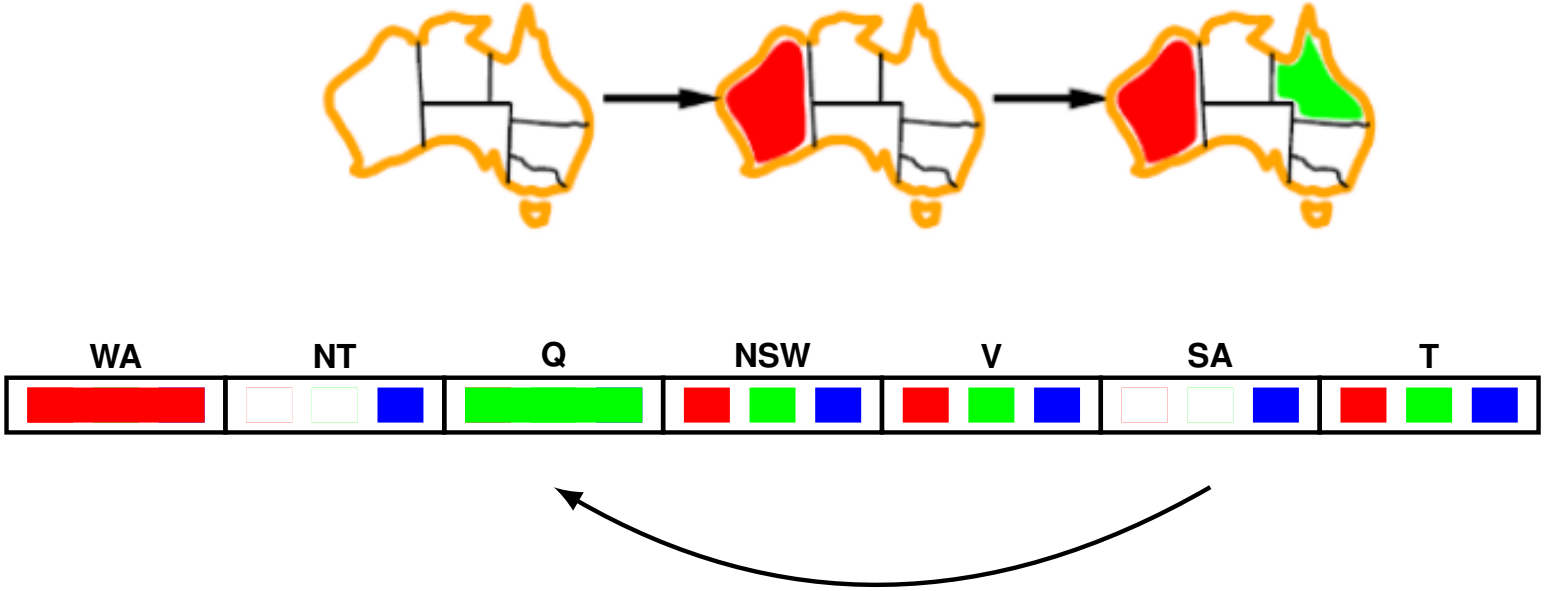

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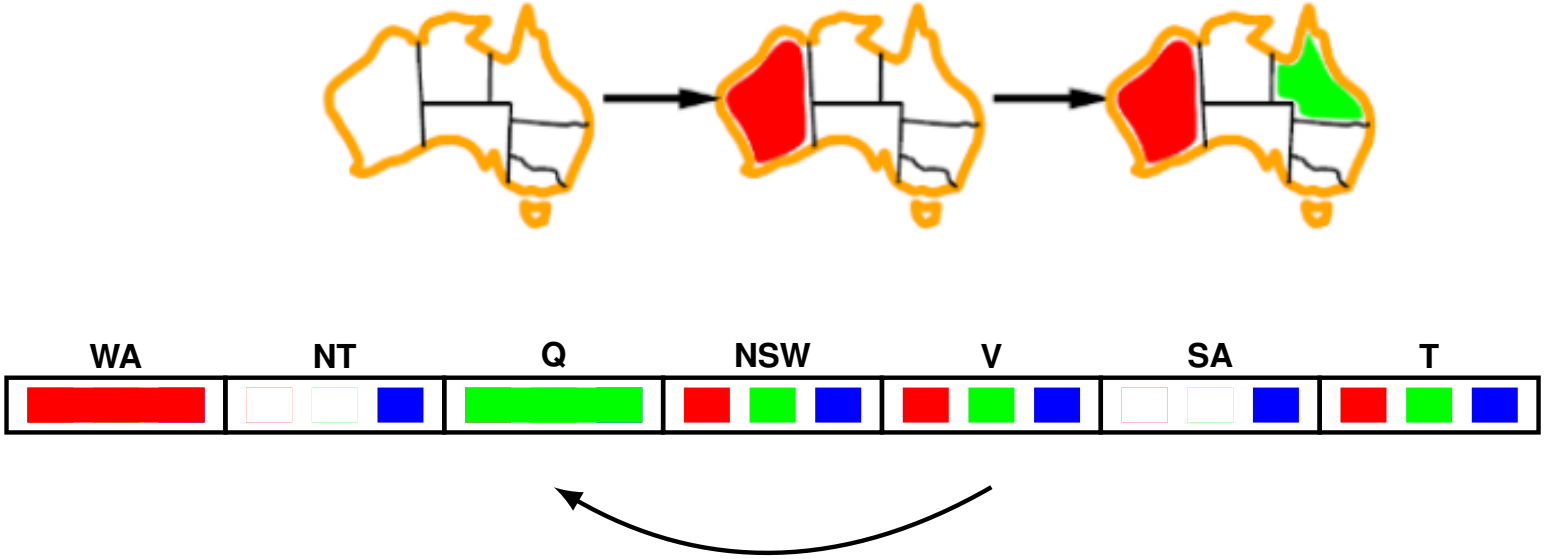

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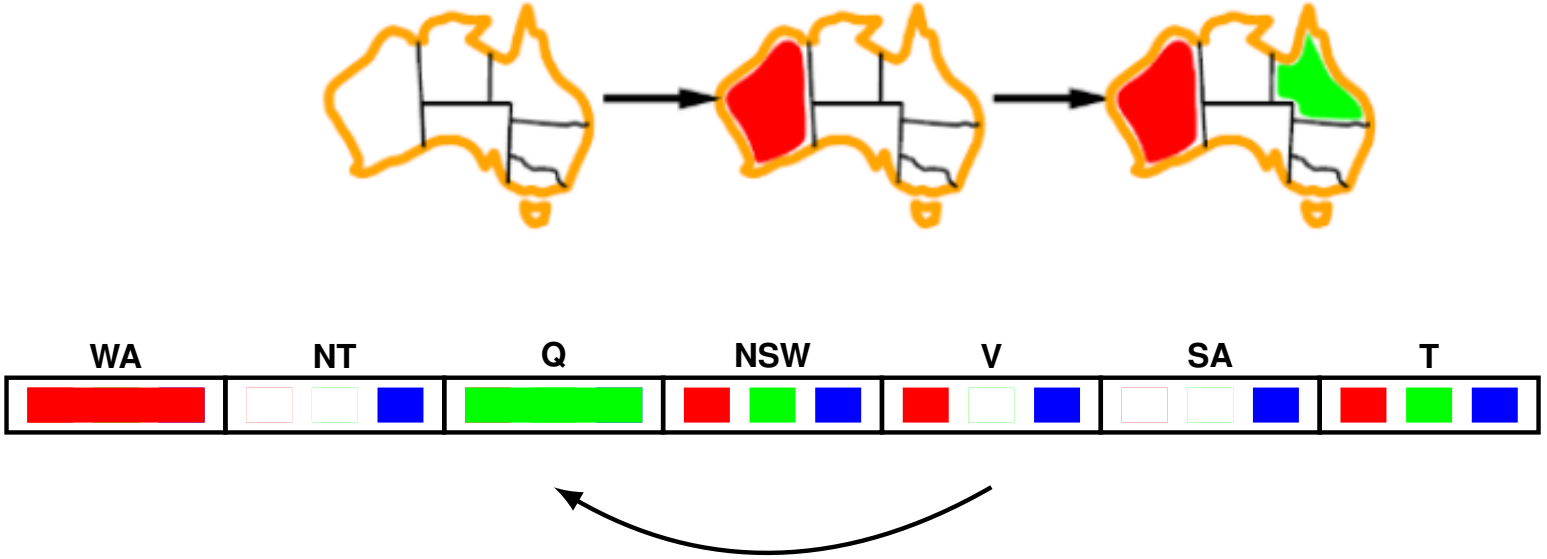

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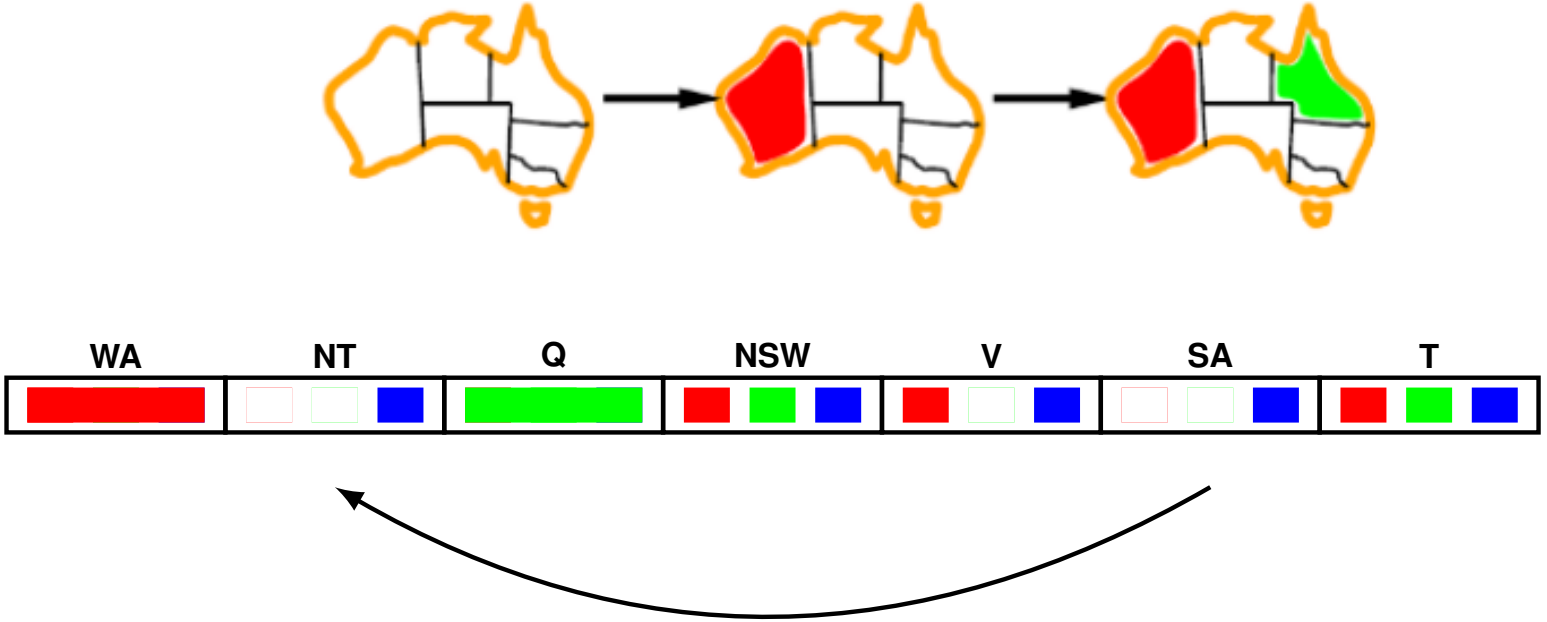

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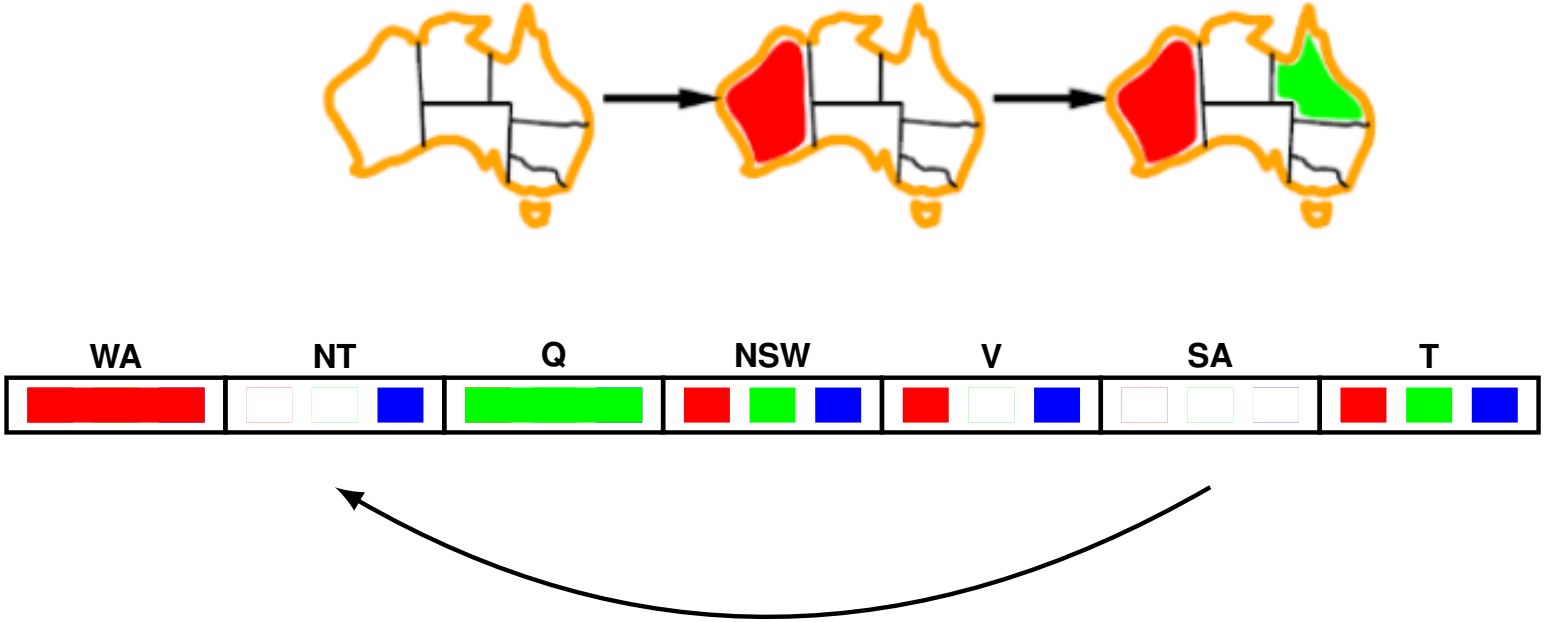

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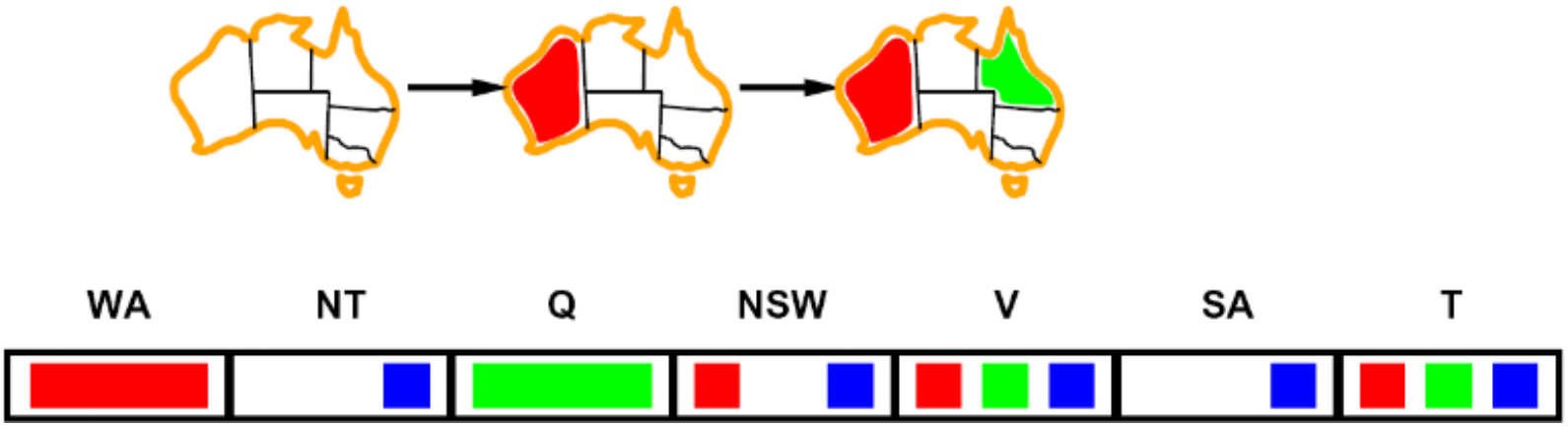

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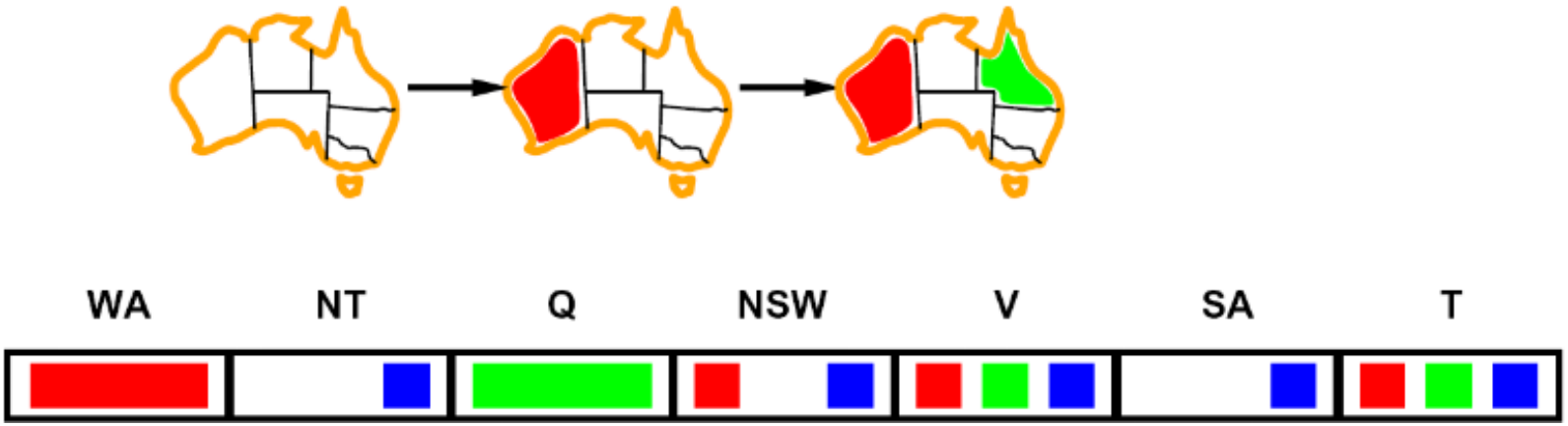

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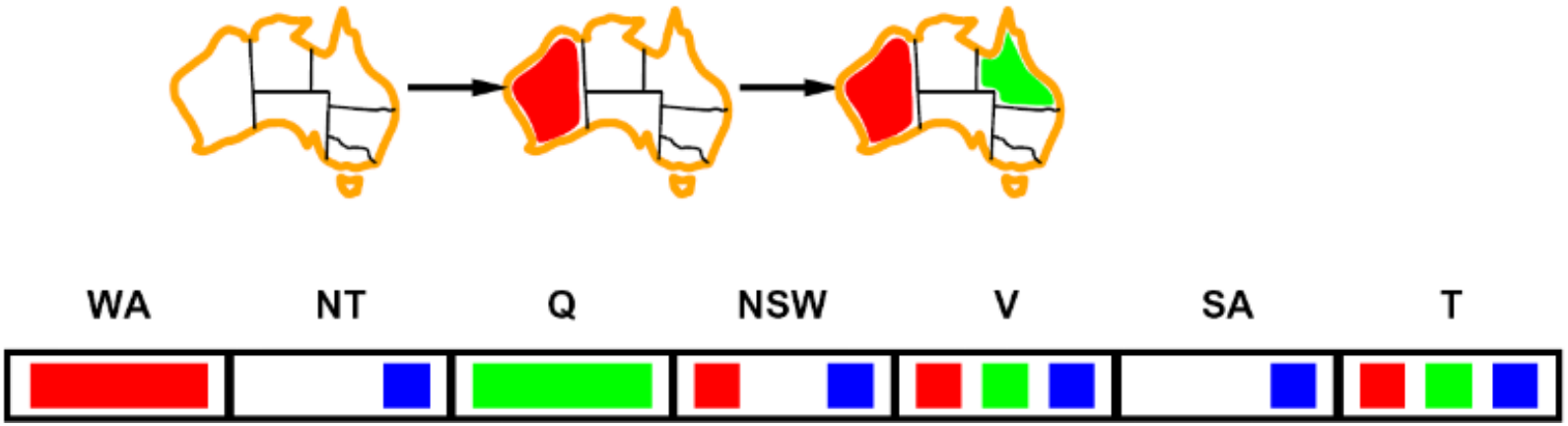
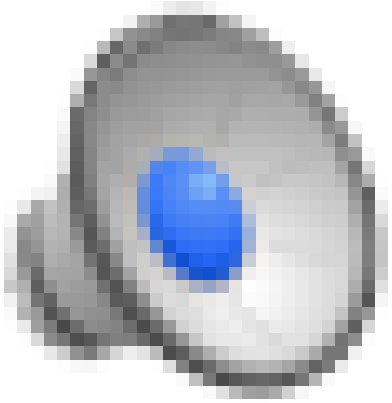


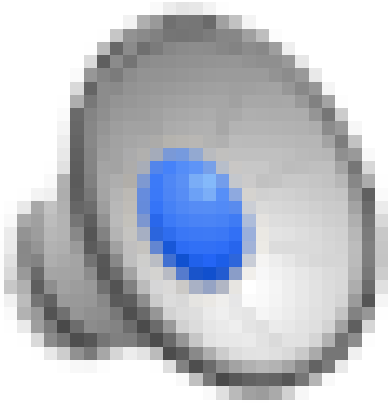
Can also eliminate Blue from NT and SA!

Can backtrack immediately.

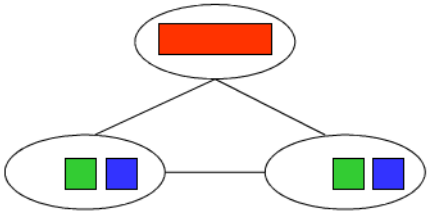
Video of Demo Arc Consistency – CSP Applet – n Queens

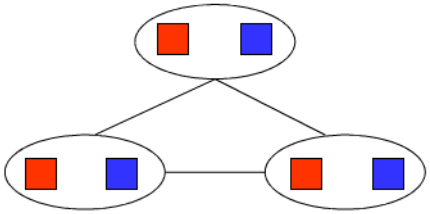


Video of Demo Coloring – Backtracking with Arc Consistency – Complex Graph



Limitations of Arc Consistency





What went wrong here?

After enforcing arc consistency:

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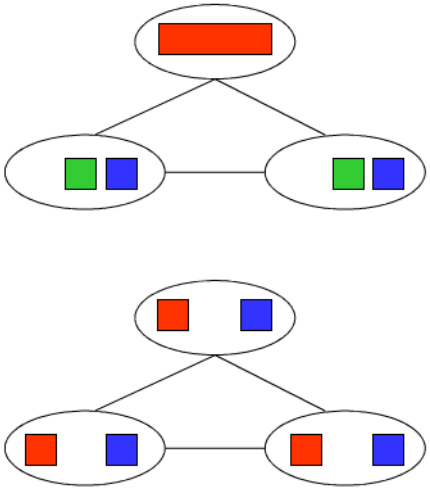

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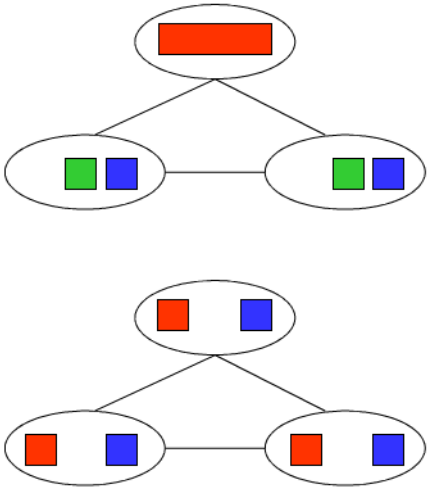

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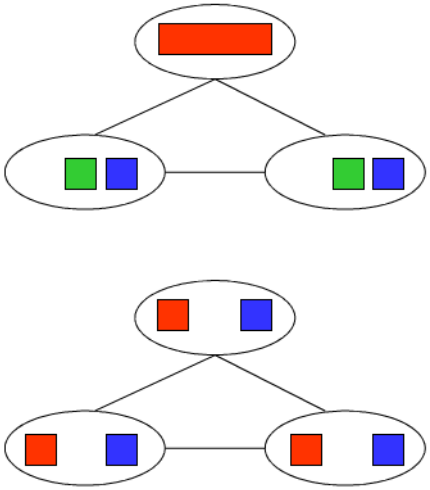

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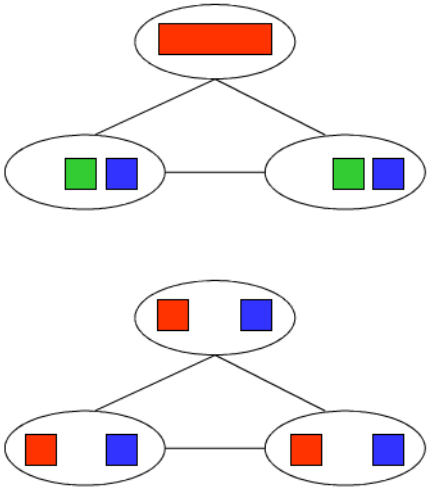

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Arc consistency still runs inside a backtracking search!

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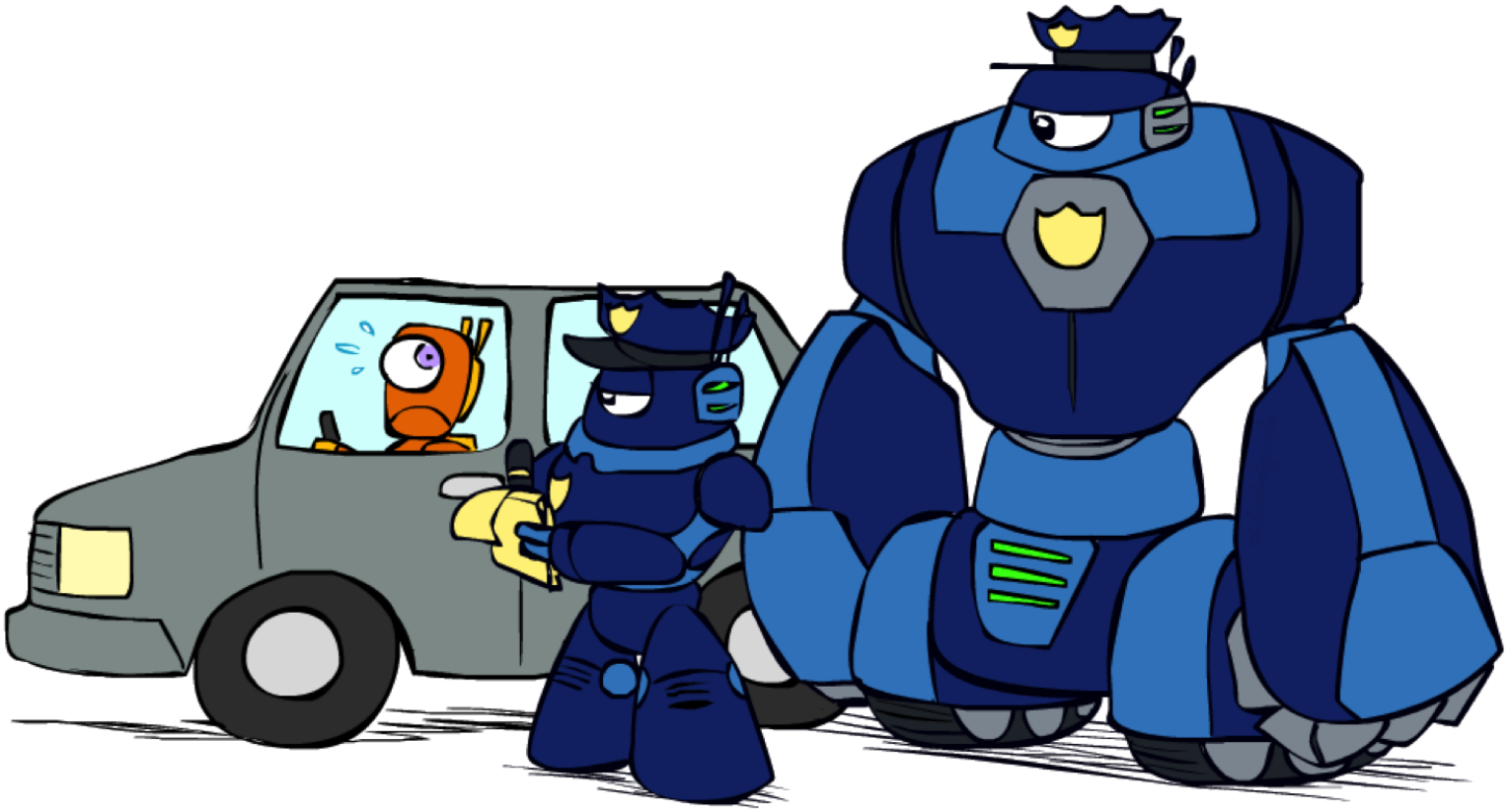

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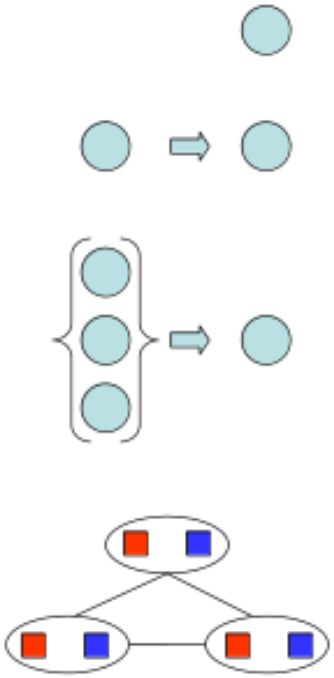

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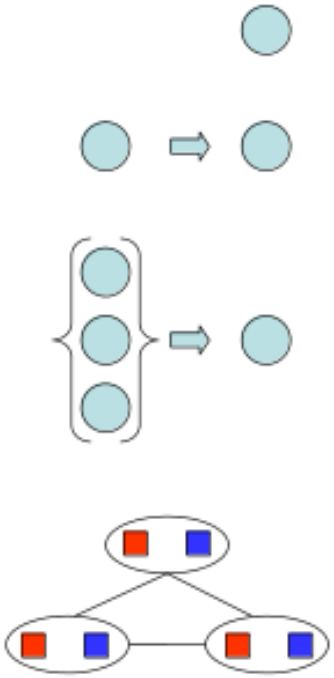

Increasing degrees of consistency

- 1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints

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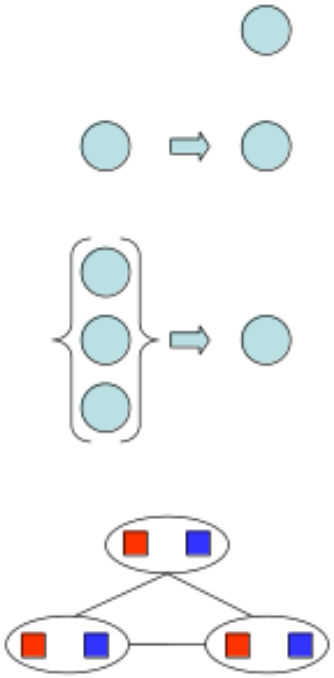

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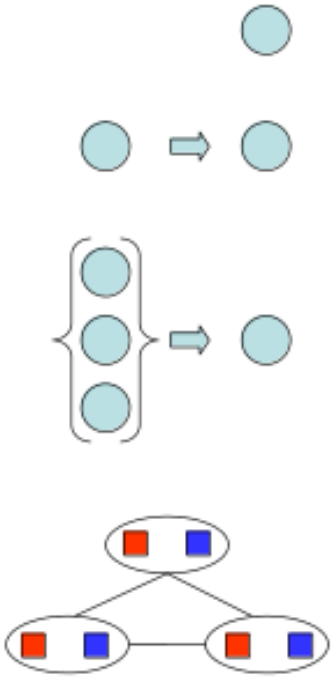

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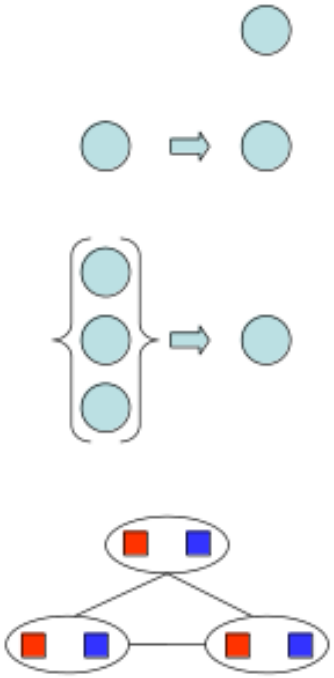

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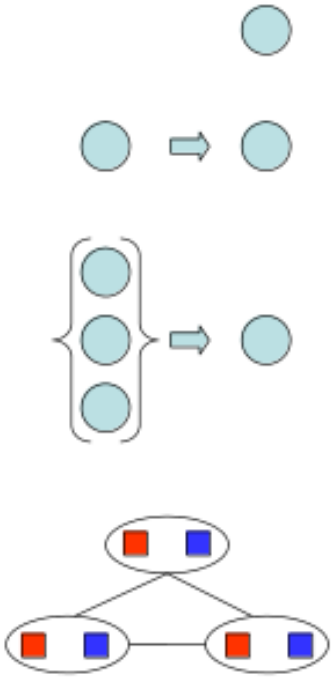

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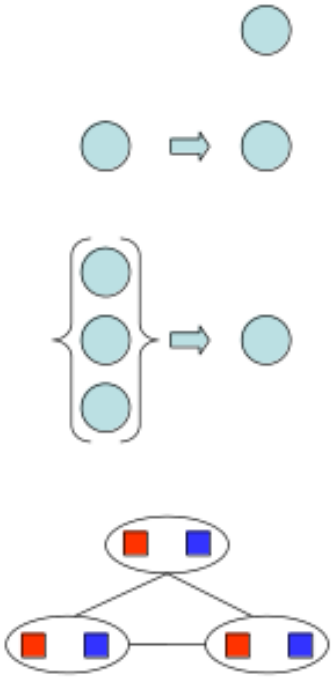

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(You need to know the k=2 case: arc consistency)

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Lots of middle ground between arc consistency and n-consistency! (e.g. k=3, called path consistency)

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General-purpose ideas give huge gains in speed

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Filtering:

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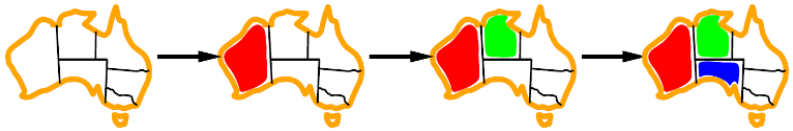
Choose the variable with the fewest legal left values in its domain



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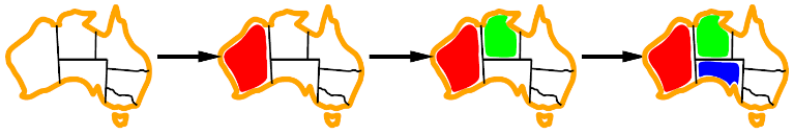

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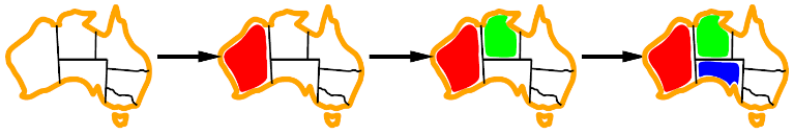

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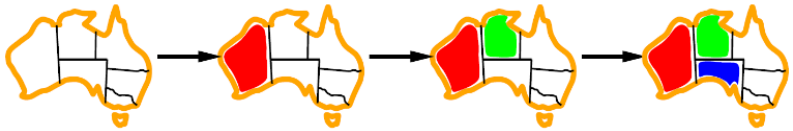

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“Fail-fast” ordering!

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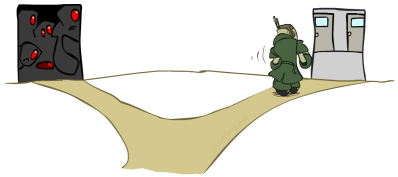
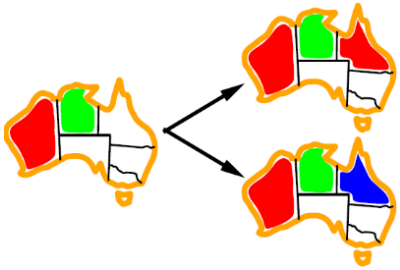
Value Ordering: Least Constraining Value.





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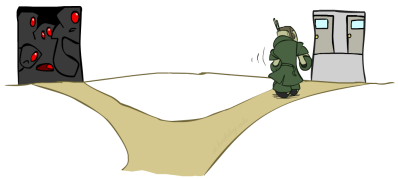

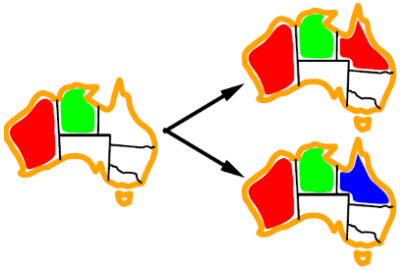

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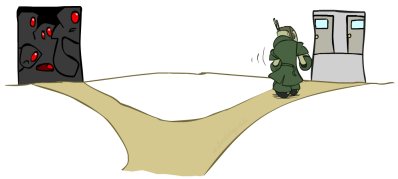

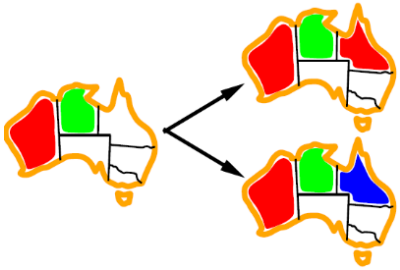

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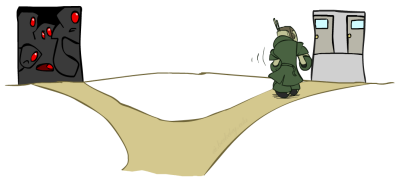

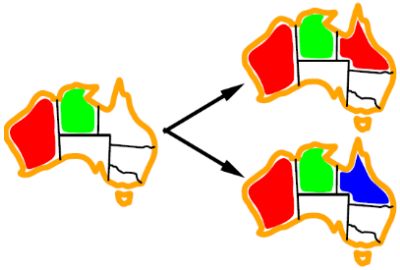

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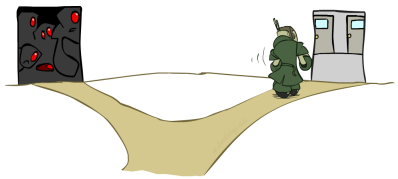

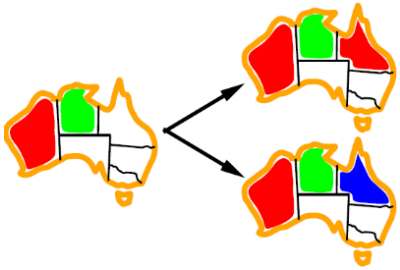

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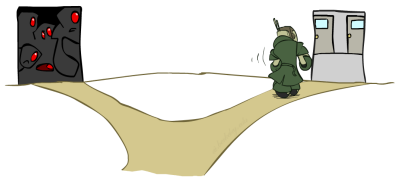

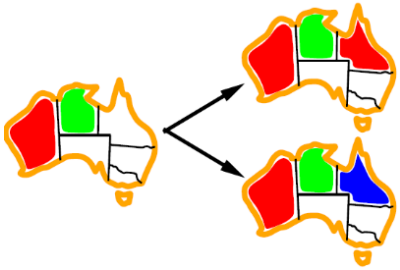

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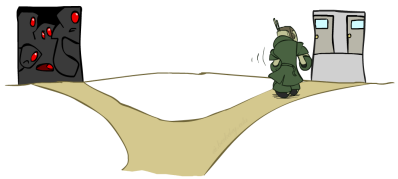

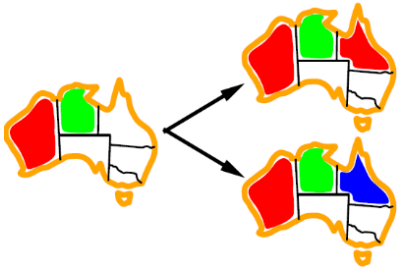

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All these ordering ideas makes 1000 queens feasible!

Demo: Coloring – Backtracking + Forward Checking + Ordering

CSP: what to know.

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