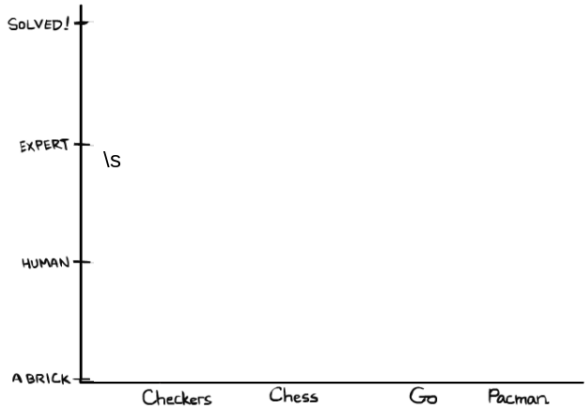
Game Playing State-of-the-Art

http://bit.ly/3GEMok9



Game Playing State-of-the-Art

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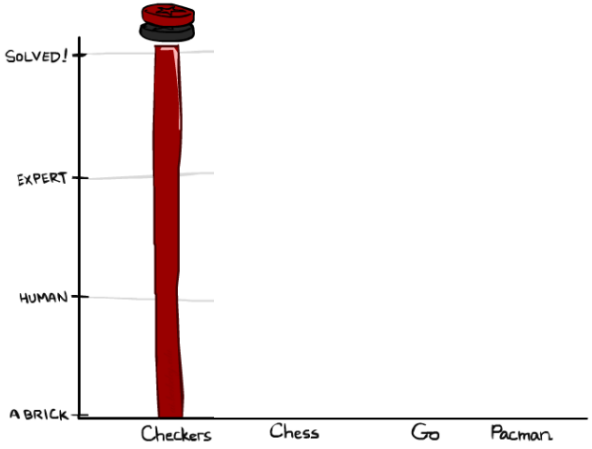
Chinook beat 40-year-reign of champion Marion Tinsley using complete 8-piece endgame.
2007: Checkers solved!



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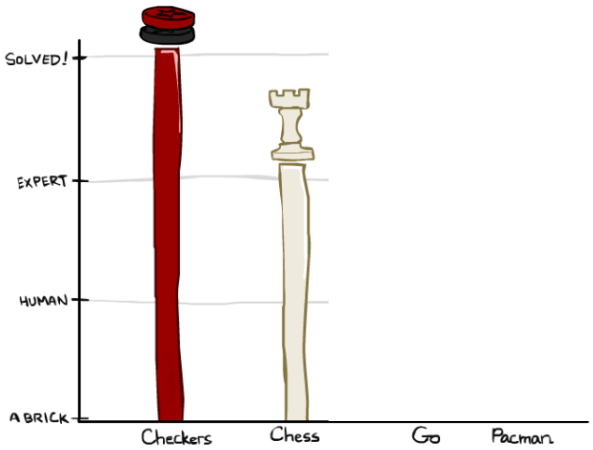

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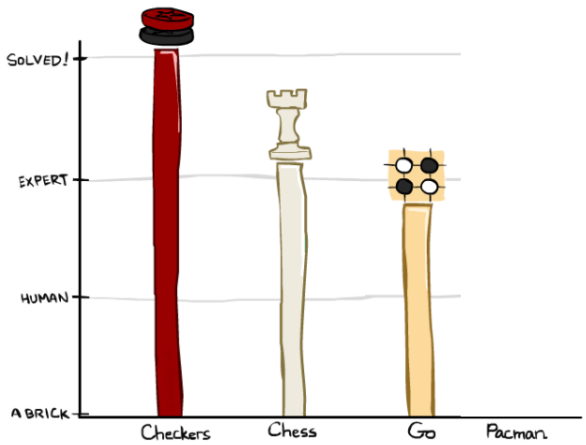

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Go: Human champions are being beaten. In go, $b > 300$! Classically use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.

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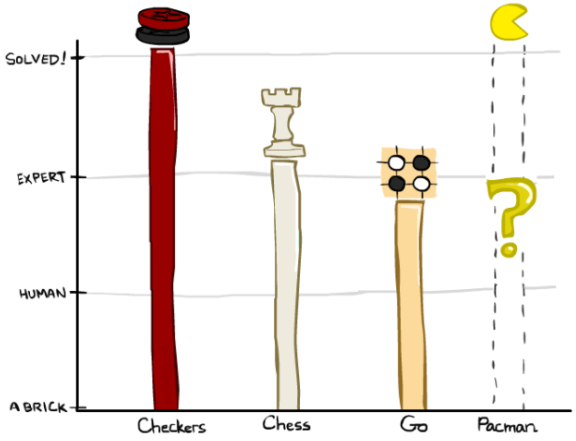

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Pacman

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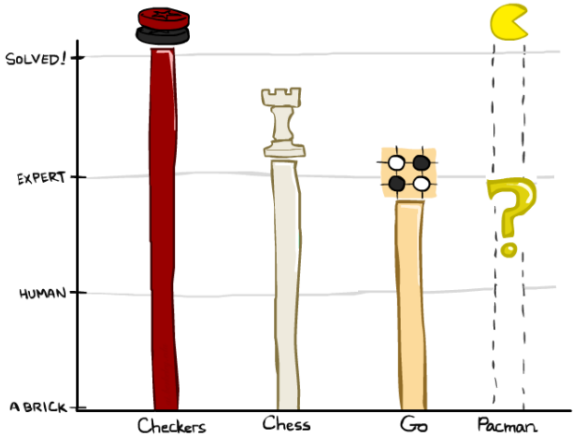

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Behavior from Computation

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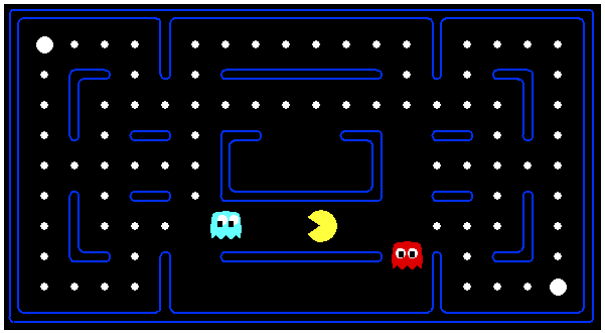

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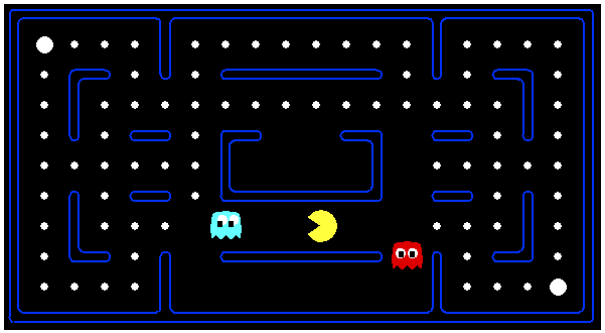
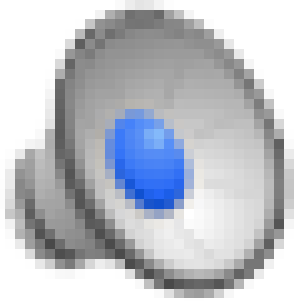


$Demo : mysterypacman(L6D1)$

Video of Demo Mystery Pacman



Adversarial Games

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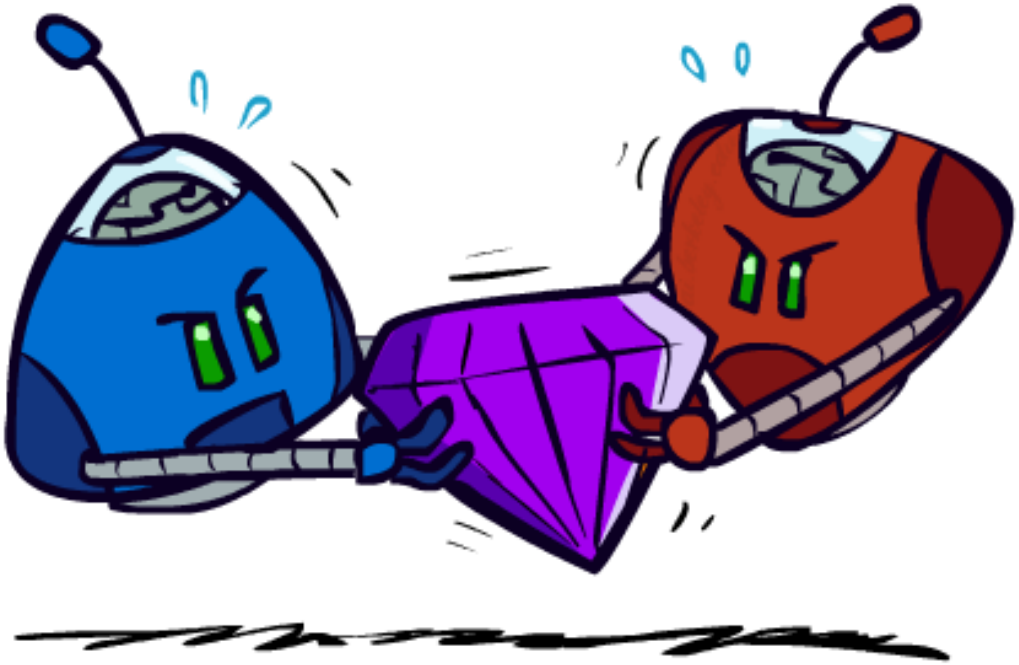

Types of Games

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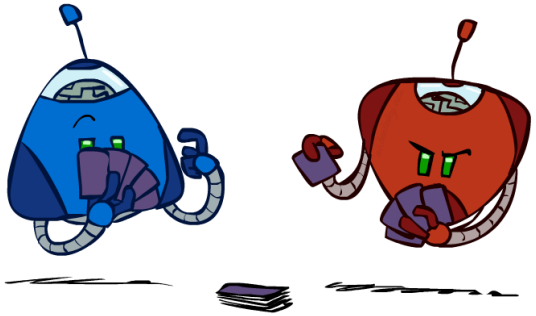

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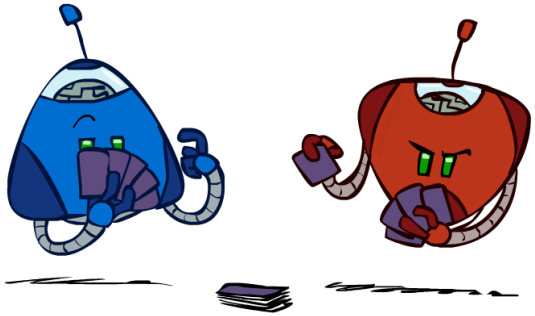

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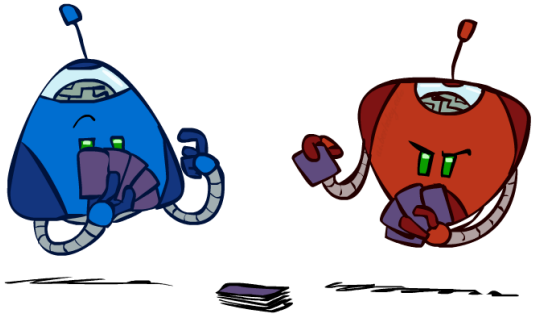

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Axes:

Types of Games

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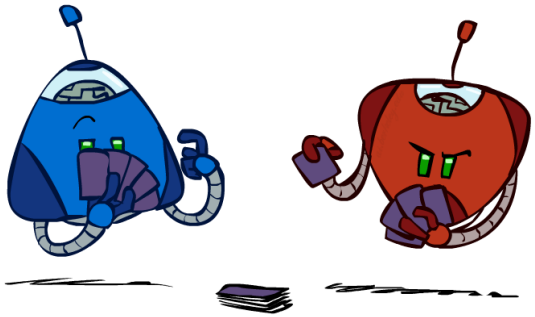

Many different kinds of games!

Axes:

- Deterministic or stochastic?

Types of Games

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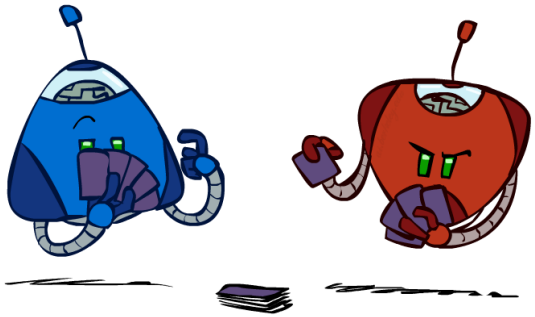

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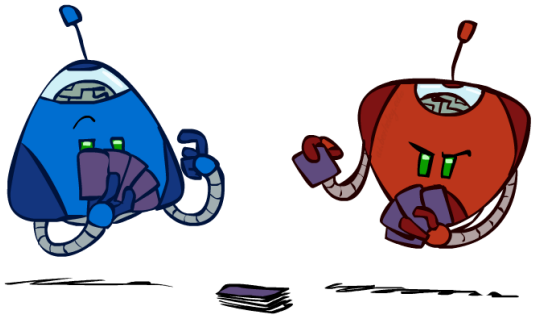

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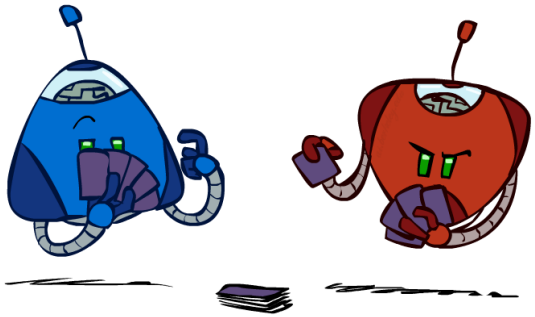

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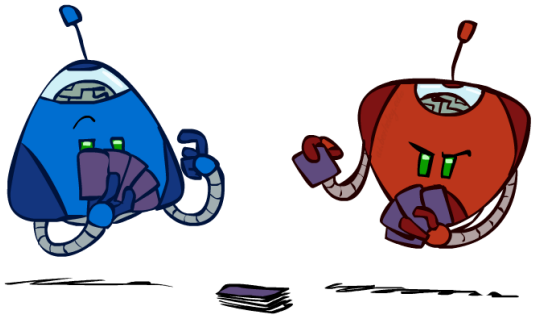

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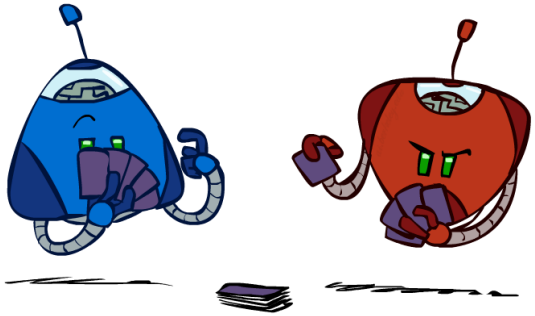

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Want algorithms for calculating a strategy (policy) which recommends a move from each state.

Deterministic Games

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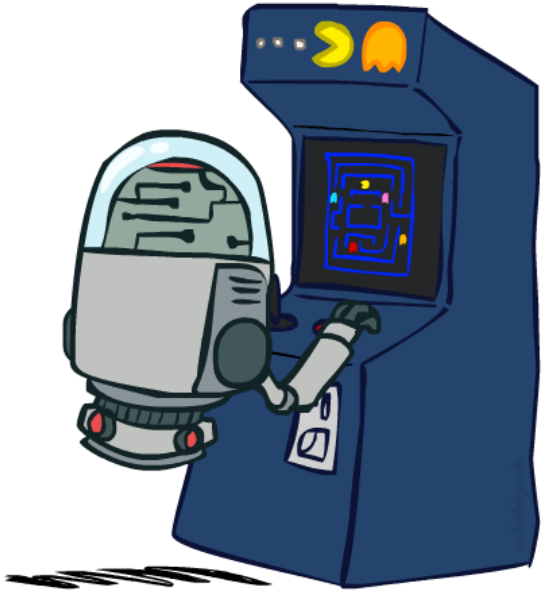

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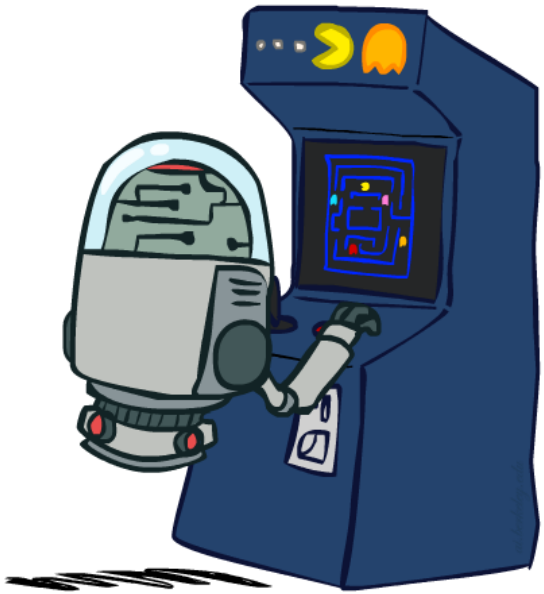

Many possible formalizations, one is:

- States: S (start at s0)

Deterministic Games

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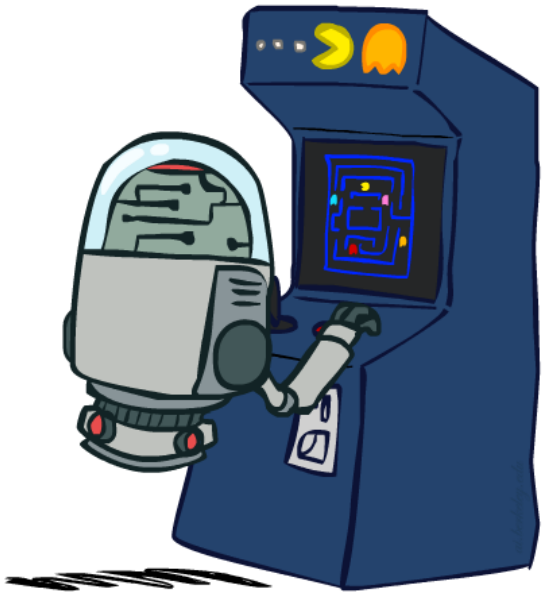

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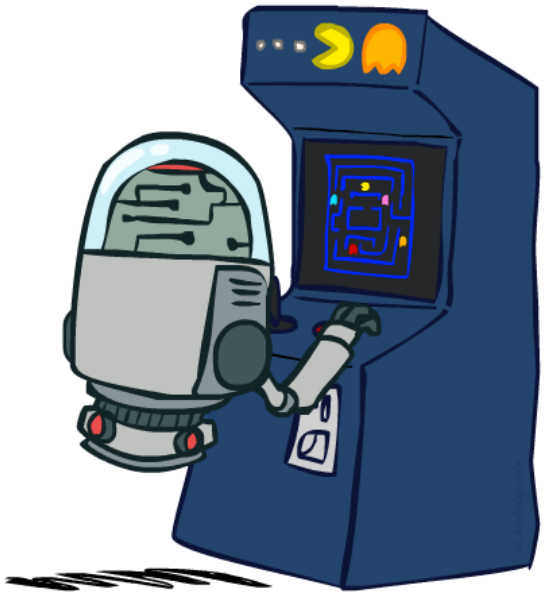

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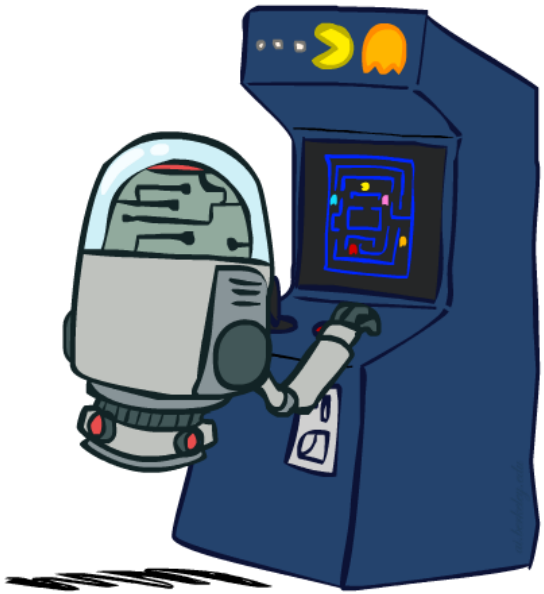

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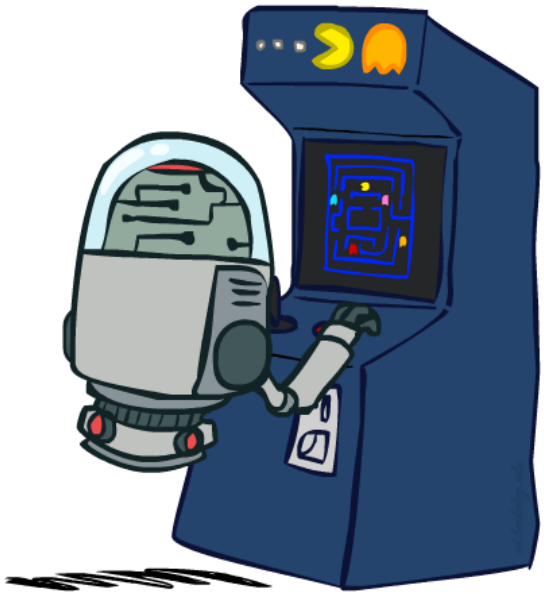

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- Transition Function: $S \times A \rightarrow S$.

Deterministic Games

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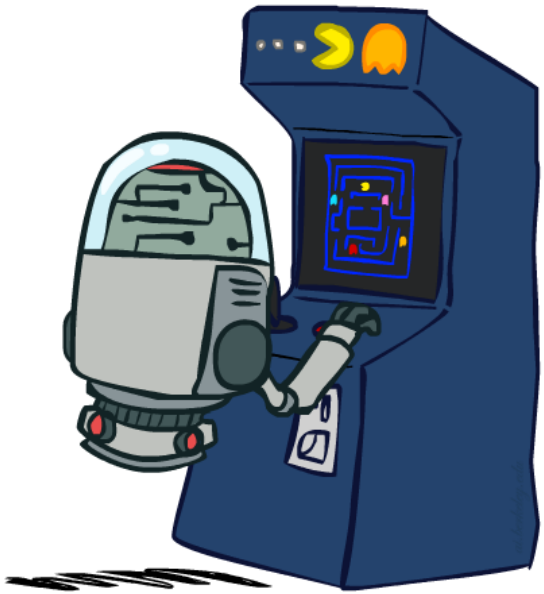

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Deterministic Games

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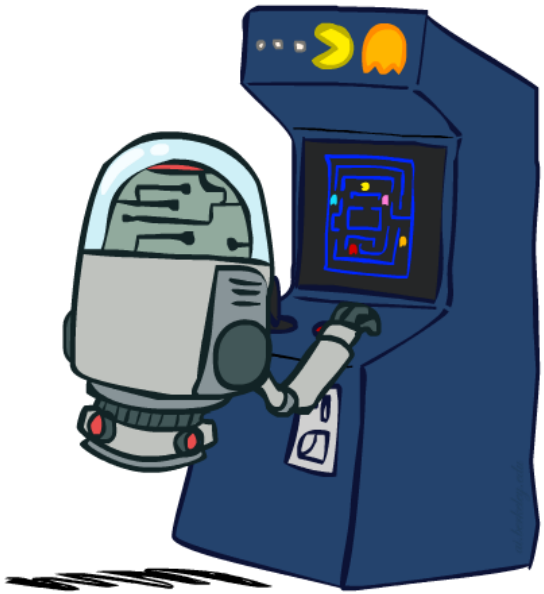

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- Terminal Utilities: $S \times P \rightarrow R$.

Deterministic Games

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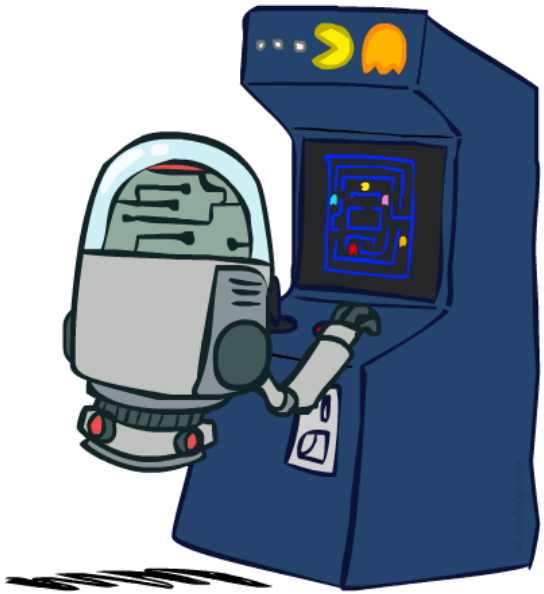

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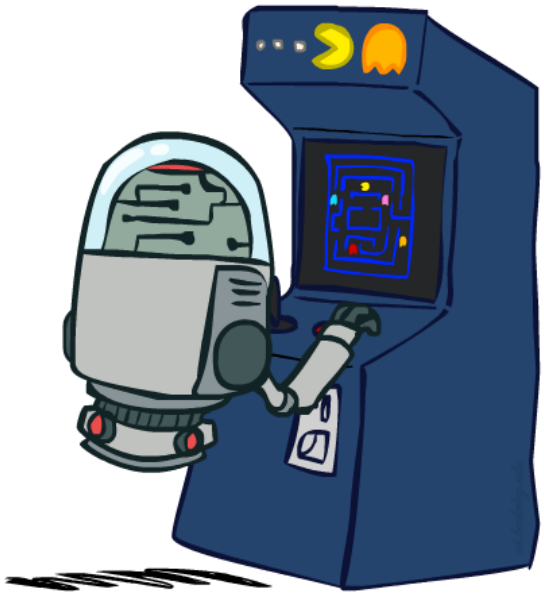

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Solution for a player is a policy: $S \rightarrow A$.

Zero-Sum Games

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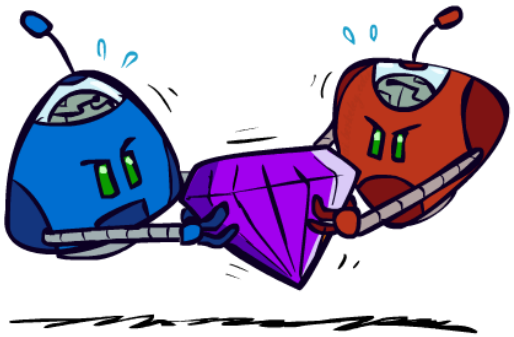

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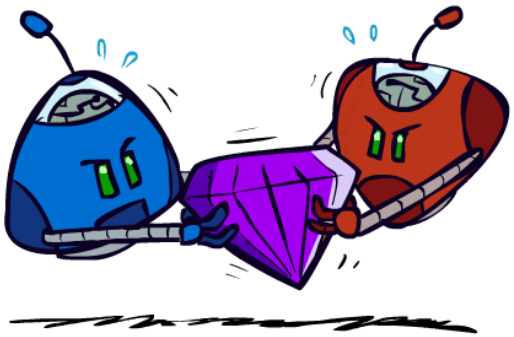

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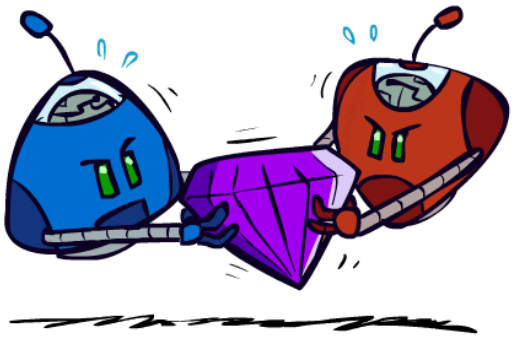

Zero-Sum Games

- Agents have opposite utilities (values on outcomes)

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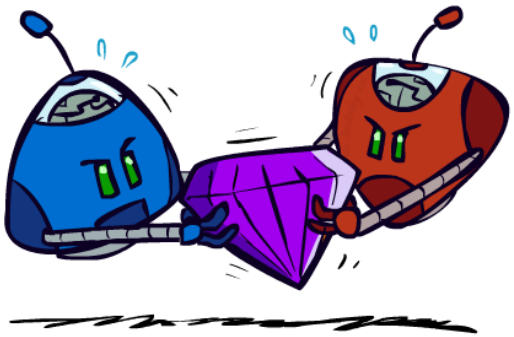

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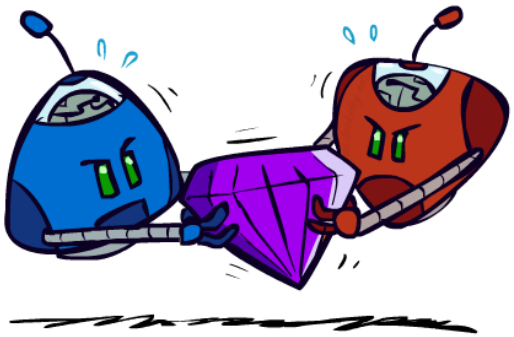

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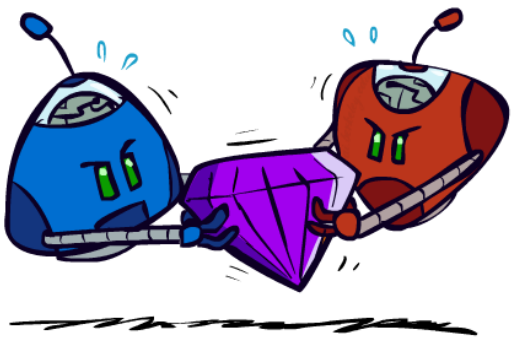

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General Games

- Agents have independent utilities (values on outcomes)

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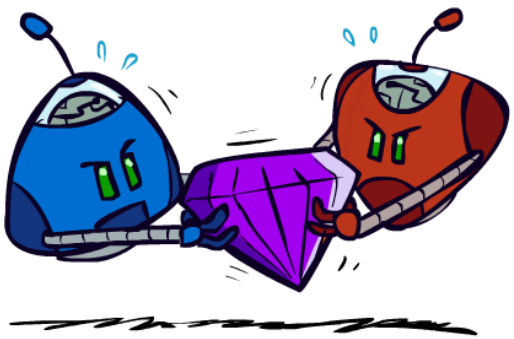

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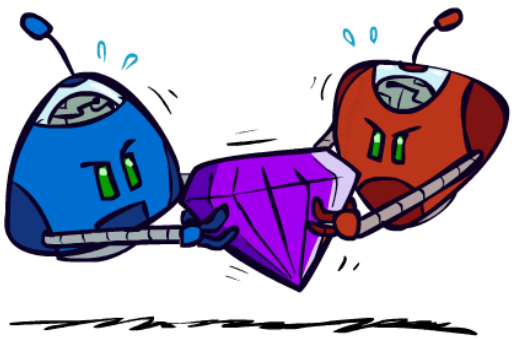

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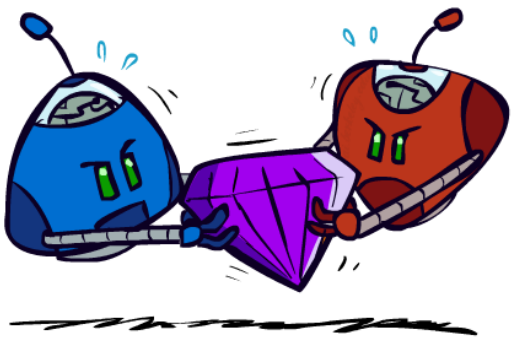

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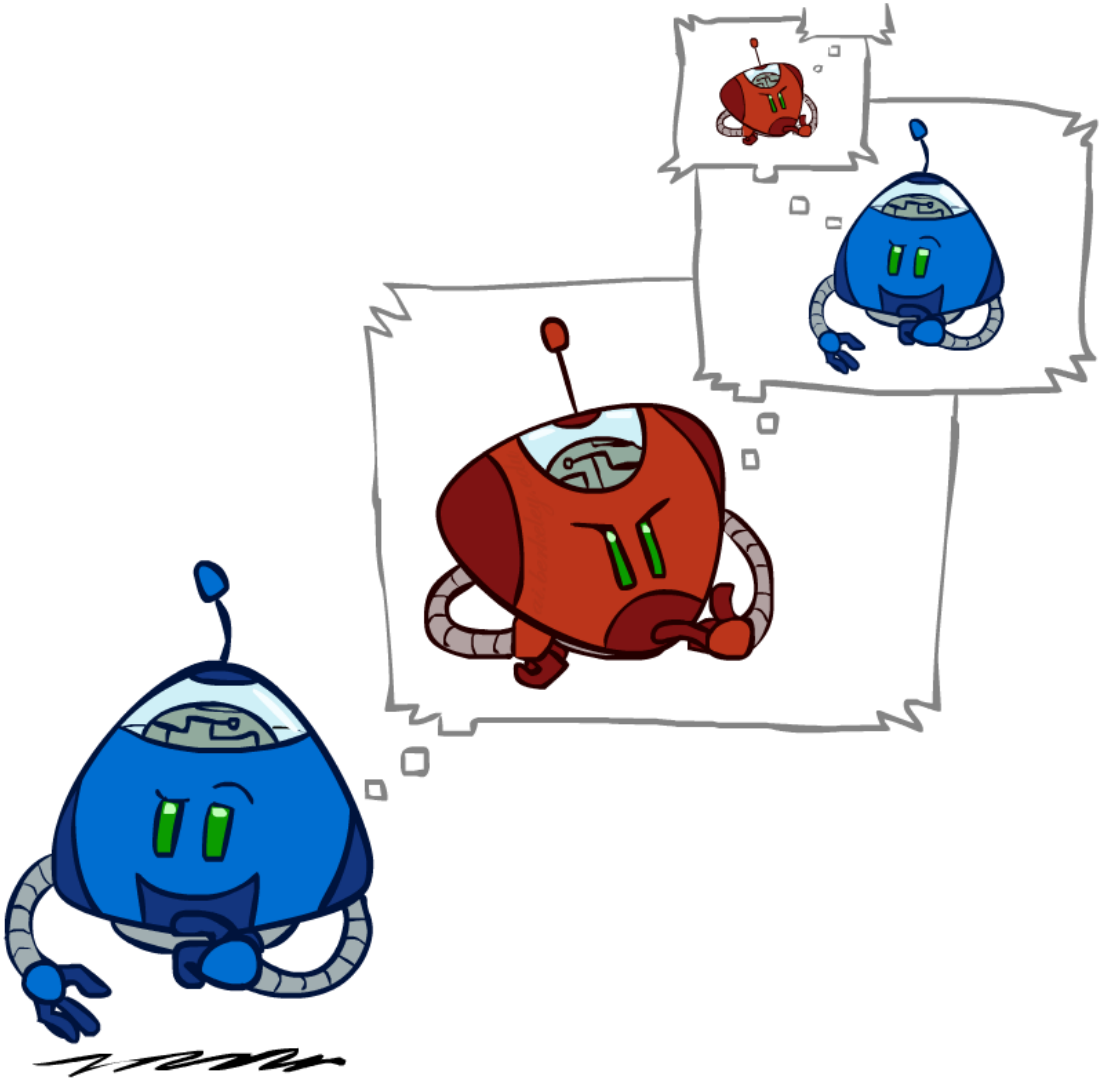
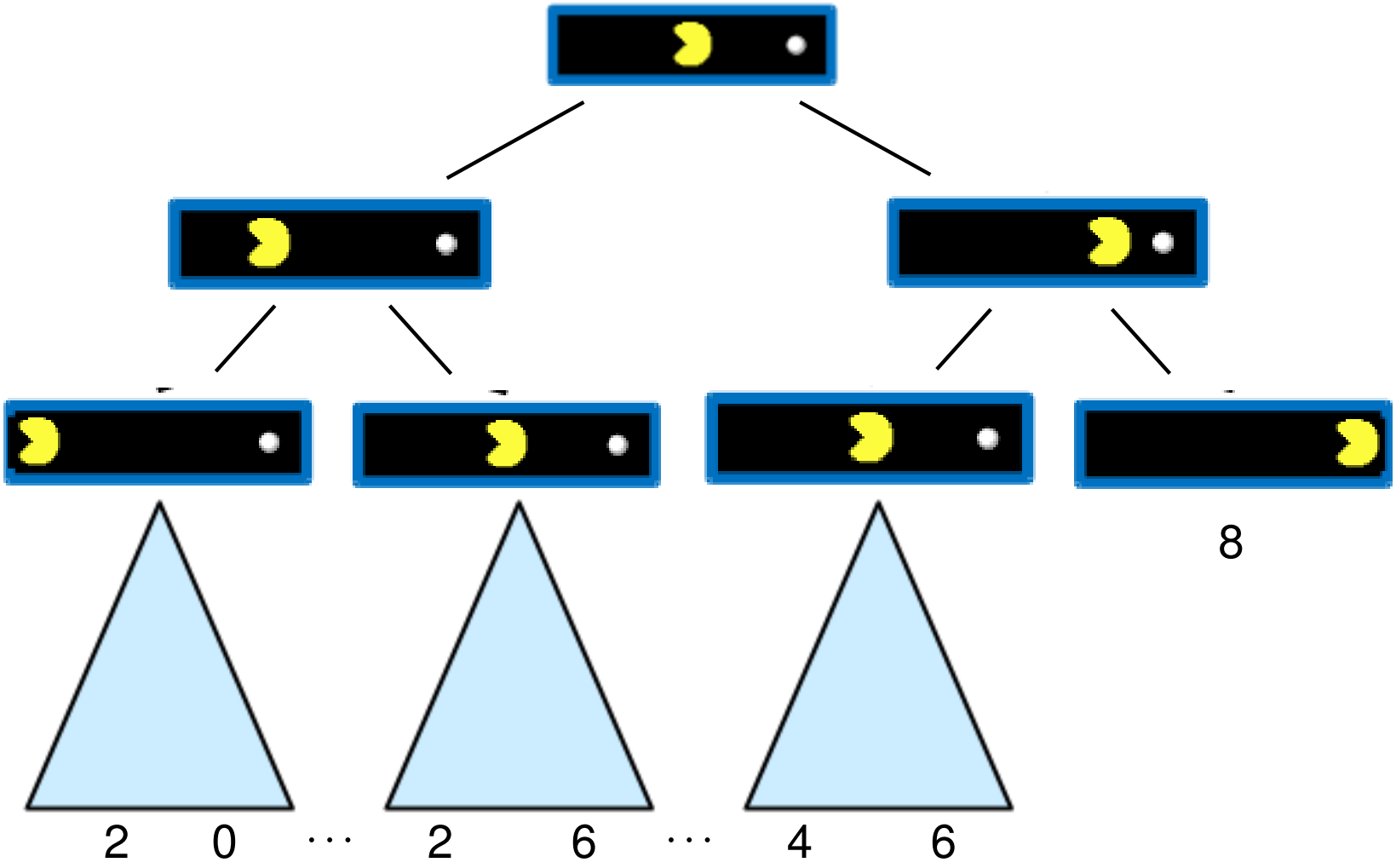
Adversarial Search



Single-Agent Trees



Value of Game Tree

Value of a State: utility of best achievable outcome from state.

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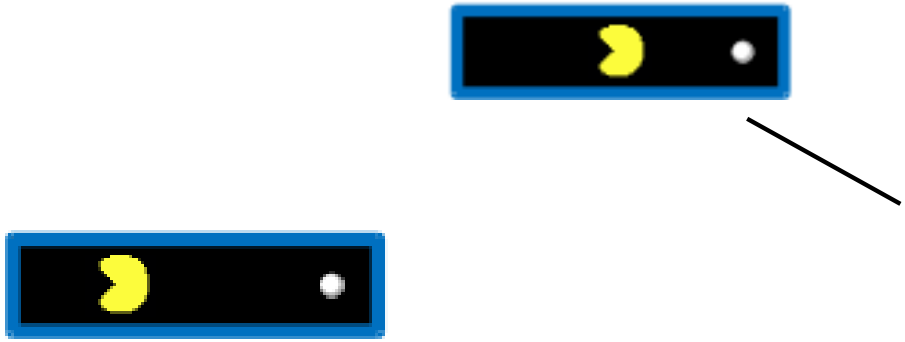

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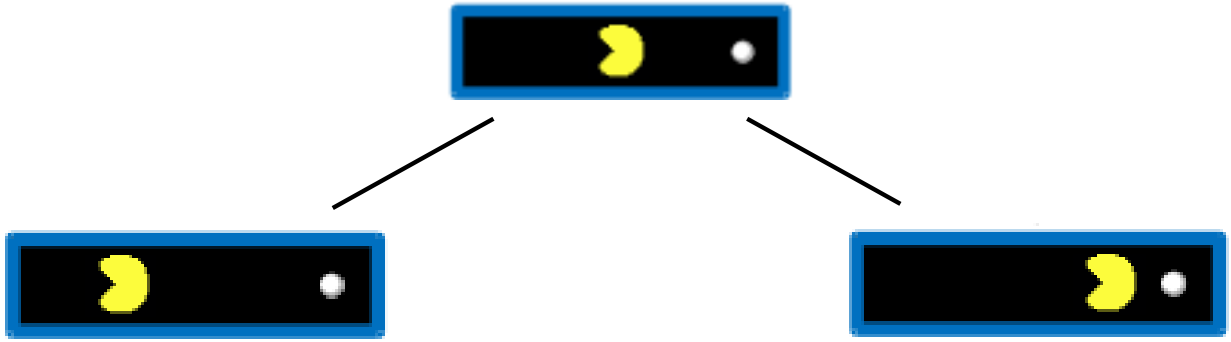
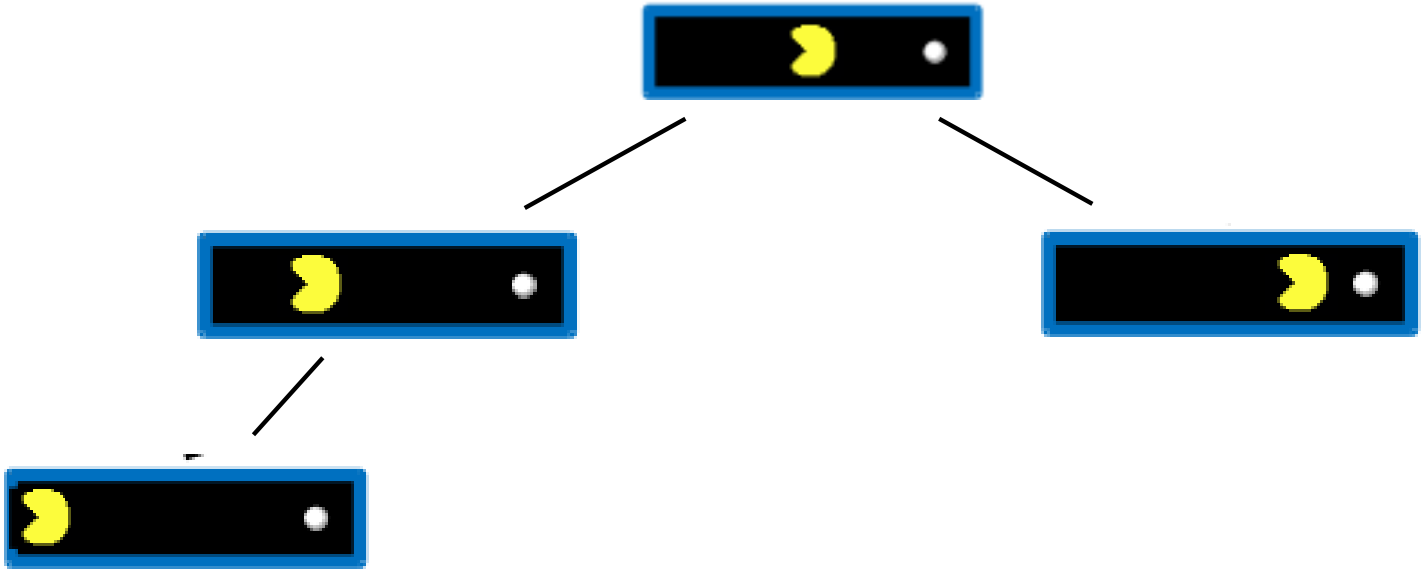


Value of Game Tree

Value of a State: utility of best achievable outcome from state.



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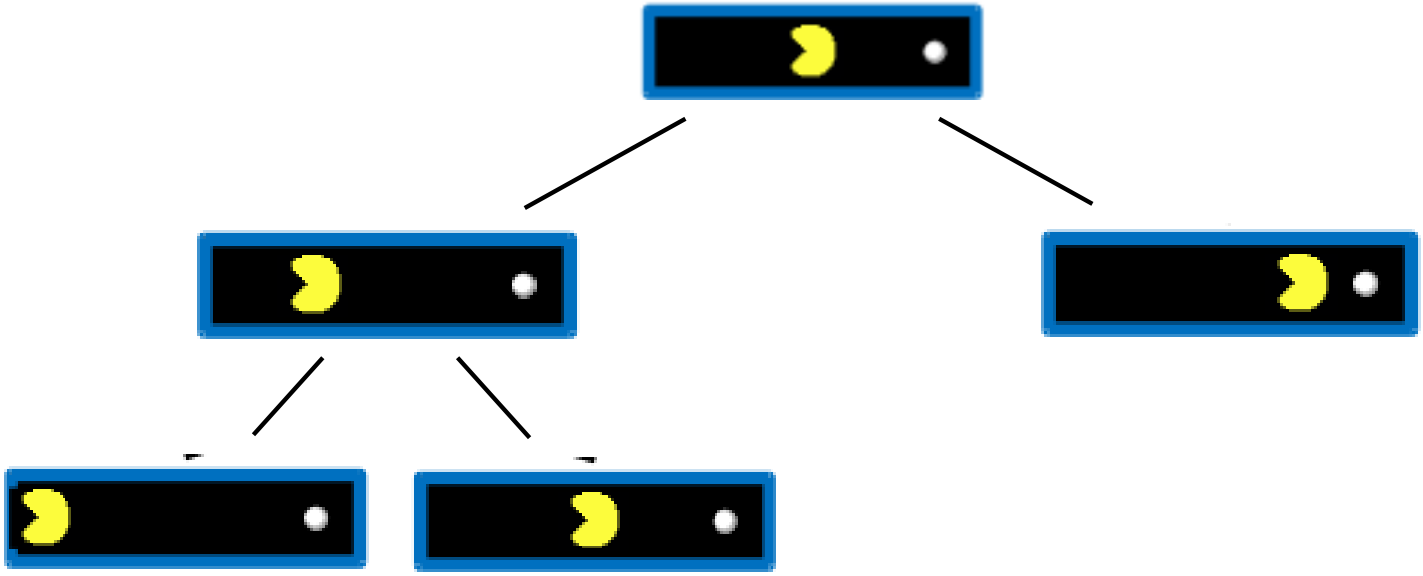

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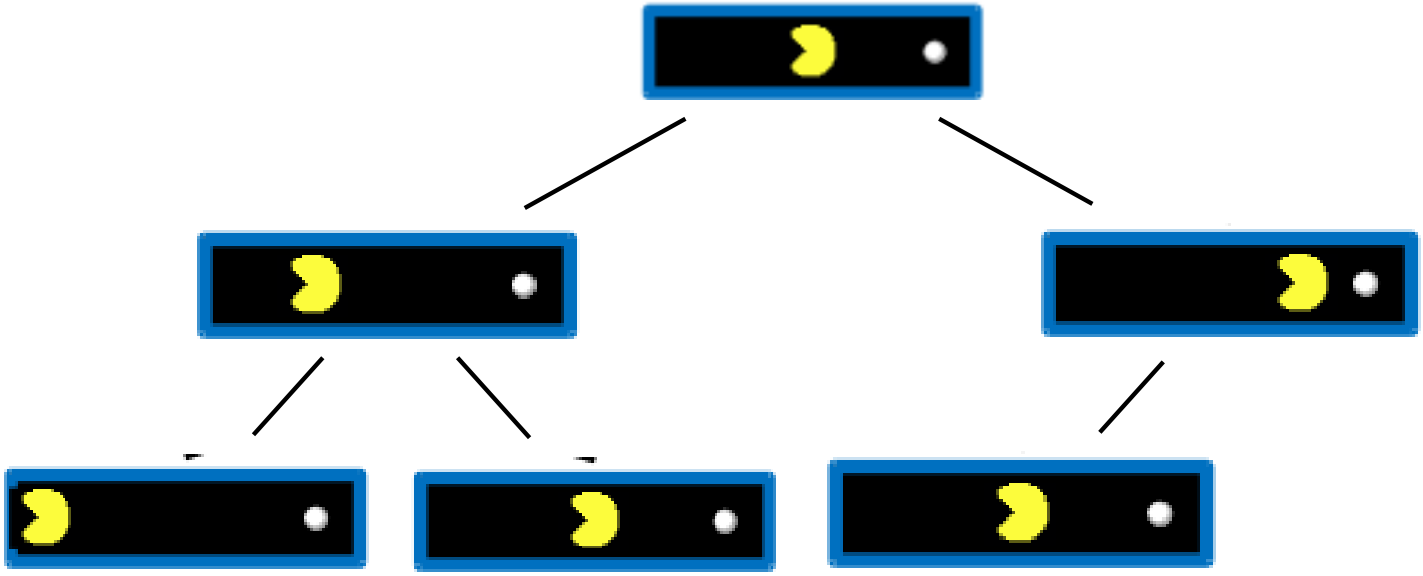

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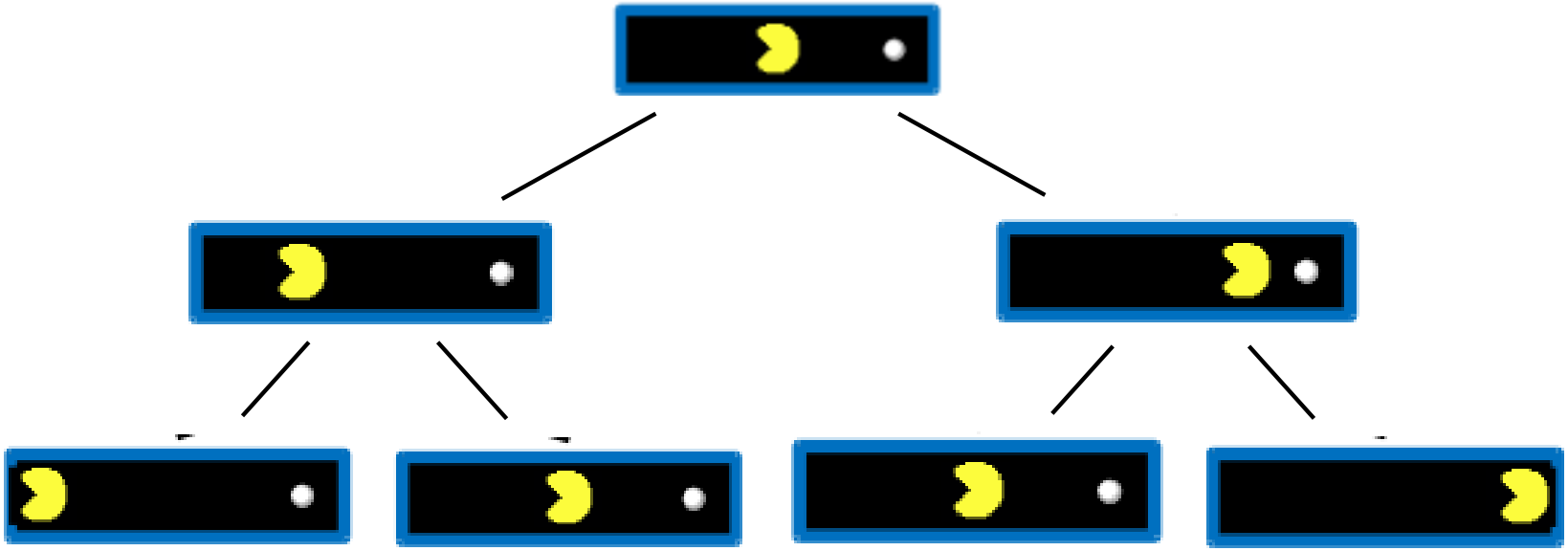

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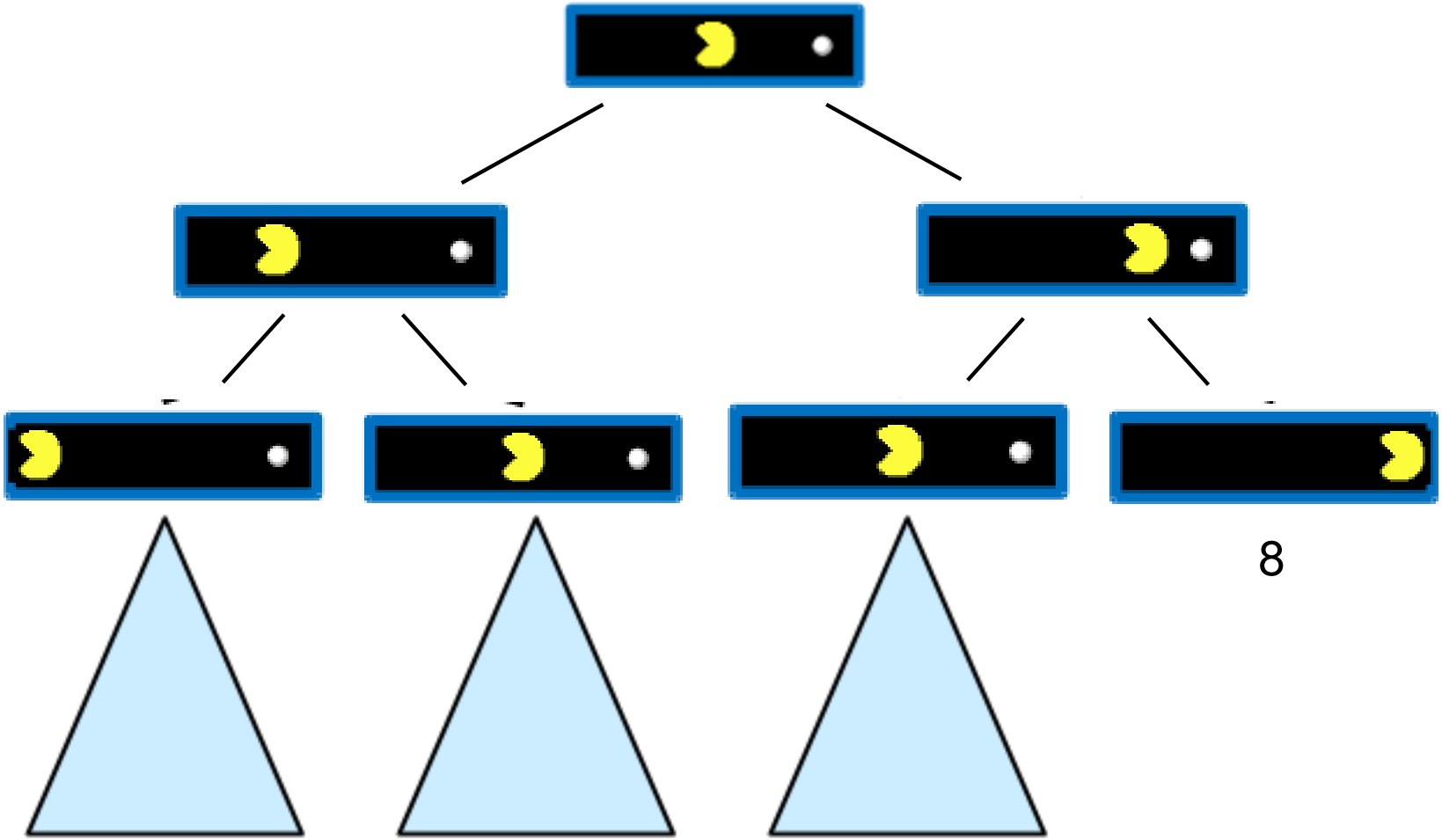

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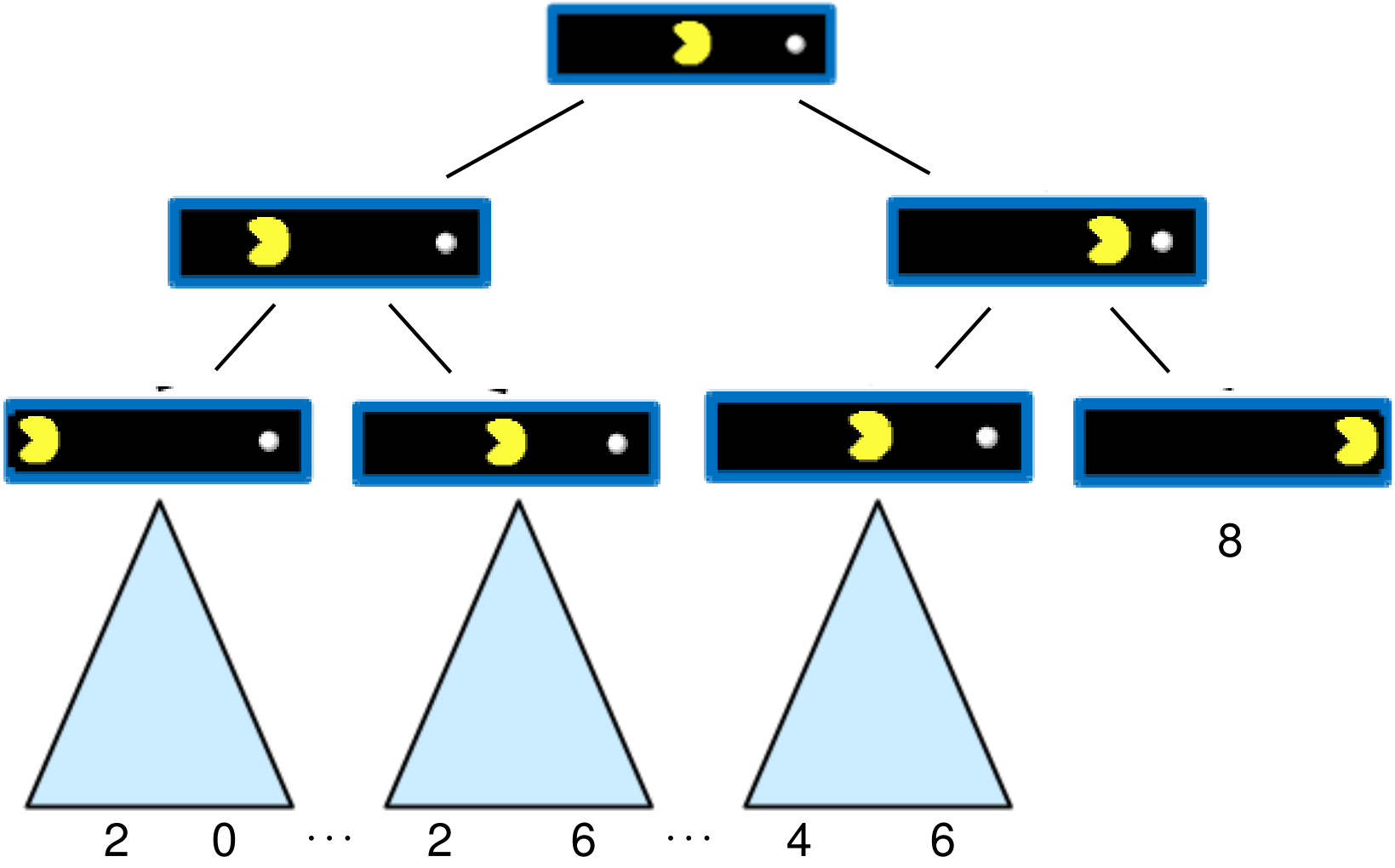

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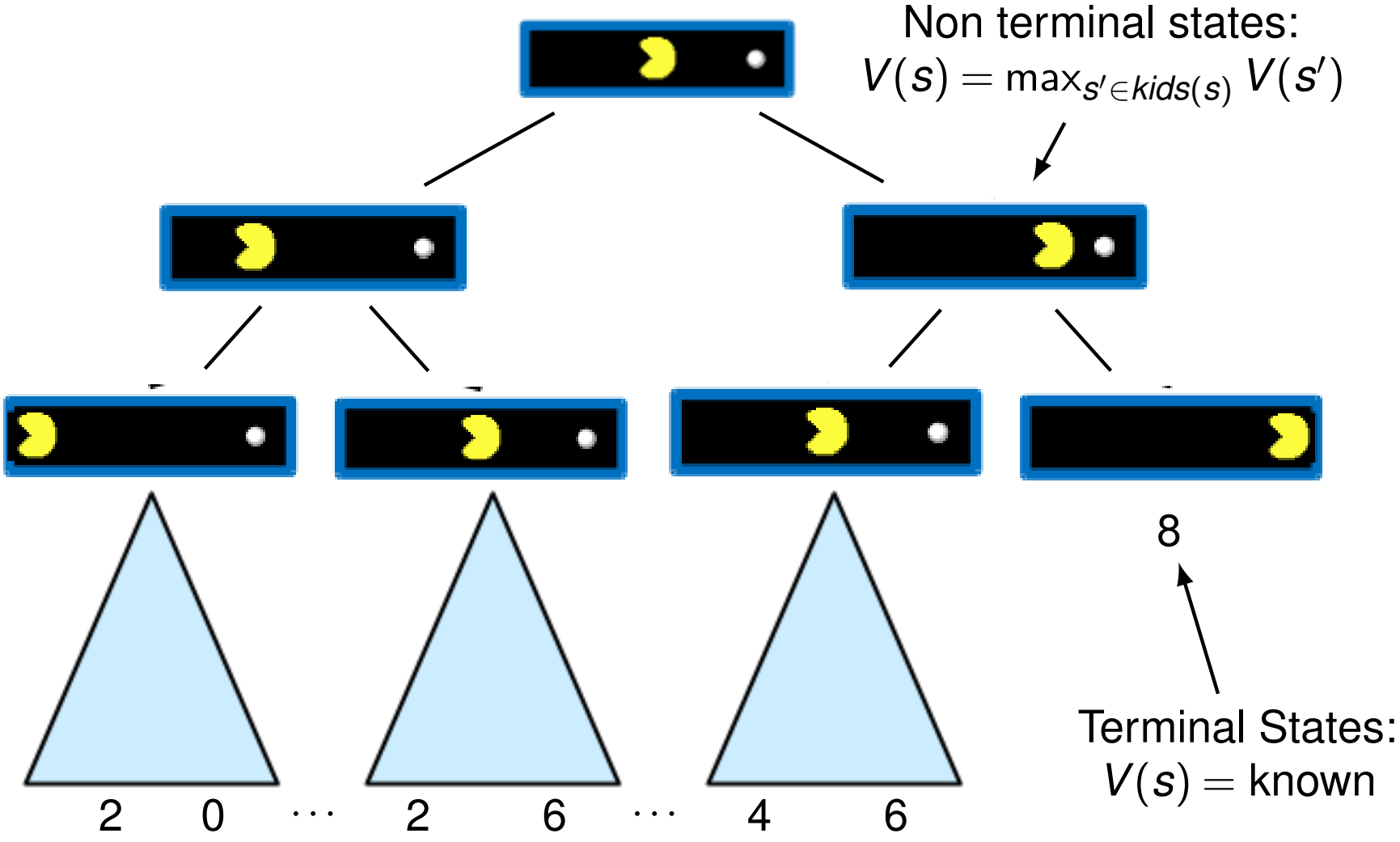

Adversarial Game Tree

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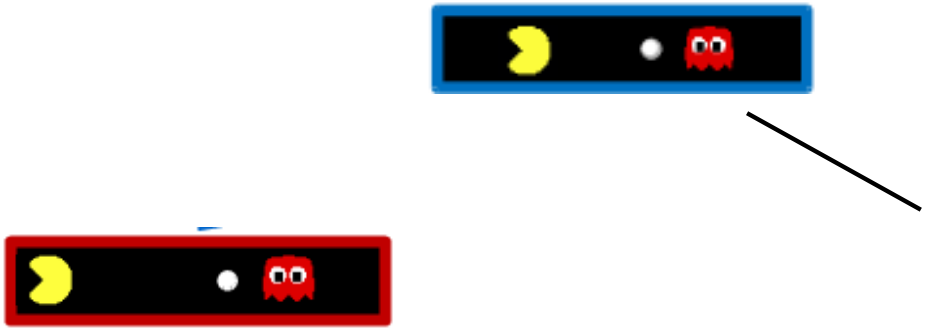
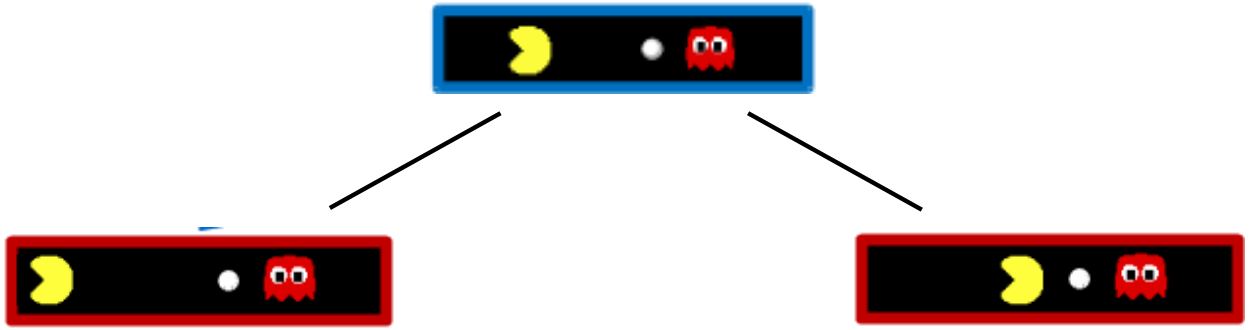


Adversarial Game Tree



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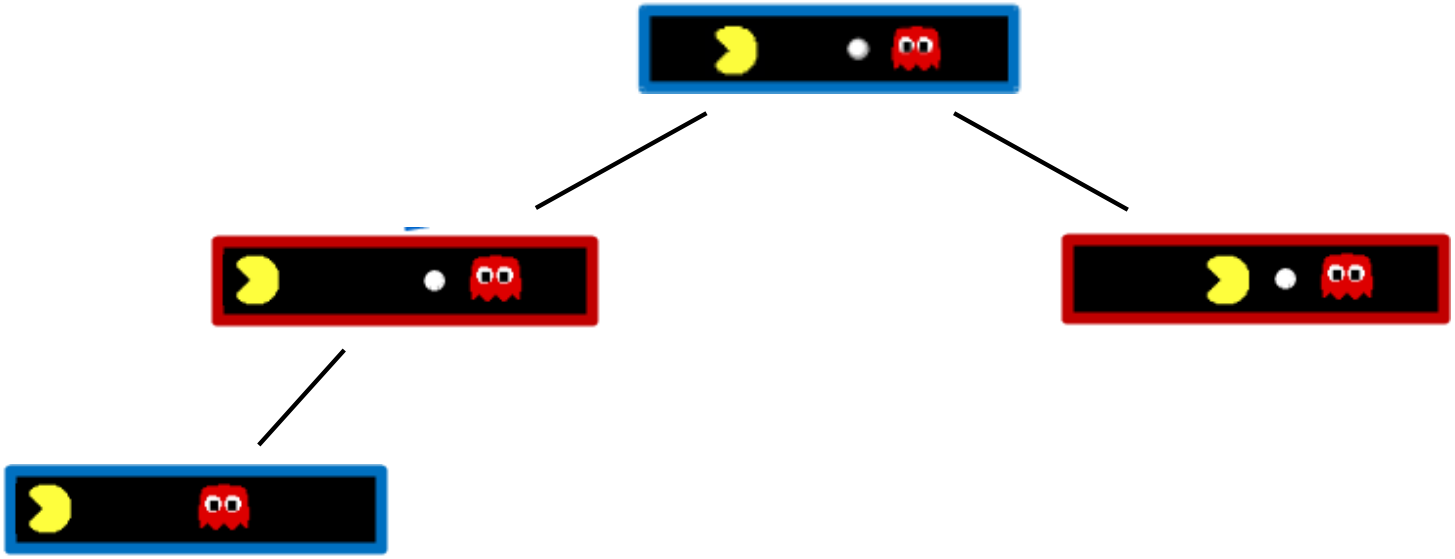

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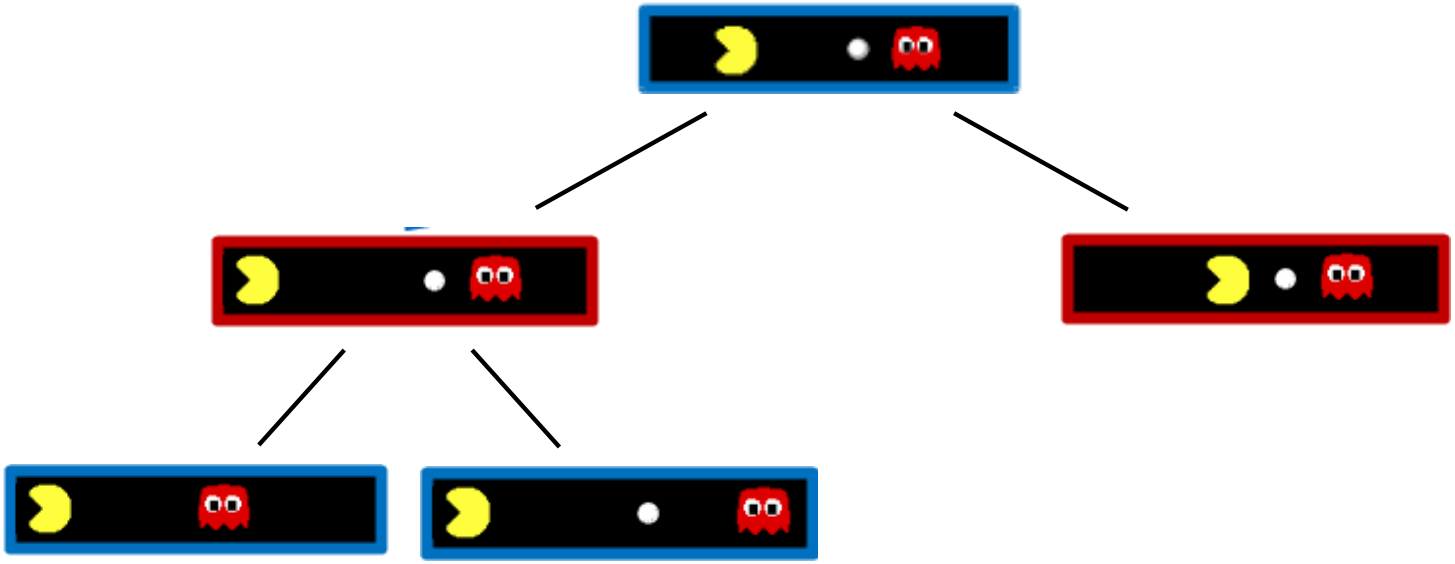

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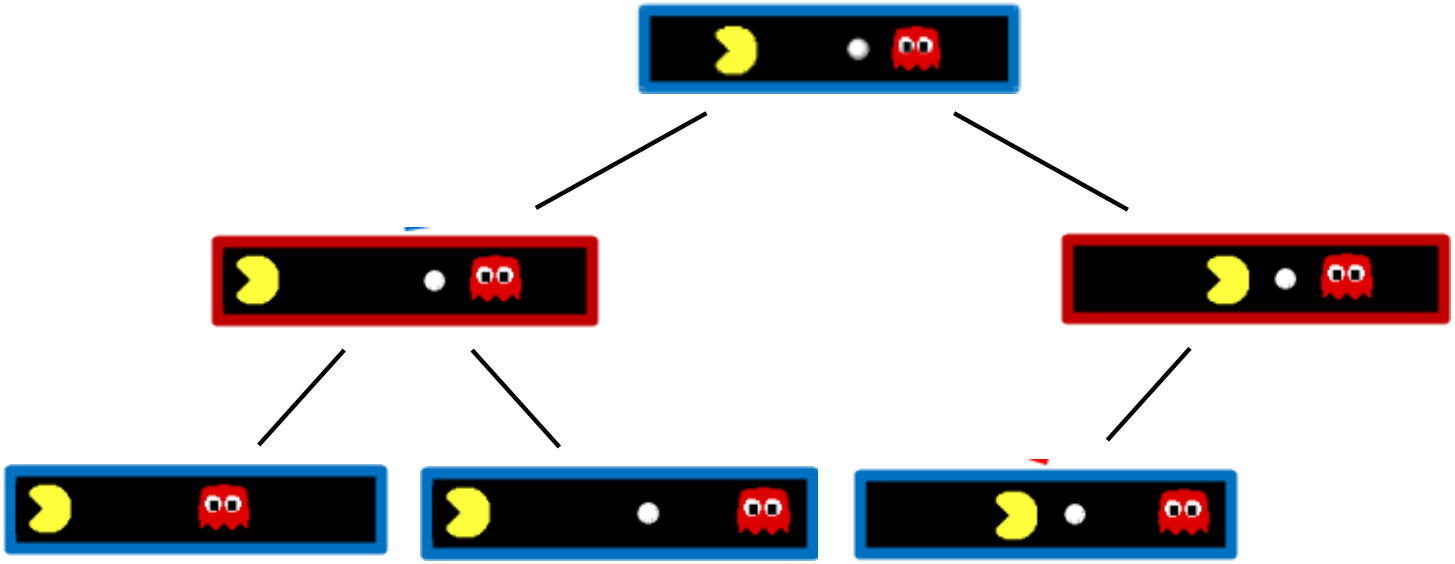

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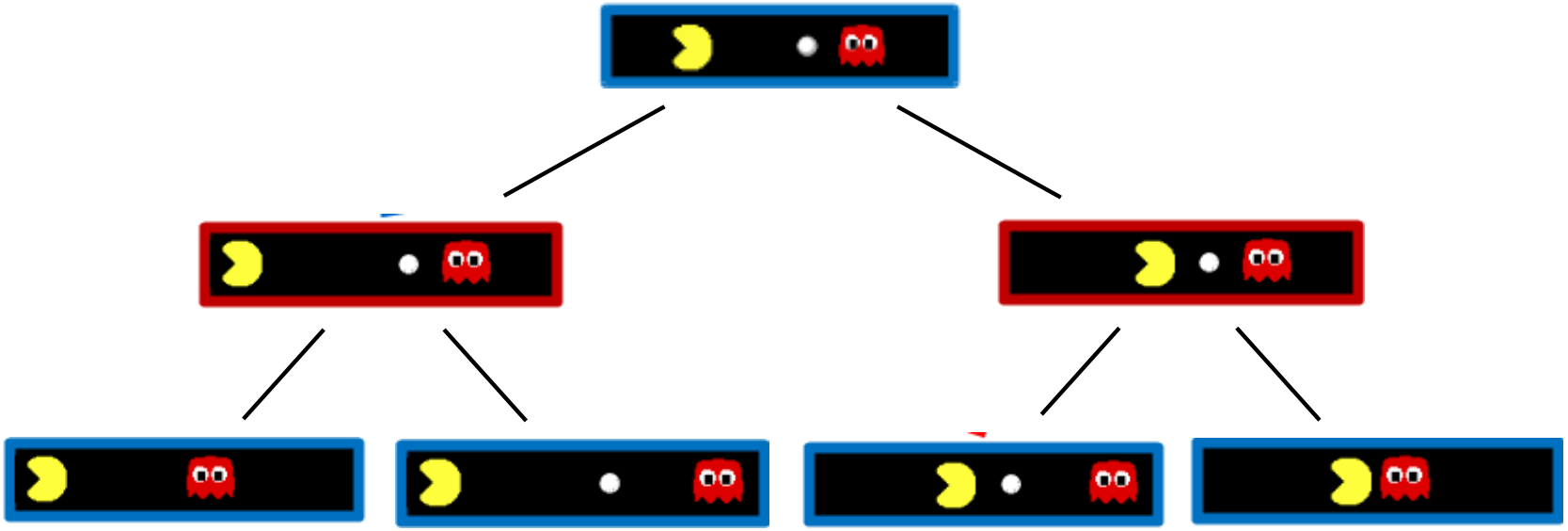
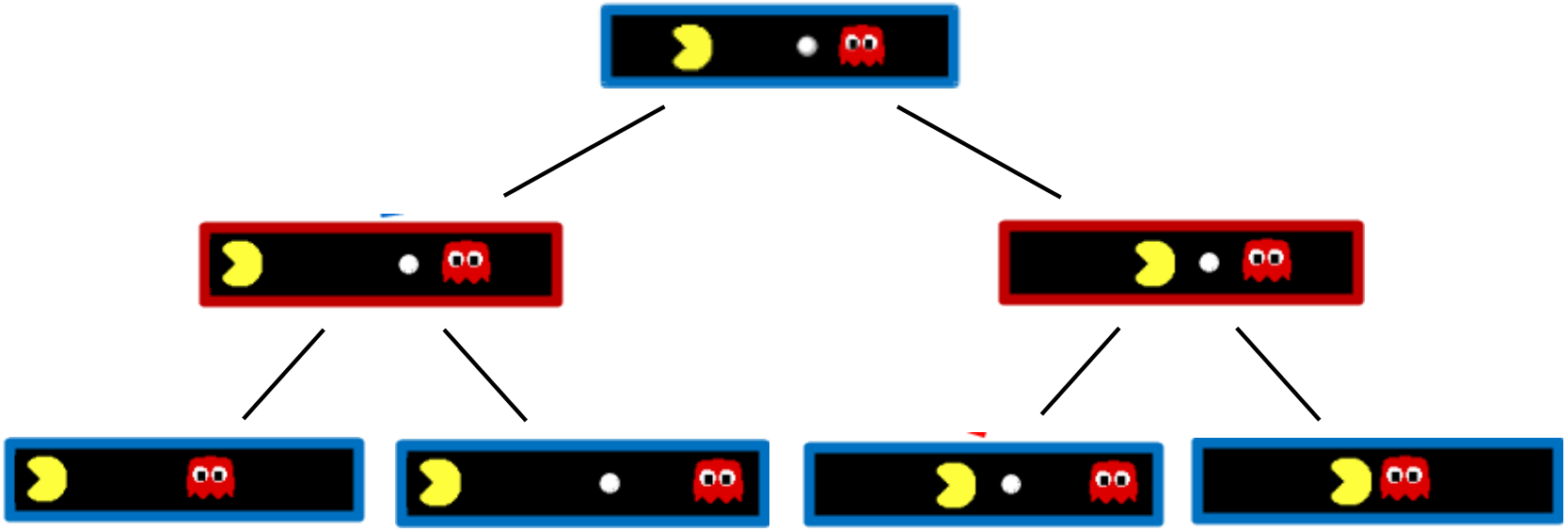


Adversarial Game Tree



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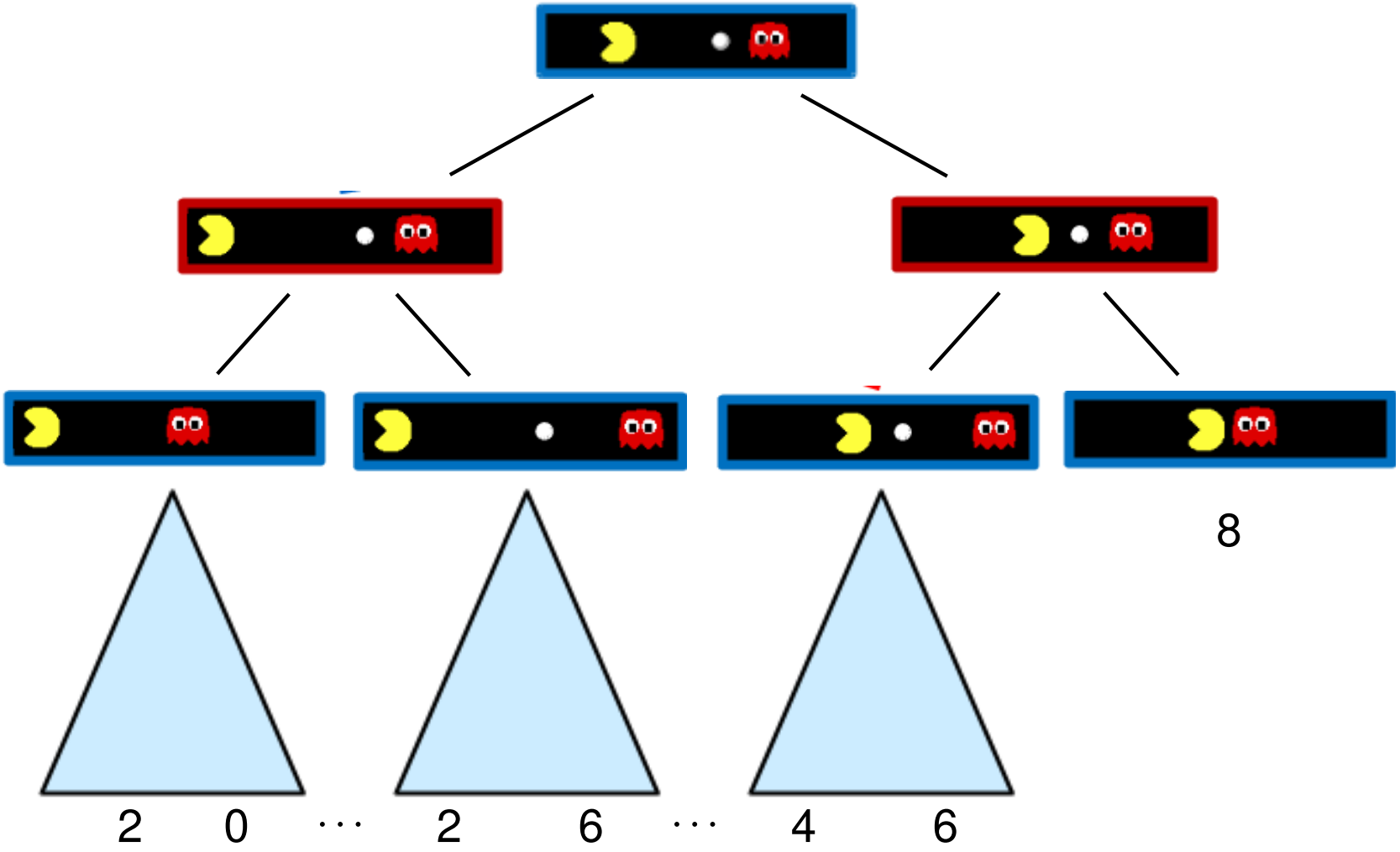
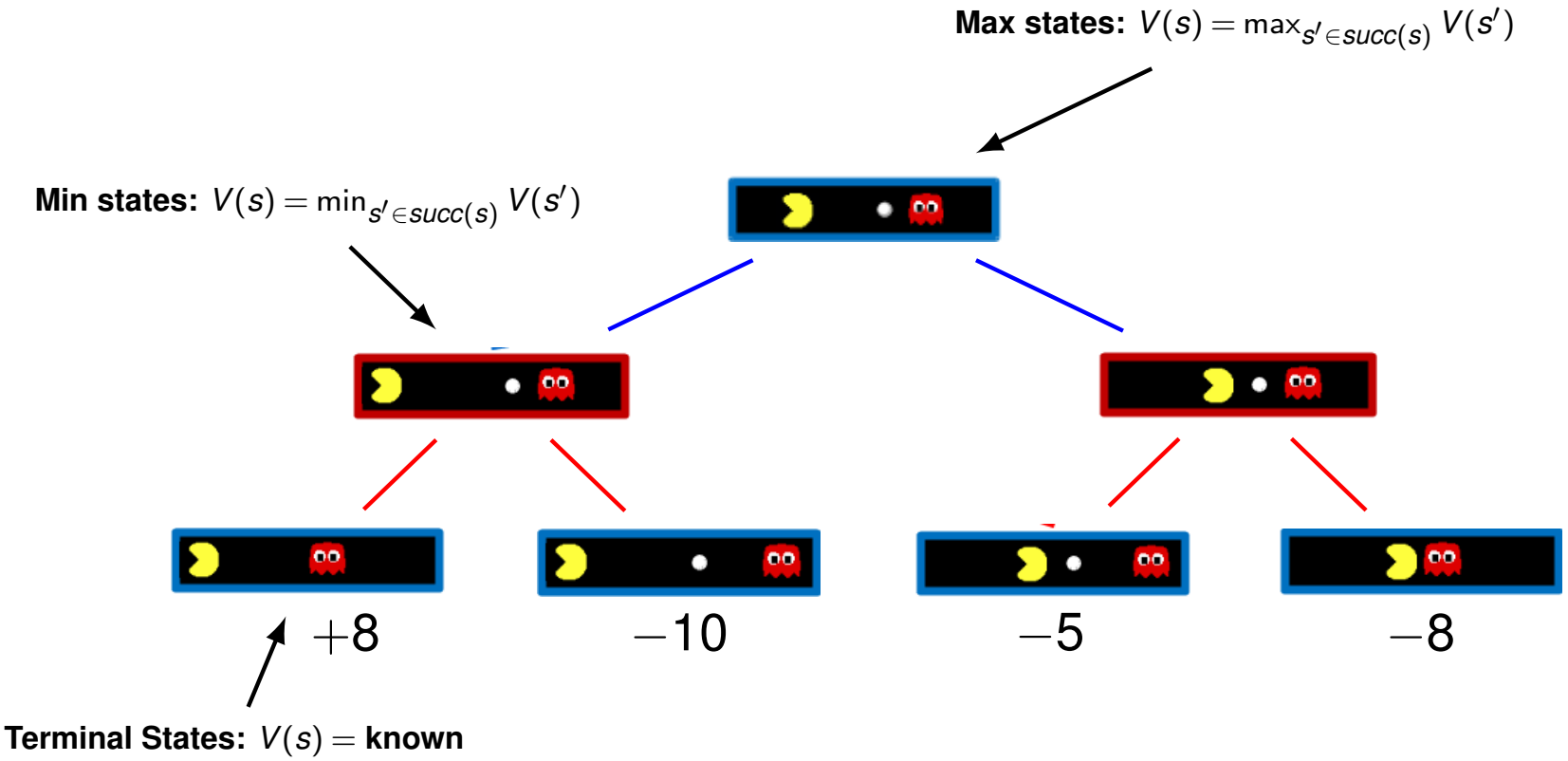


Minimax Values



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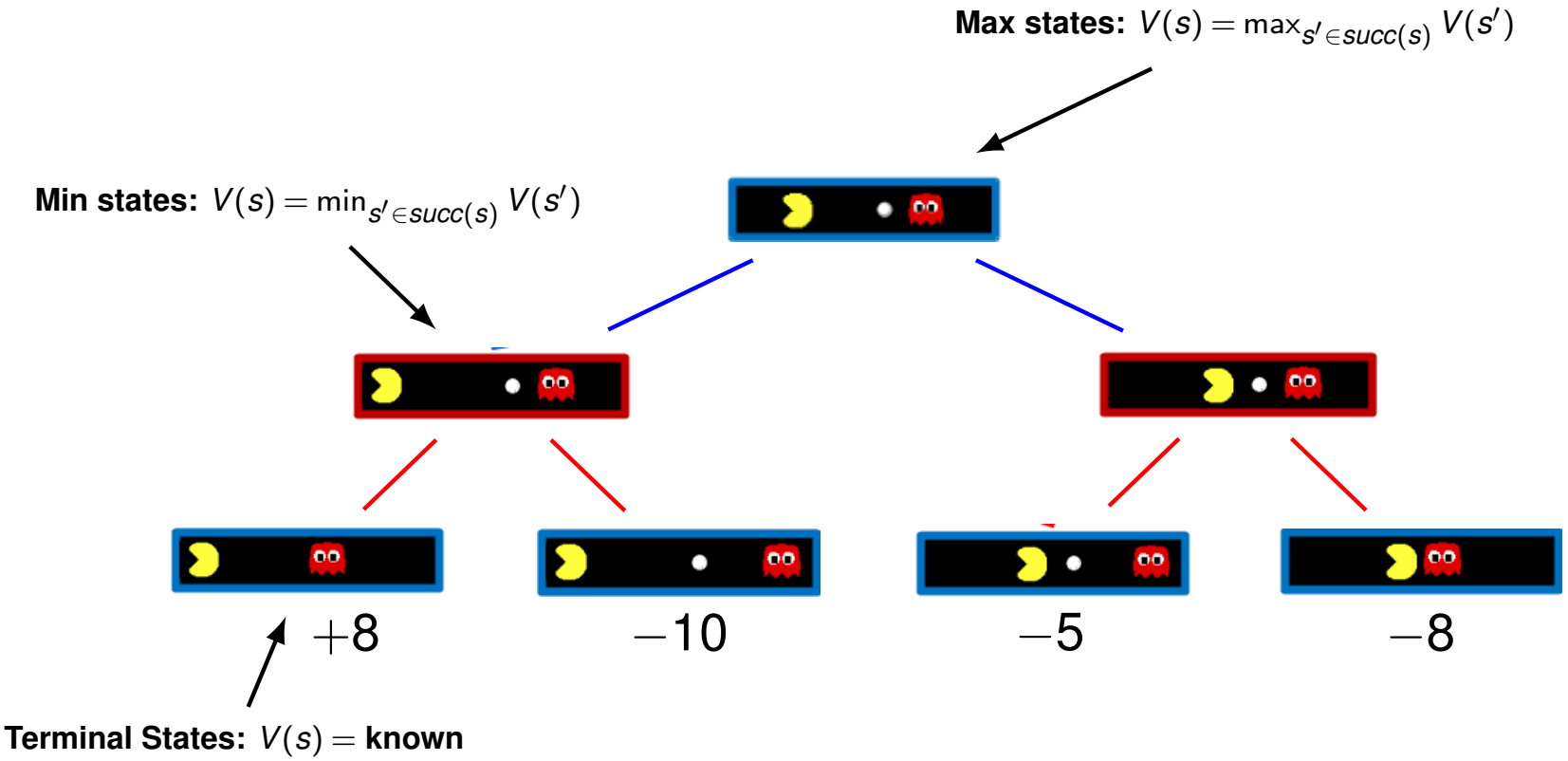
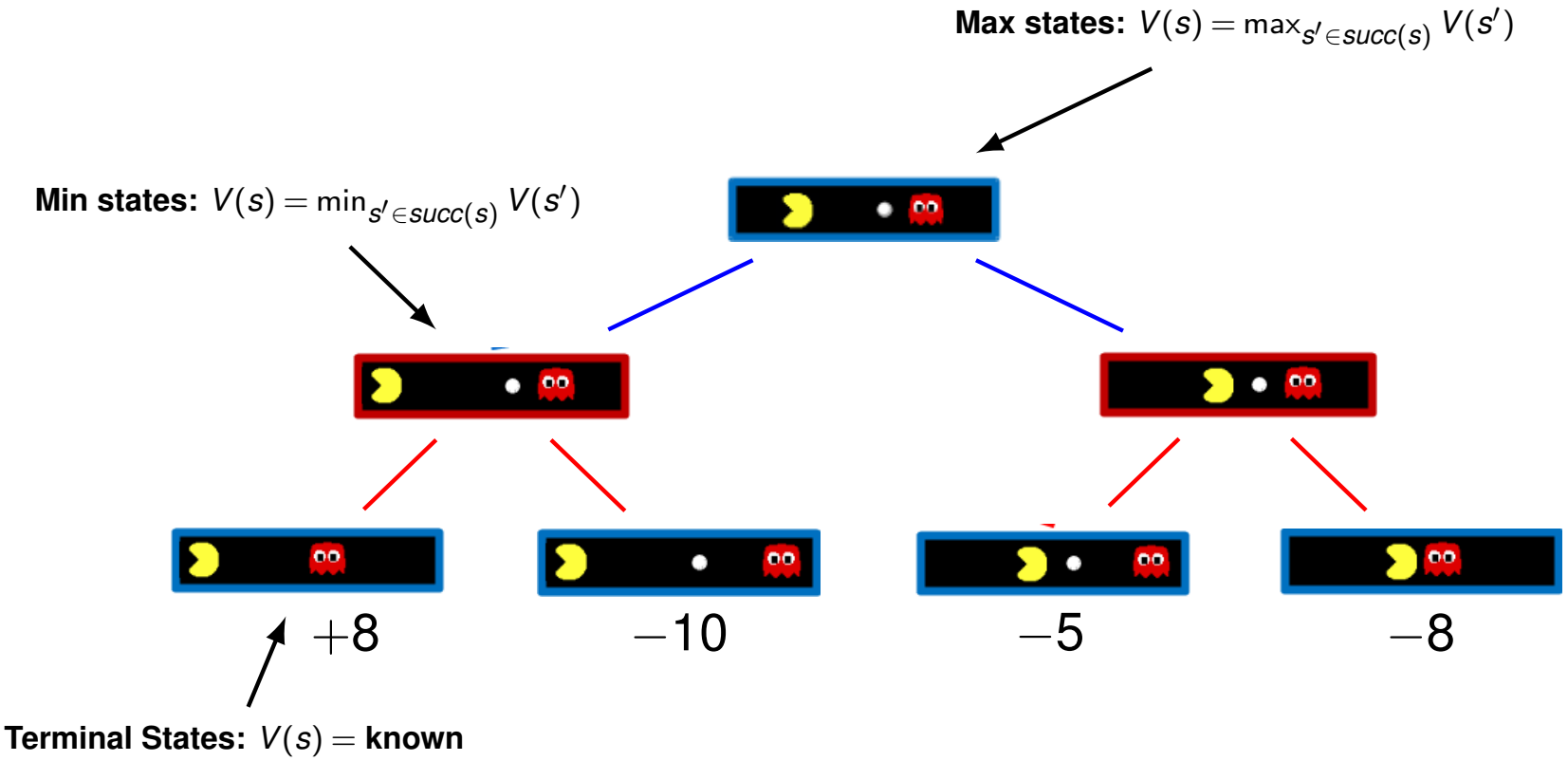


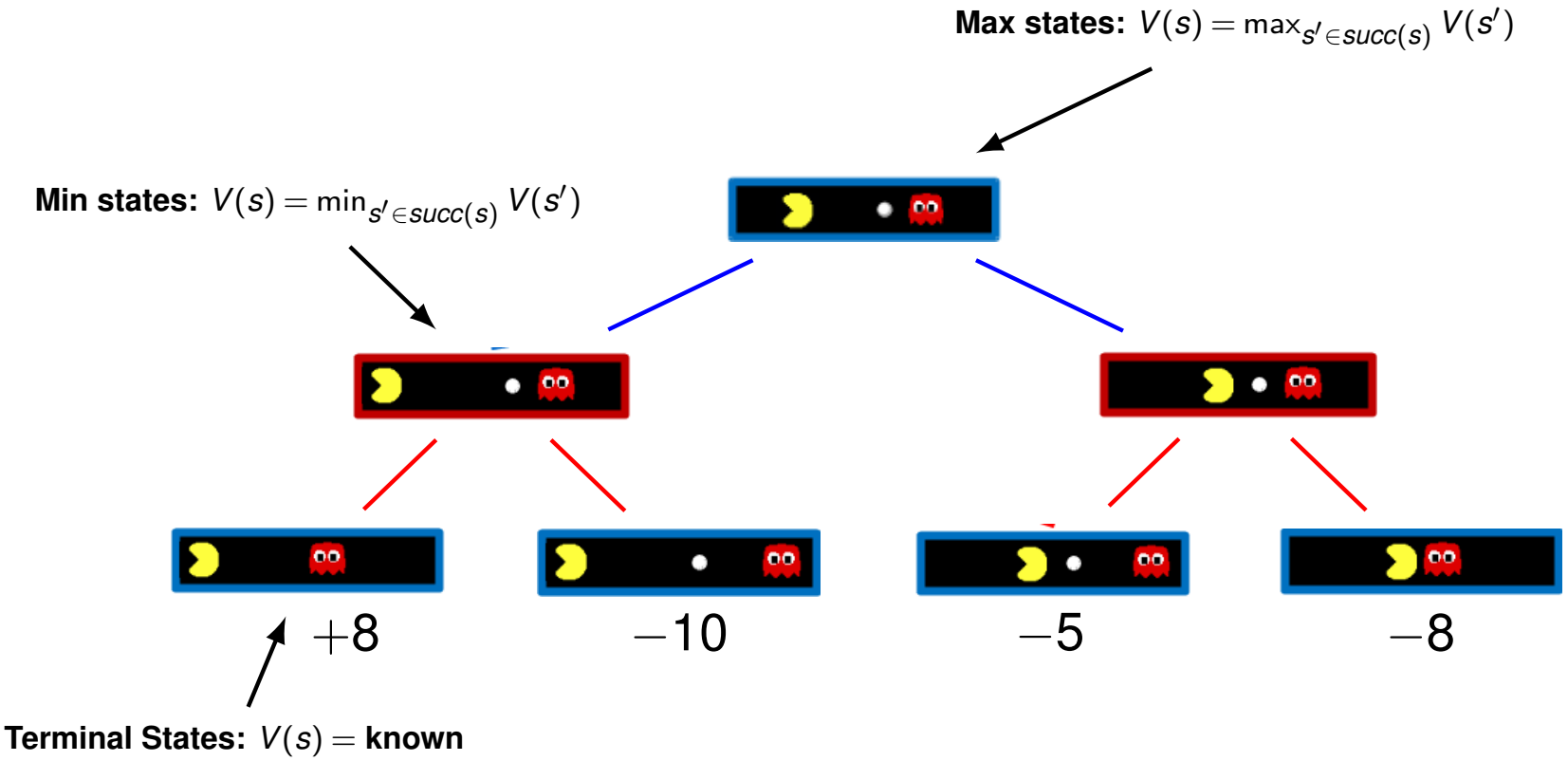
States Under Agent's Control.

Minimax Values



States Under Agent's Control. Max.

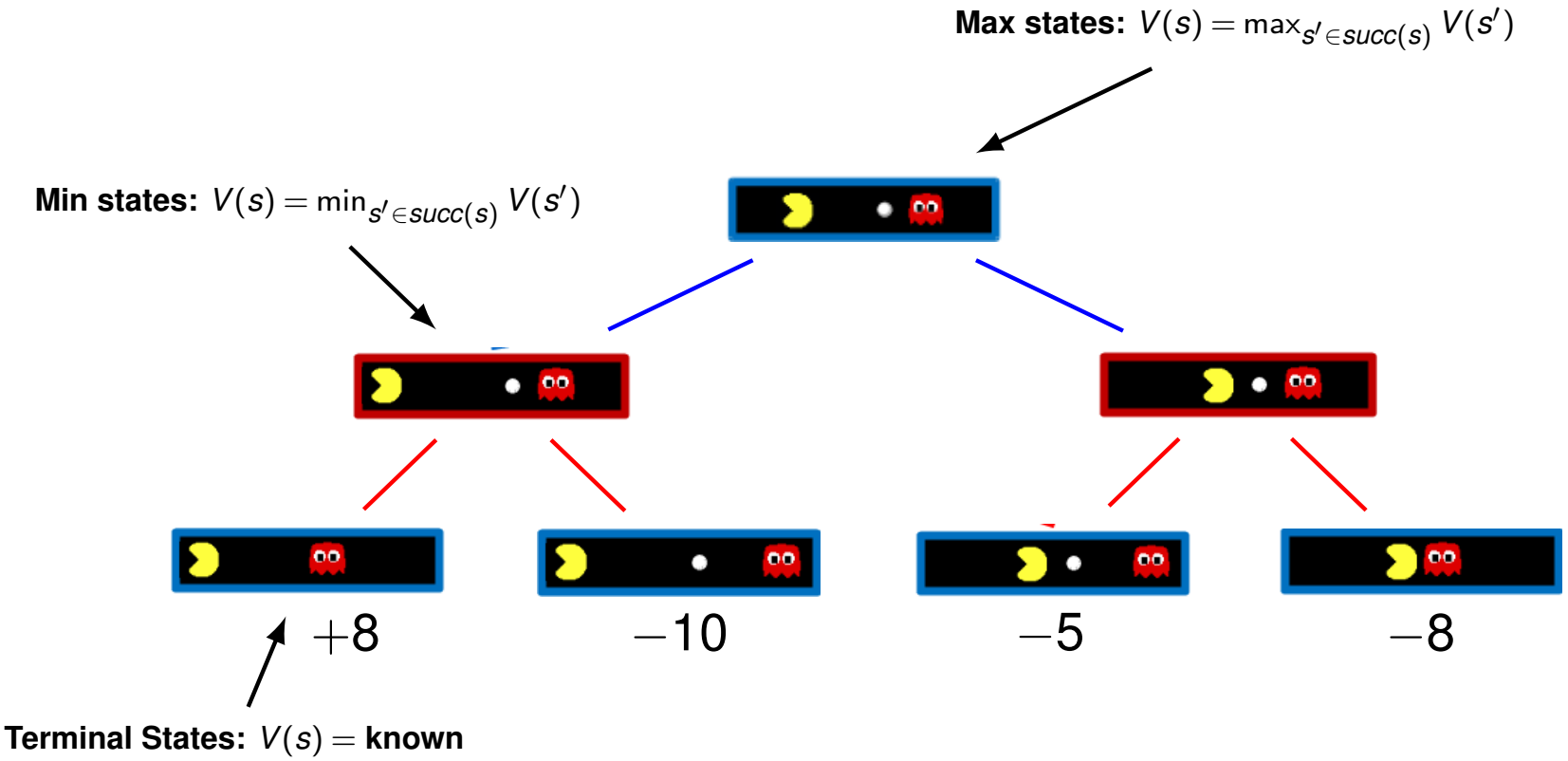
Minimax Values



States Under Agent's Control. Max.

Terminal States.

Minimax Values

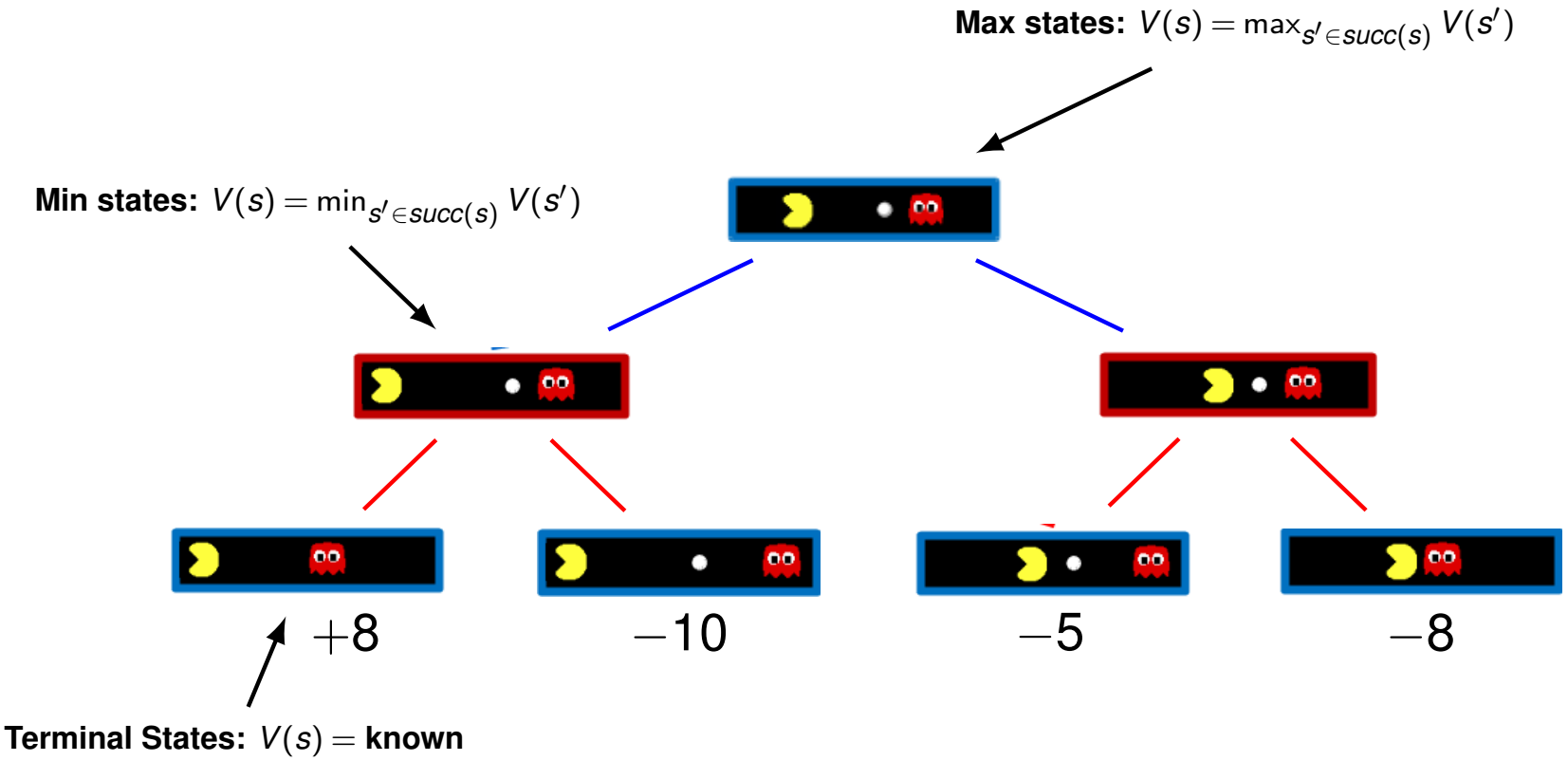


States Under Agent's Control. Max.

Terminal States.

States Under Opponent's Control.

Minimax Values

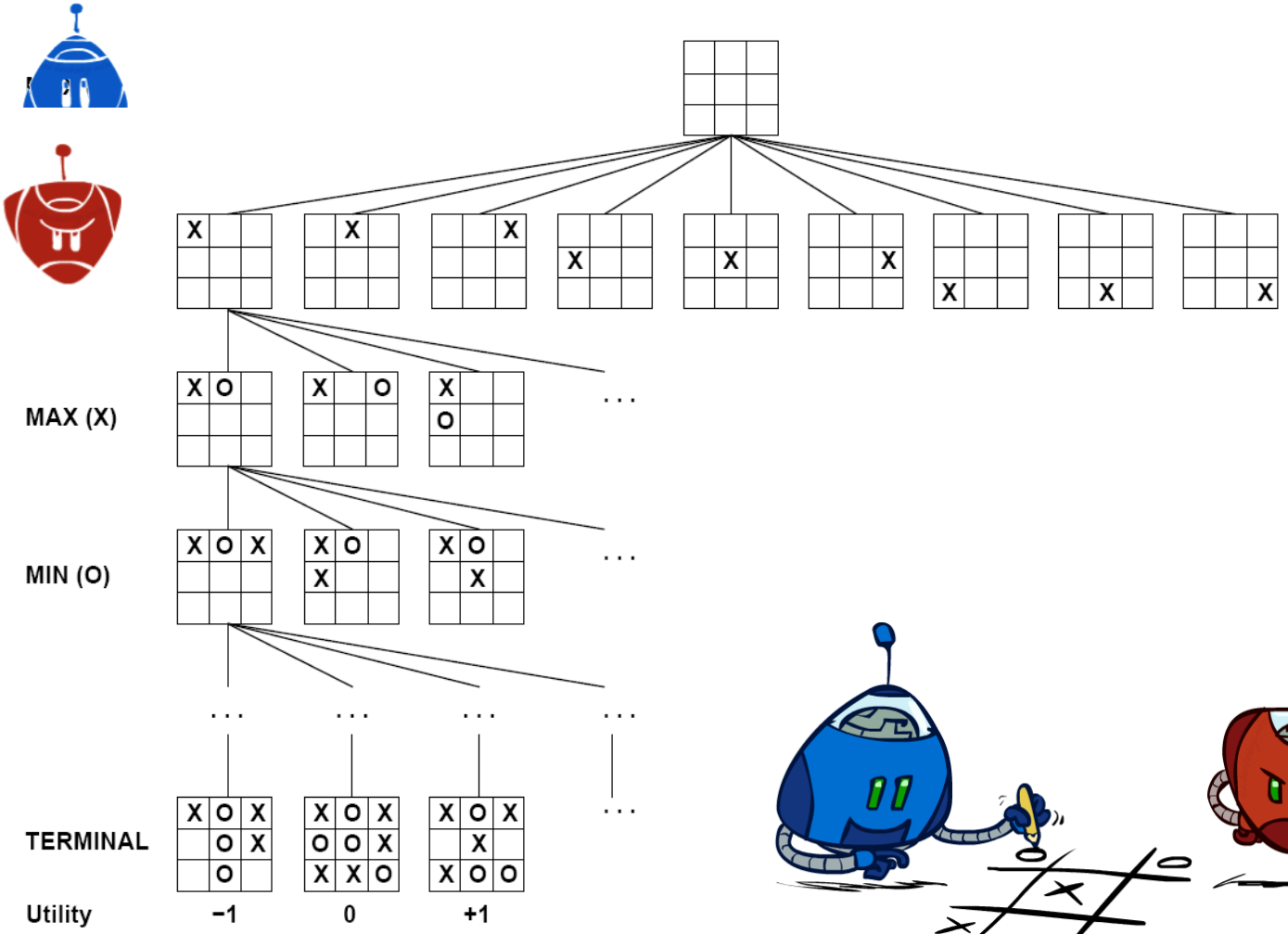


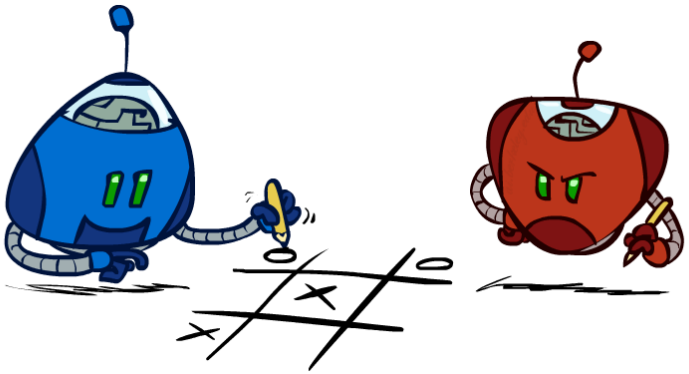
States Under Agent's Control. Max.

Terminal States.

States Under Opponent's Control. Min.

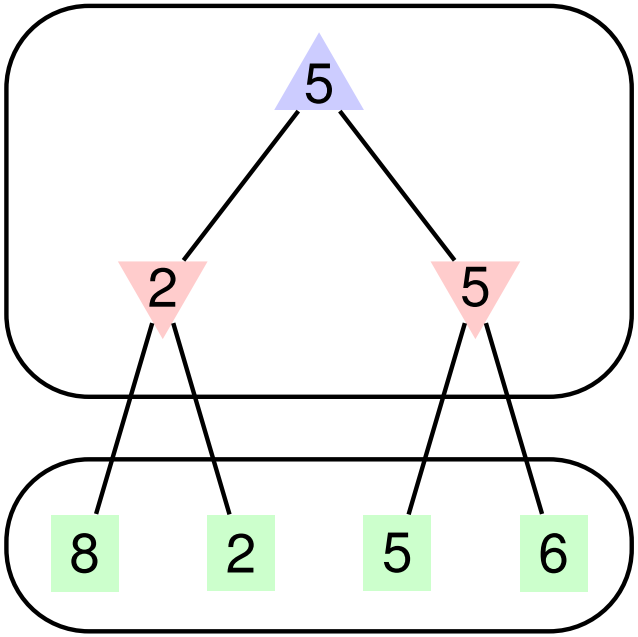
Tic-Tac-Toe Game Tree





Adversarial Search (Minimax)

Minimax values:computed recursively.



Terminal values:part of the game.

Adversarial Search (Minimax)

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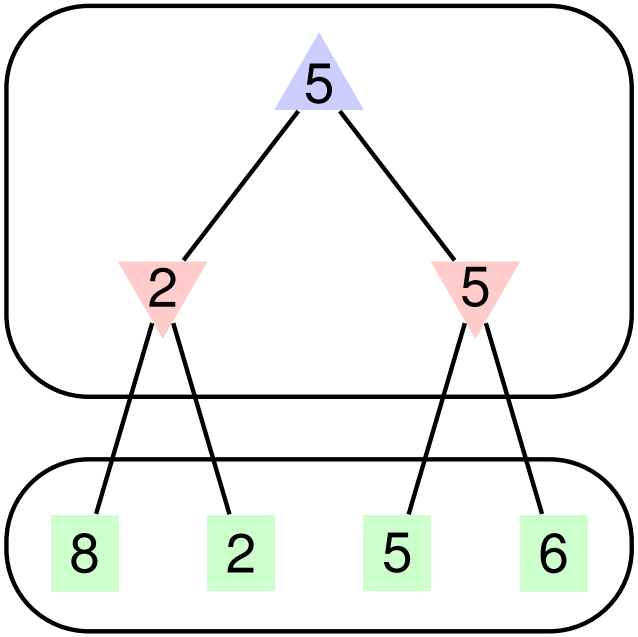

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Deterministic, zero-sum games:

- Tic-tac-toe, chess, checkers

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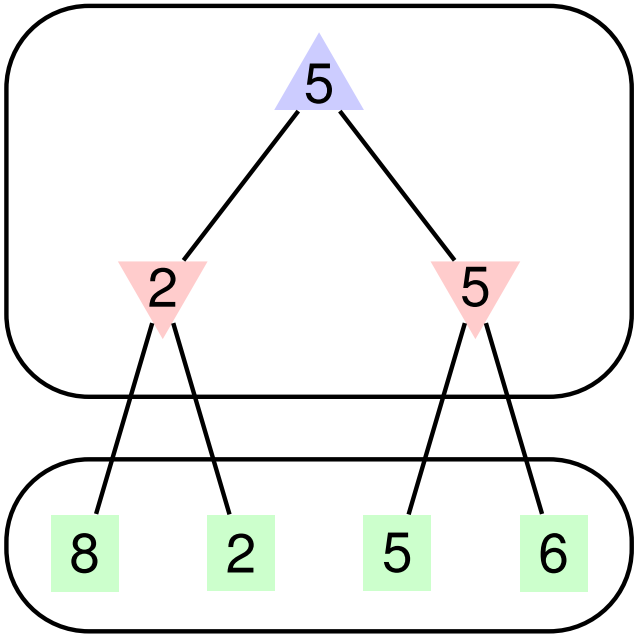

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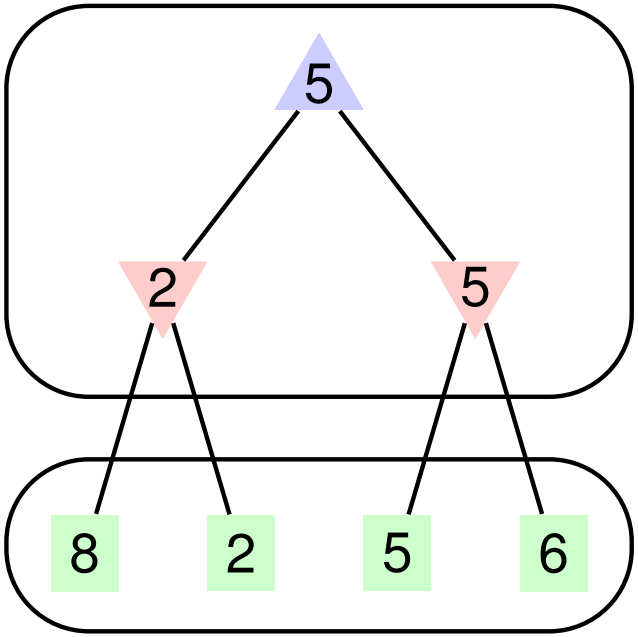

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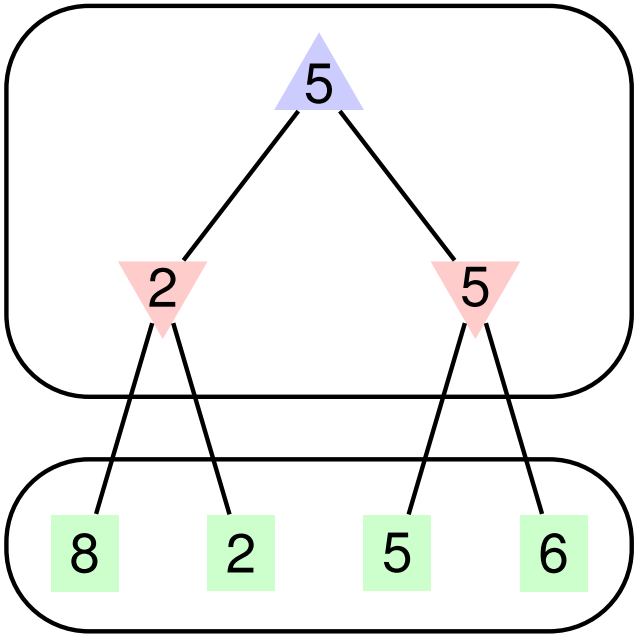

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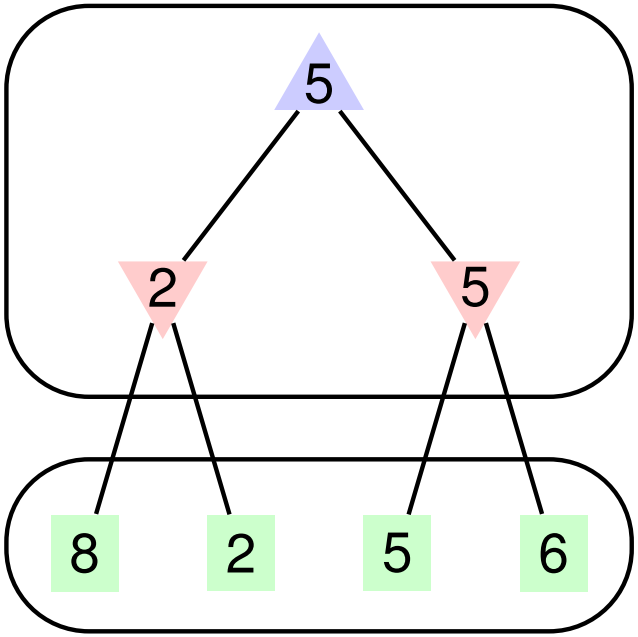

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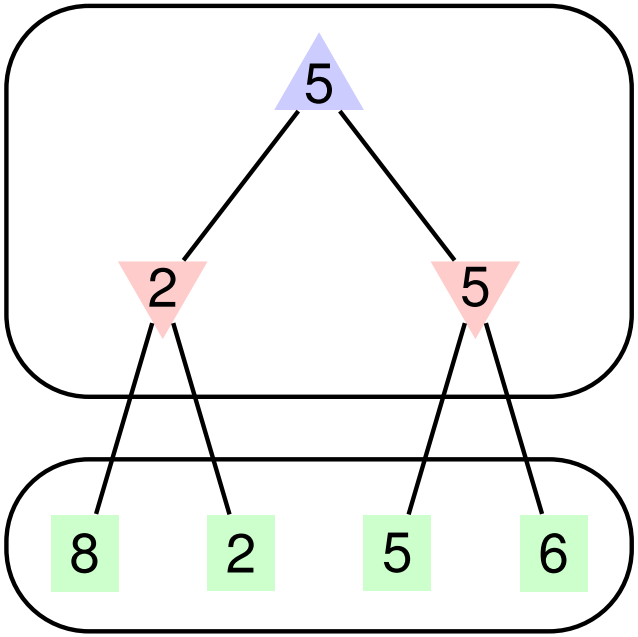
Minimax search:

- A state-space search tree

Adversarial Search (Minimax)

Minimax values:computed recursively.



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Minimax search:

- A state-space search tree
- Players alternate turns

Adversarial Search (Minimax)

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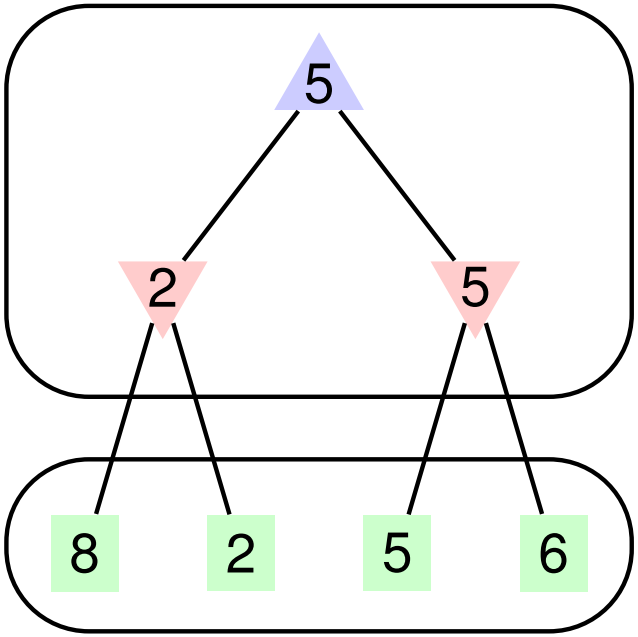

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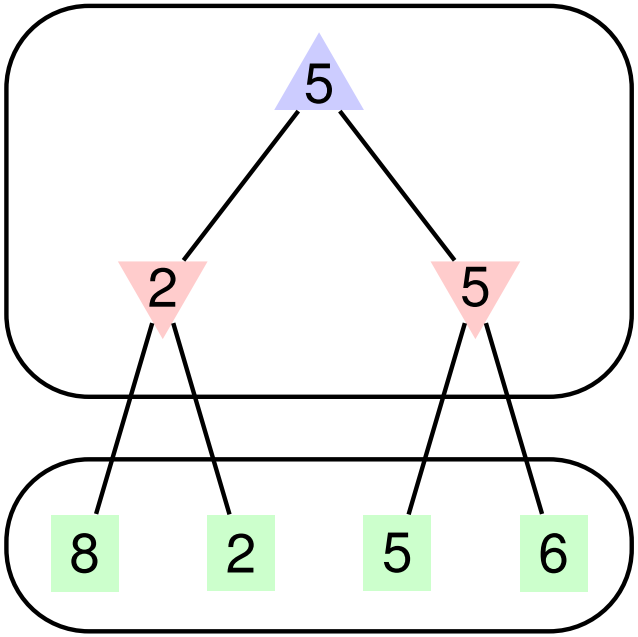
Minimax search:

- A state-space search tree
- Players alternate turns
- Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary

Adversarial Search (Minimax)

Minimax values:computed recursively.



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- A state-space search tree
- Players alternate turns
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Minimax Implementation

$V(s) = \min_{s' \in succ(s)} V(s')$

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def min-value(state):

Minimax Implementation

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Minimax Implementation

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def min-value(state):
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for each successor s **of state:**

Minimax Implementation

$V(s) = \min_{s' \in succ(s)} V(s')$

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def min-value(state):
    initialize v = +∞
    for each successor s of state:
        v = min(v, max-value(s))
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Minimax Implementation

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def min-value(state):
    initialize v = +∞
    for each successor s of state:
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Minimax Implementation

$V(s) = \min_{s' \in succ(s)} V(s')$

$V(s') = \max_{s \in succ(s')} V(s)$

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def min-value(state):
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Minimax Implementation

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def max-value(state):
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def value(state):
```

Minimax Implementation

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def value(state):
  if the state is terminal: return the state's utility
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def max-value(state):
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  for each successor s of state:
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  return v
```

```
def value(state):
  if the state is terminal: return the state's utility
  if the next agent is MAX: return max-value(state)
```

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```
def min-value(state):
  initialize v = +∞
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```
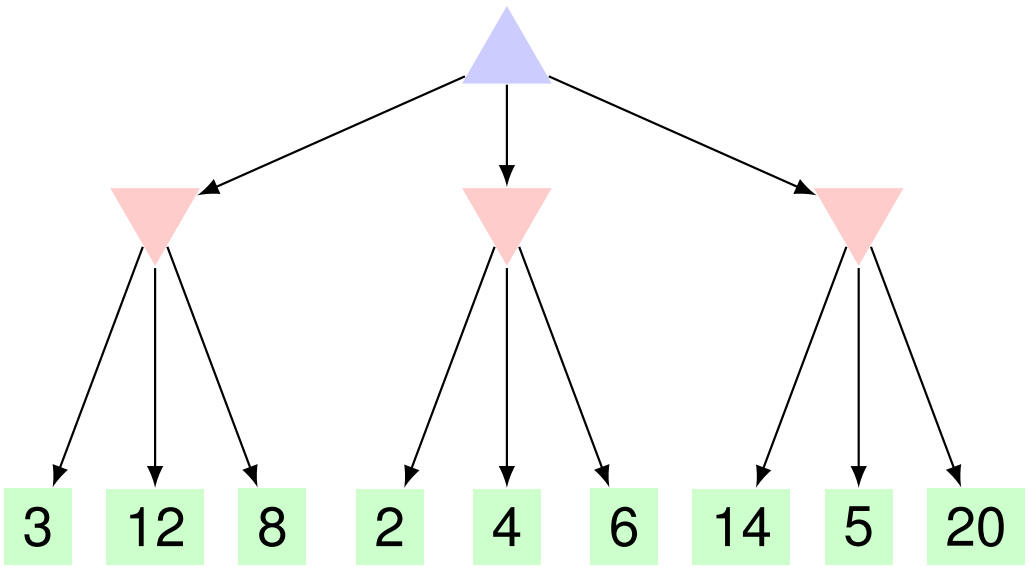
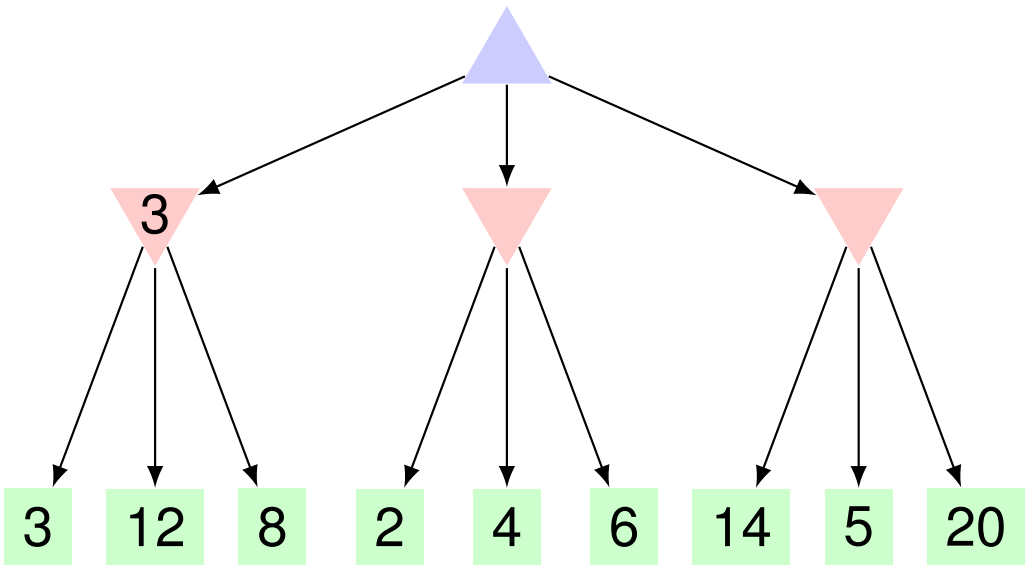
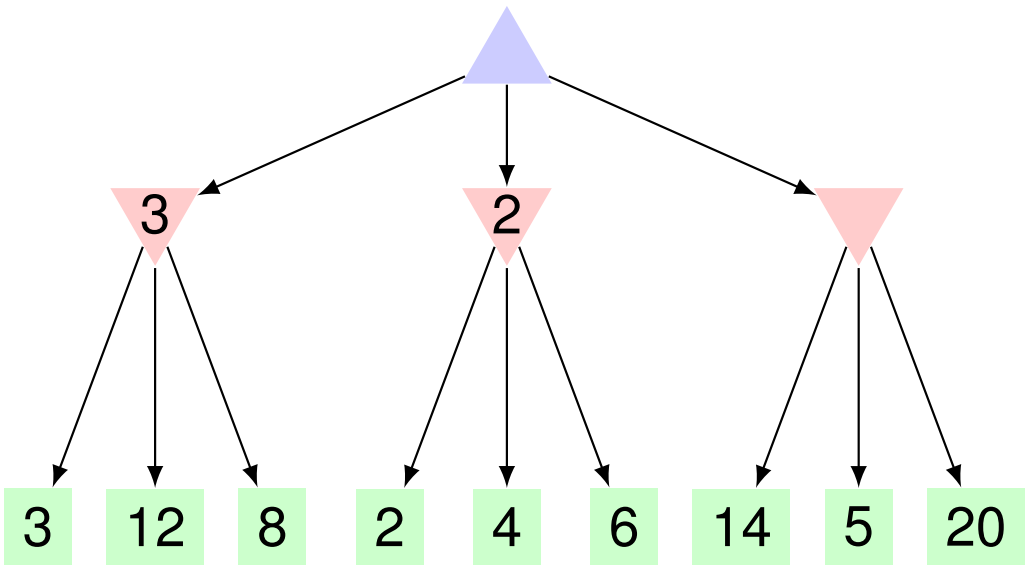

Minimax Example



Minimax Example



Minimax Example



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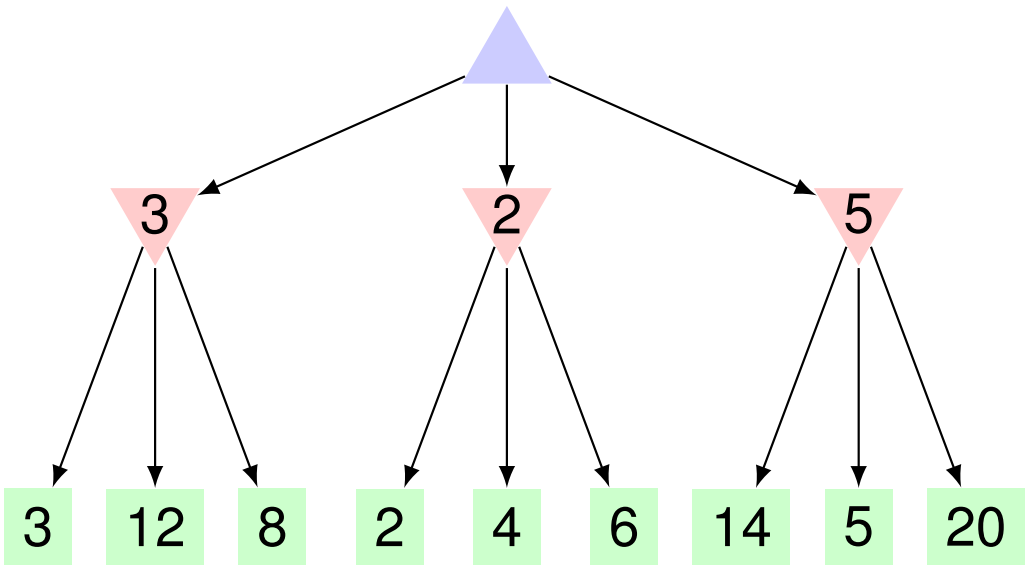
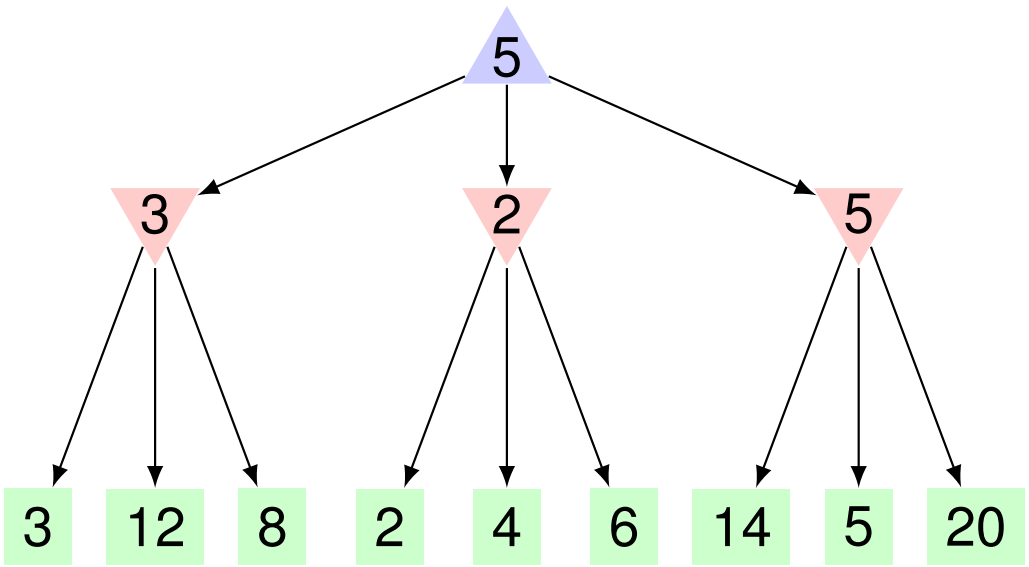
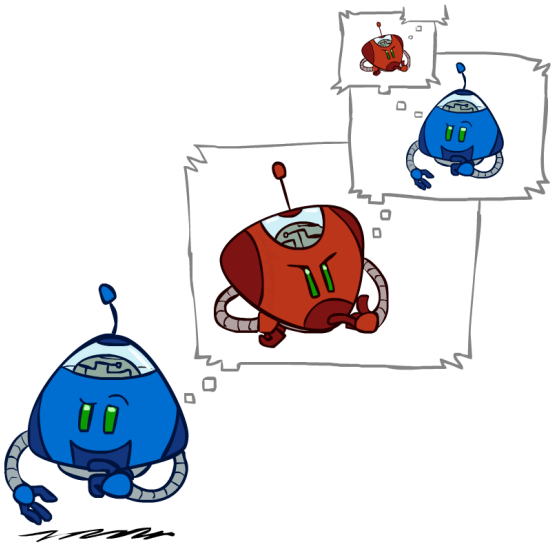


Minimax Example



Minimax Efficiency



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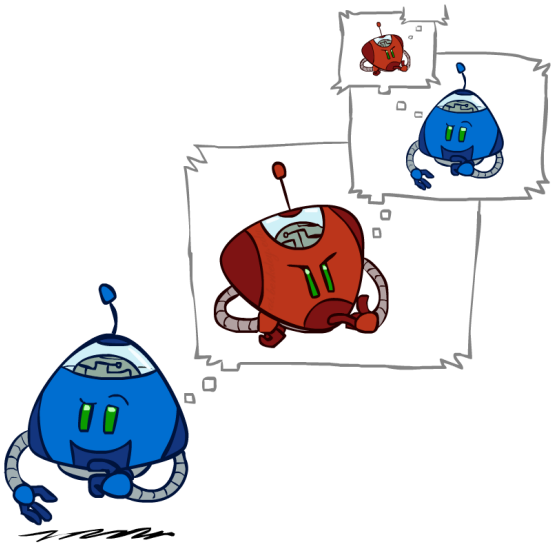

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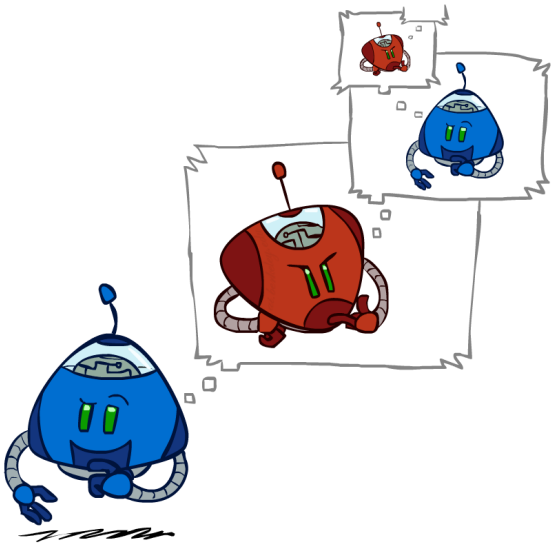

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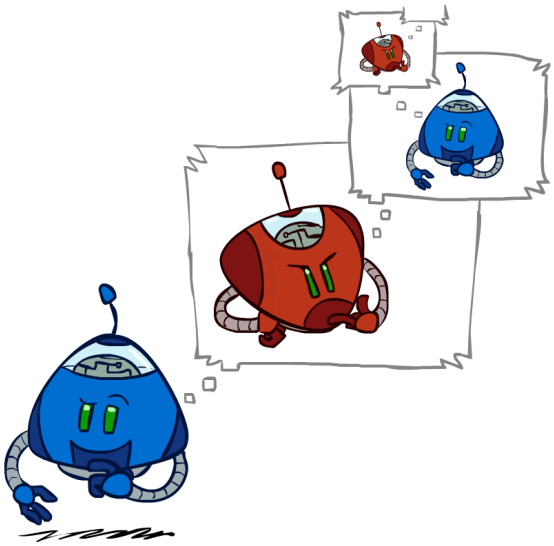

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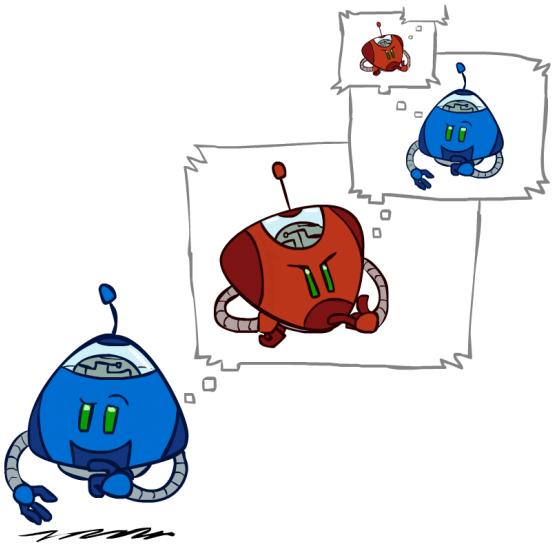

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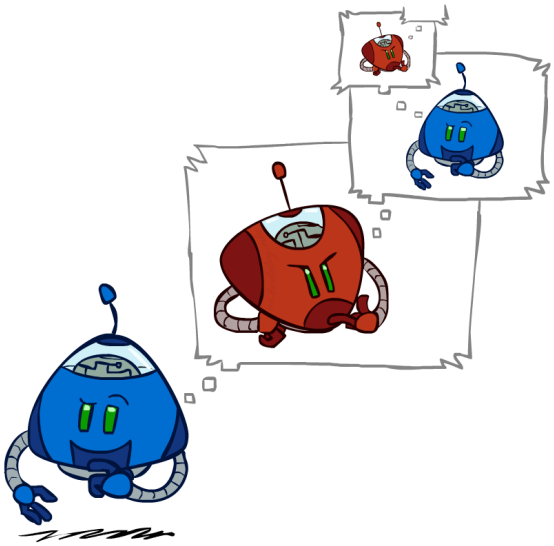

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- Exact solution is completely infeasible.

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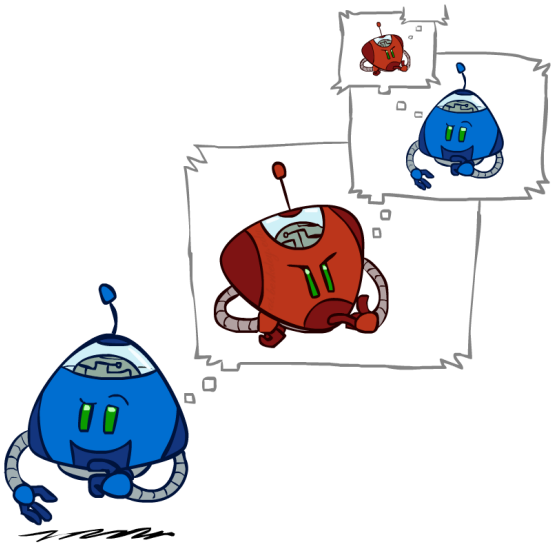

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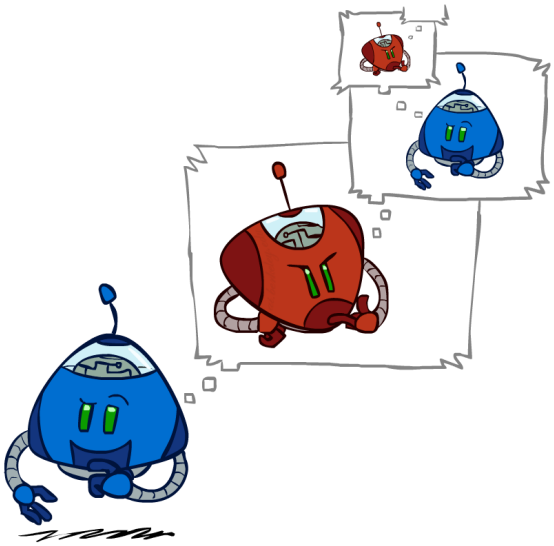

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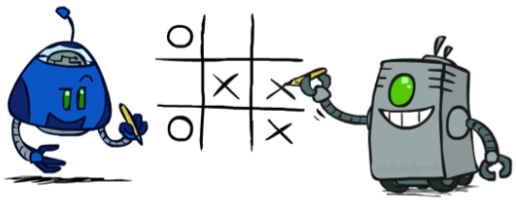

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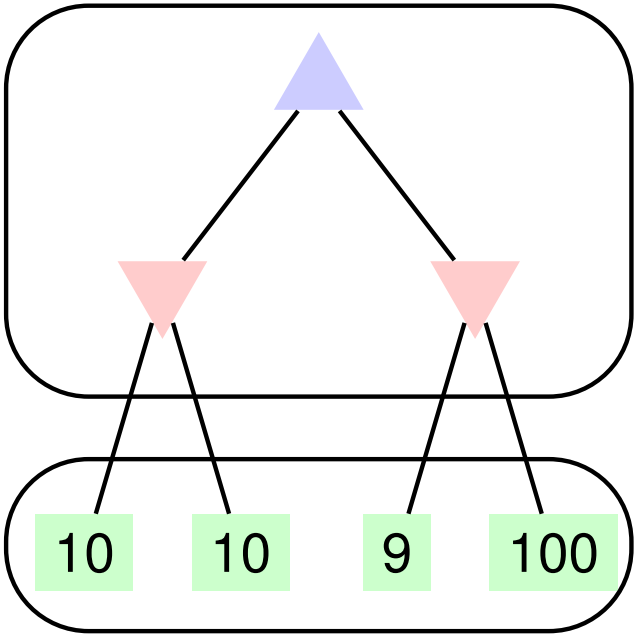

Terminal values:part of the game.

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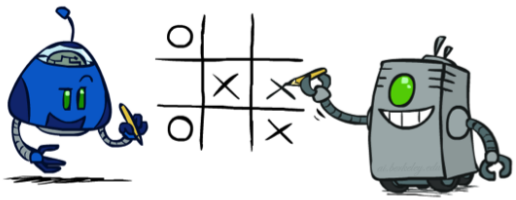

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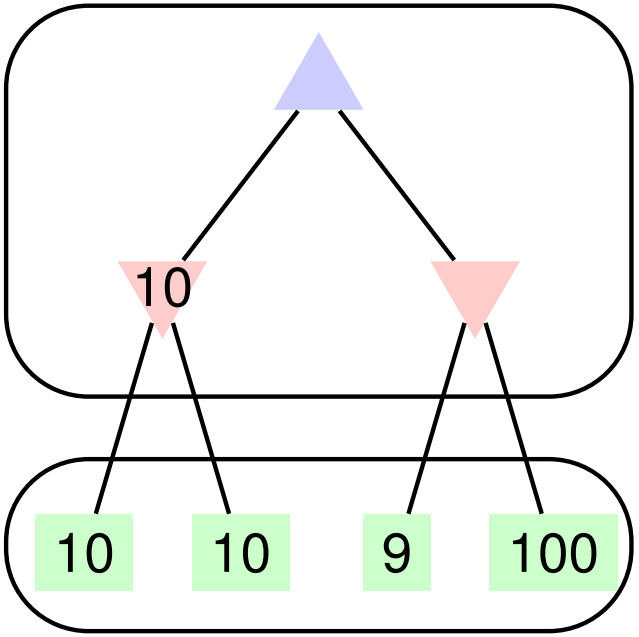

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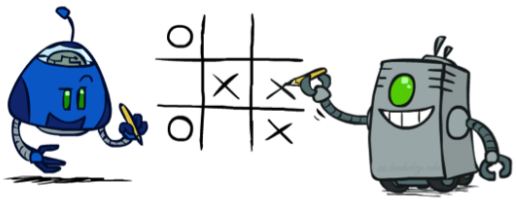

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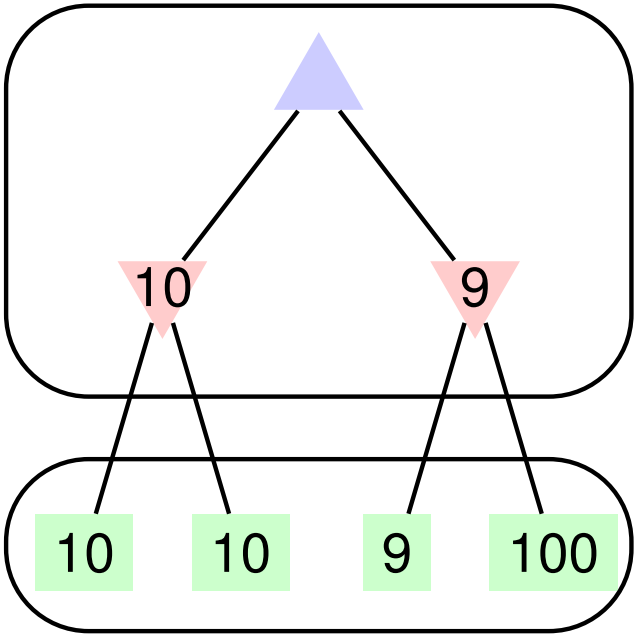

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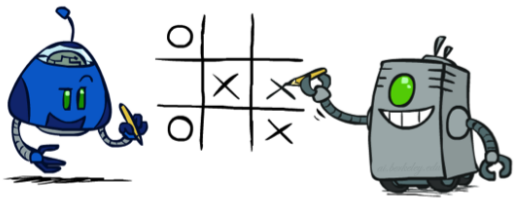

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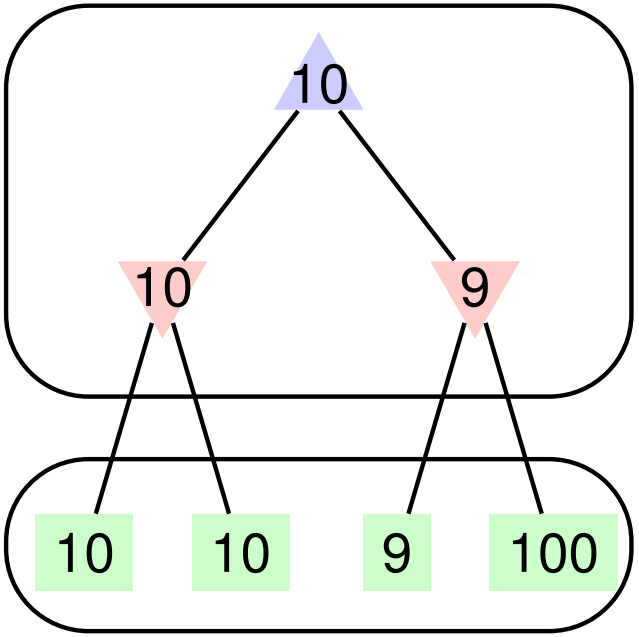

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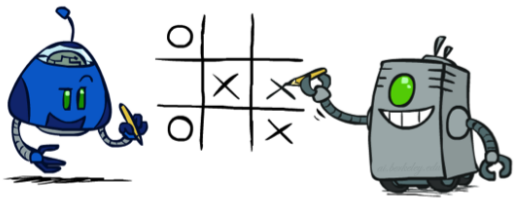

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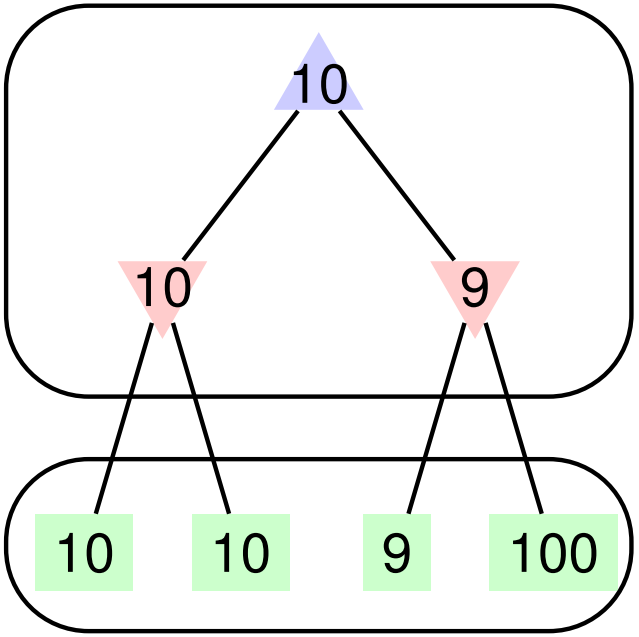

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Optimal against a perfect player.

Minimax Properties

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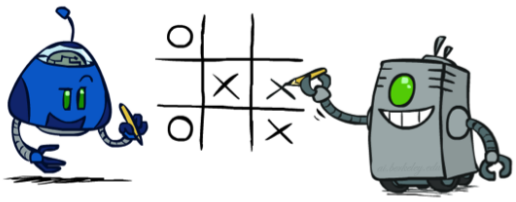

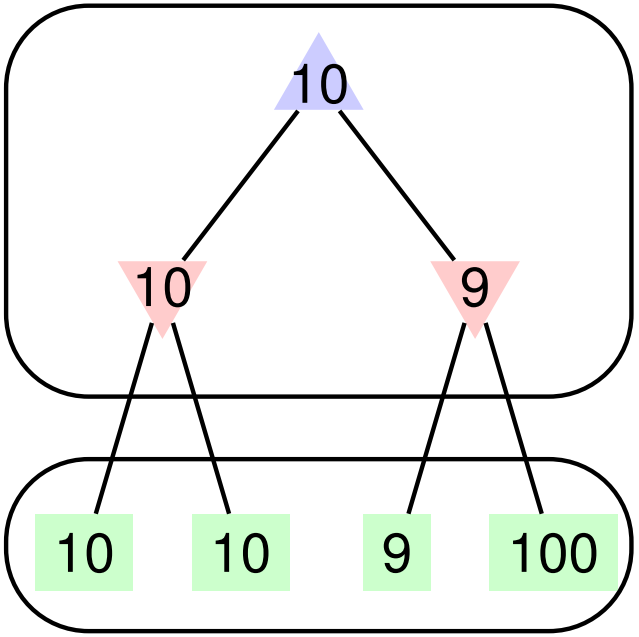

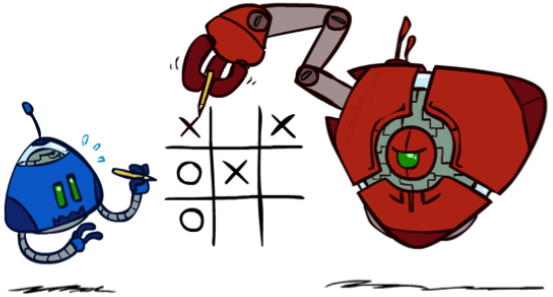

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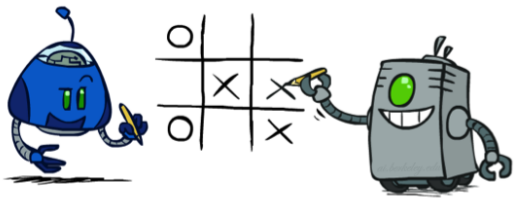

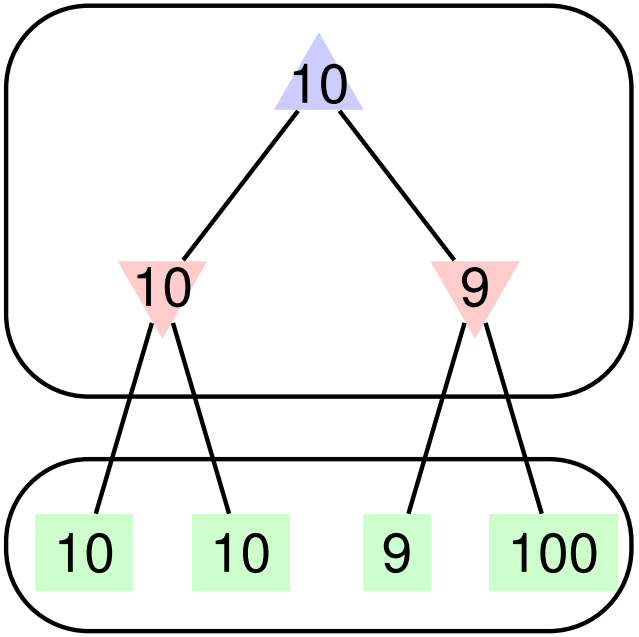

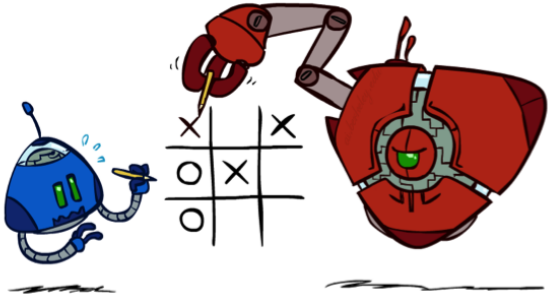

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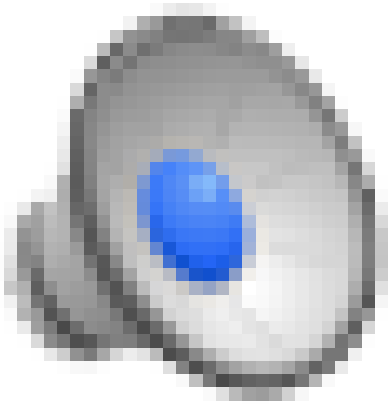
$$Demo : minvsexp(L6D2, L6D3)$$

Video of Demo Min vs. Exp (Min)



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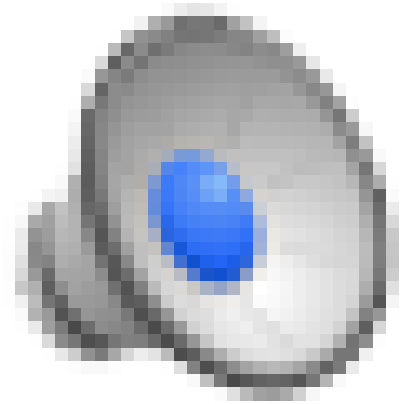

Resource Limits

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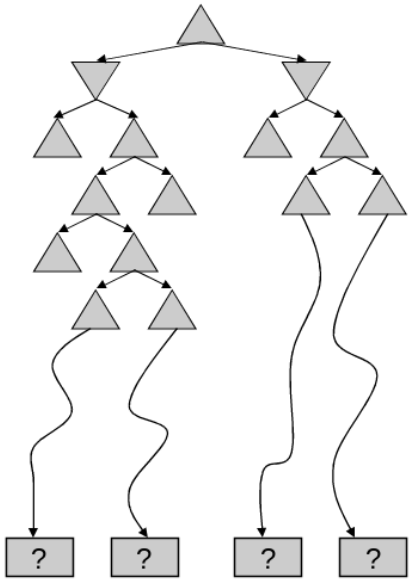

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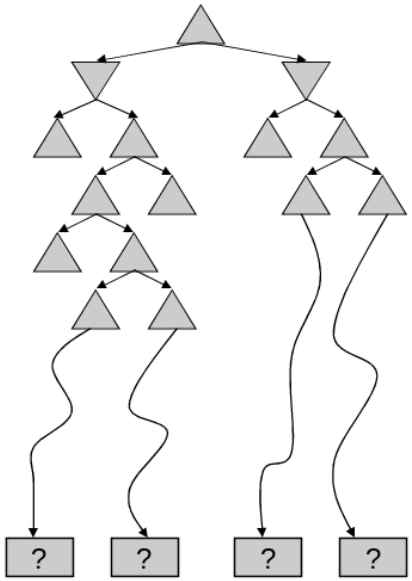

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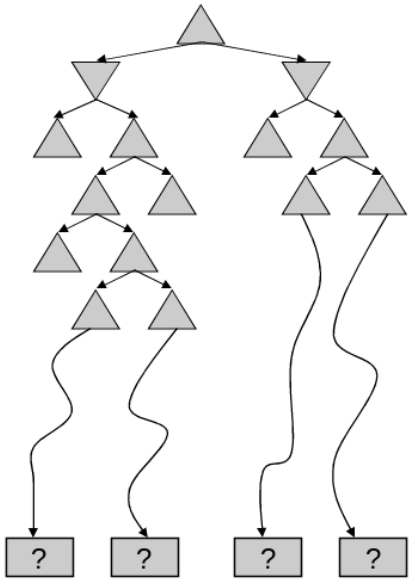

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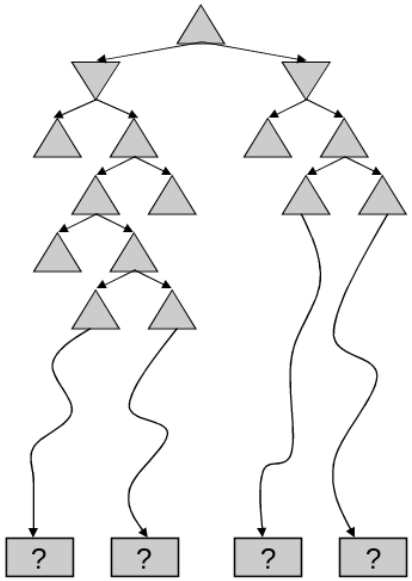

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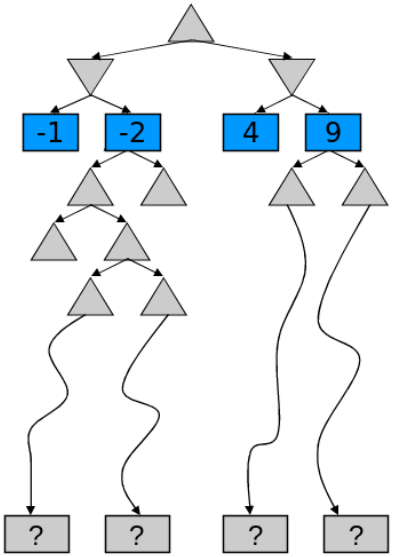

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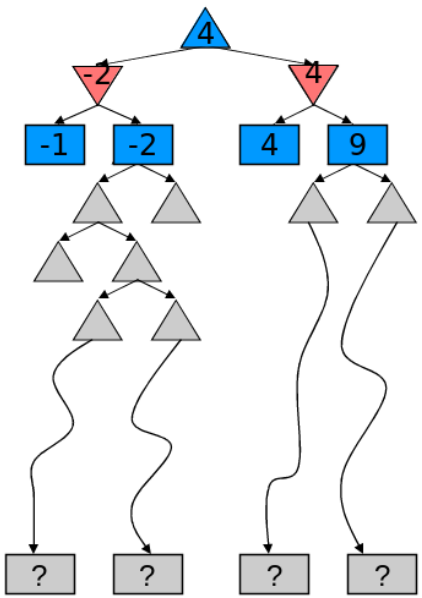

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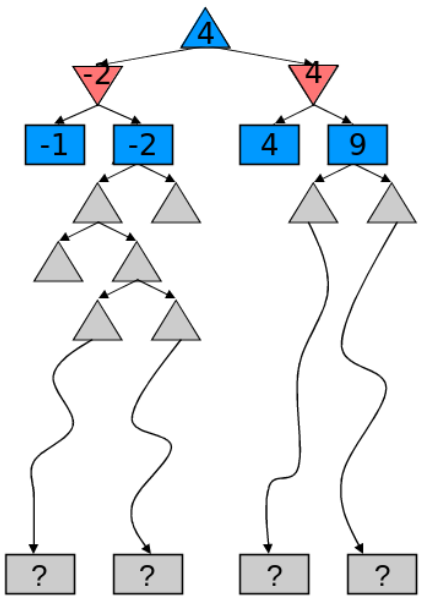

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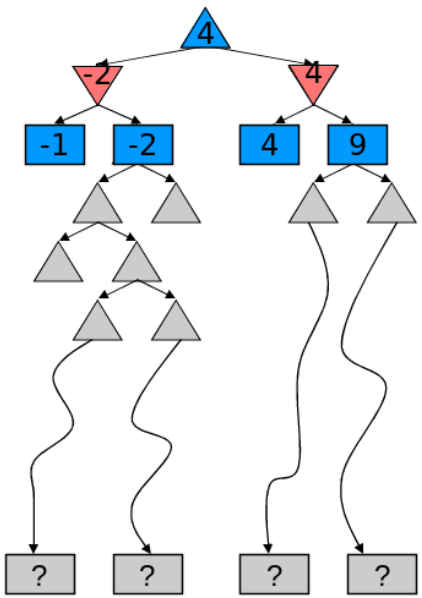

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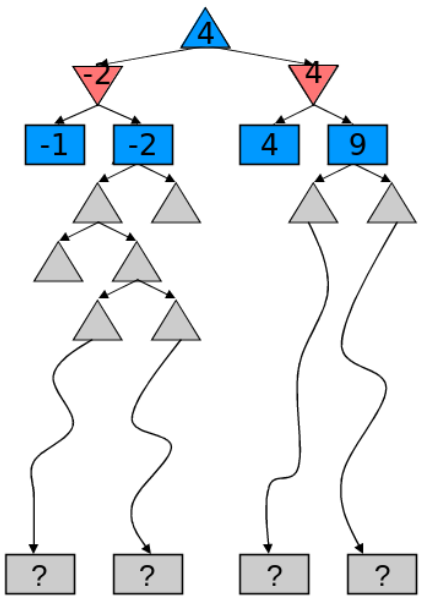

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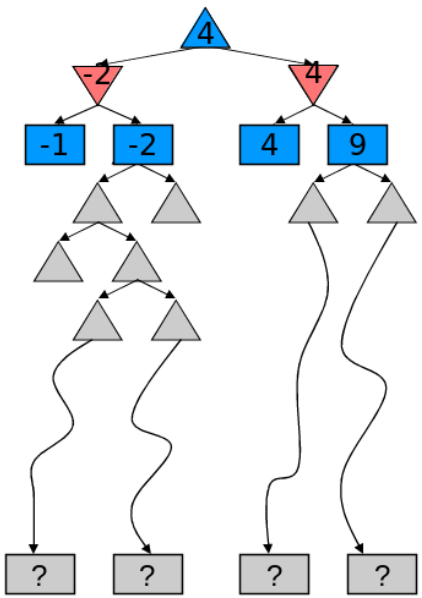

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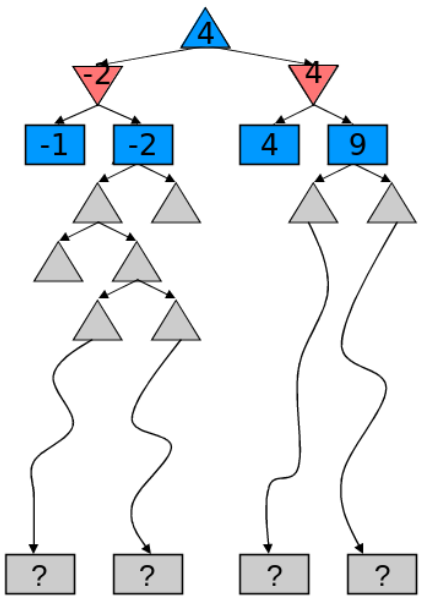

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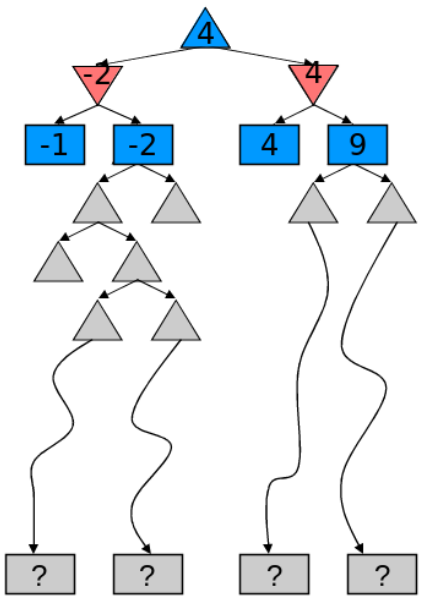

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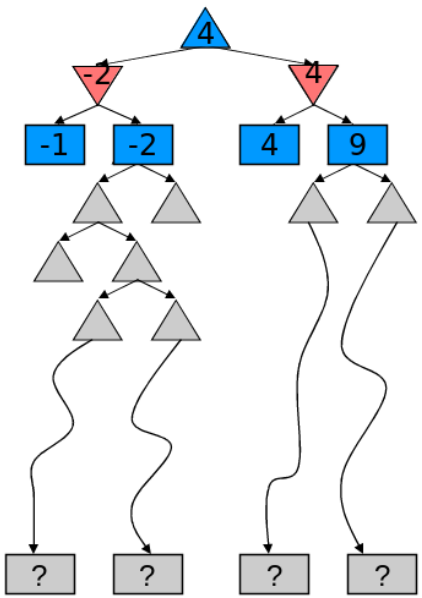

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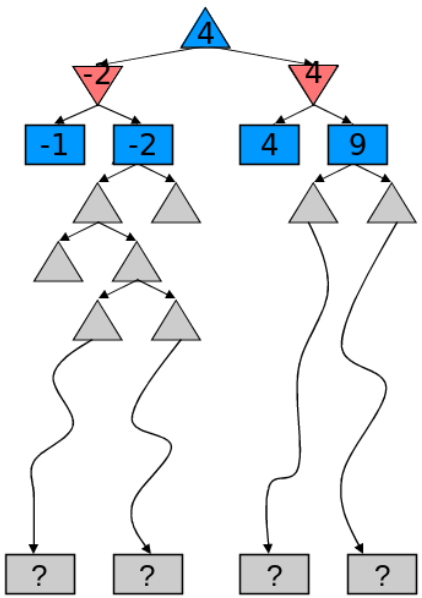

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Use iterative deepening for an anytime algorithm

Depth Matters

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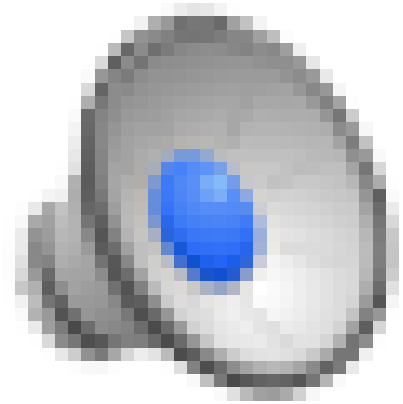
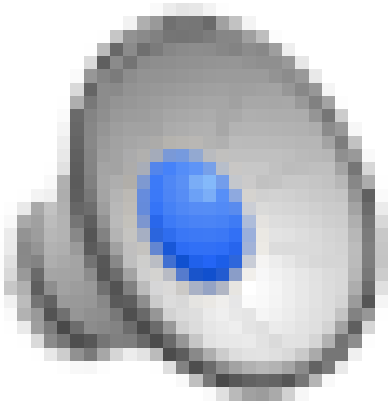
$$Demo : depthlimited(L6D4, L6D5)$$

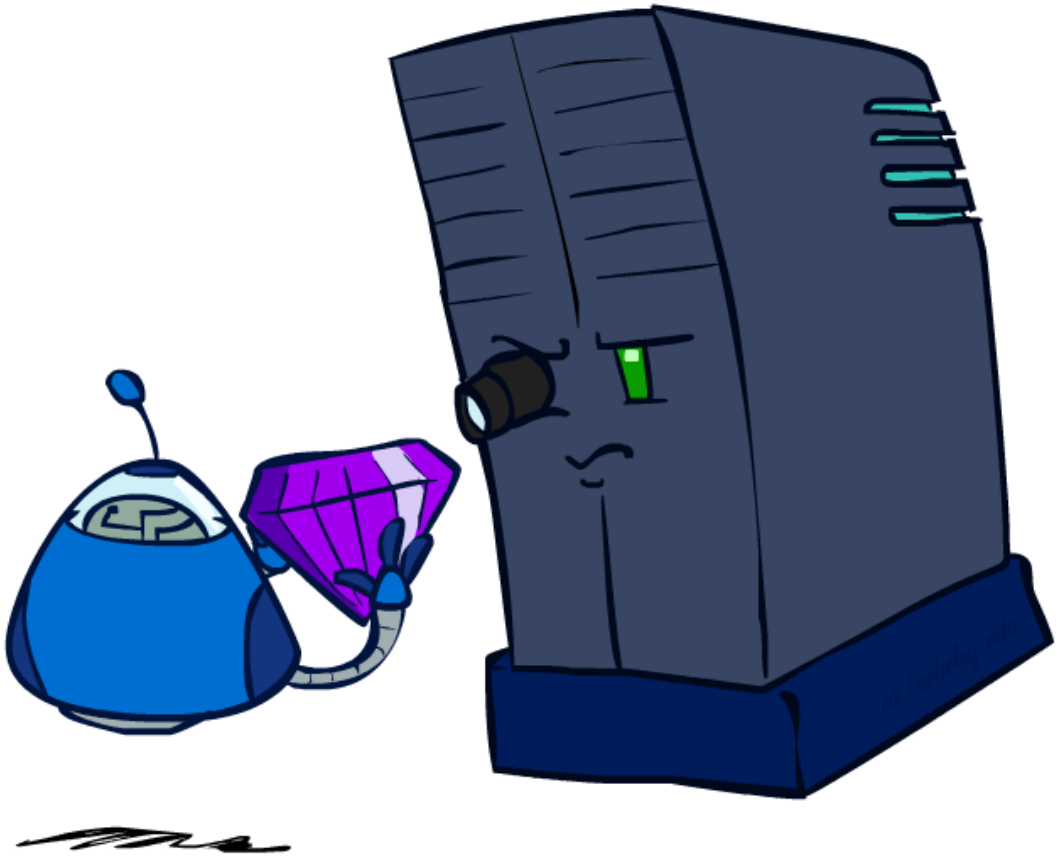
Video of Demo Limited Depth (2)



Video of Demo Limited Depth (10)



Evaluation Functions



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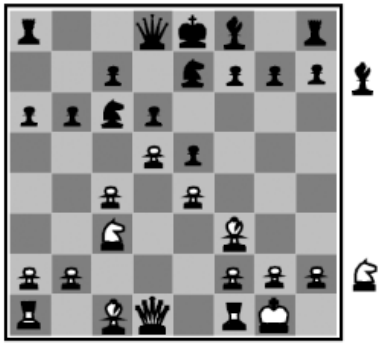
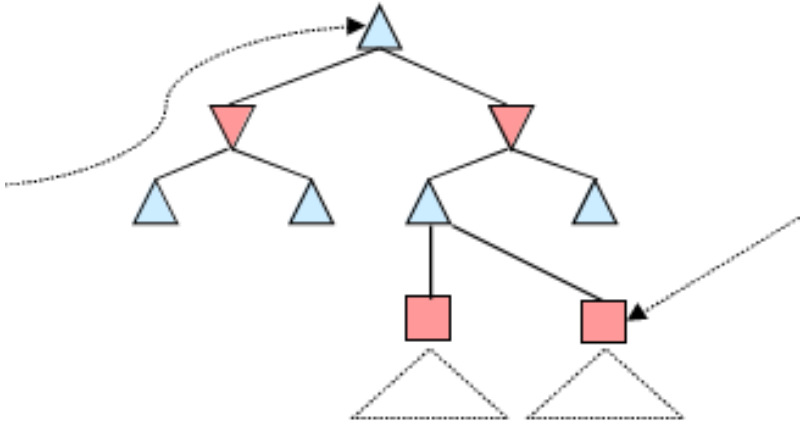
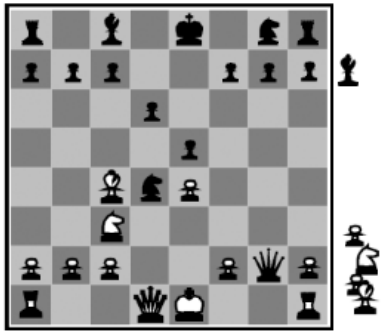


Black to move

White slightly better

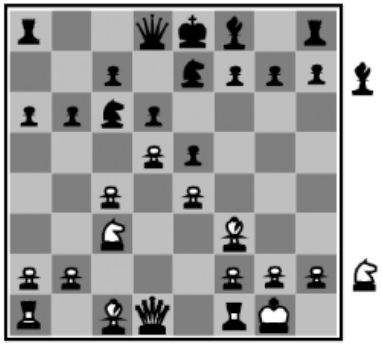




White to move

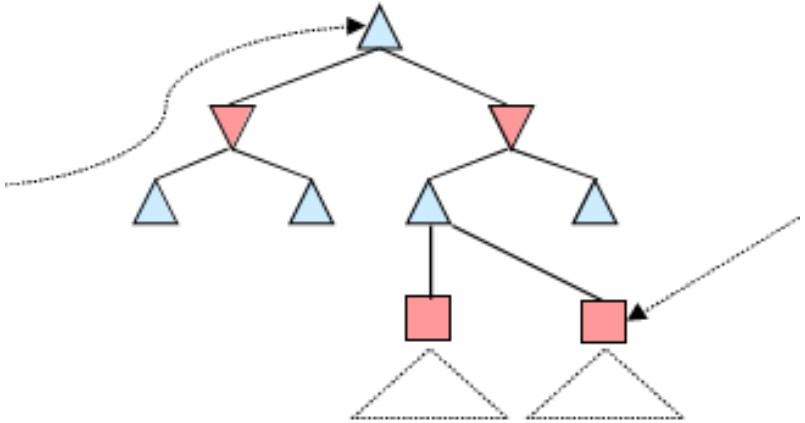
Black winning

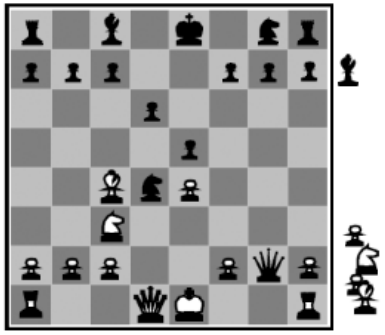
Evaluation Functions



Black to move

White slightly better



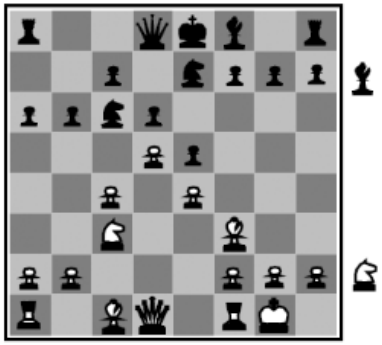


White to move

Black winning

Evaluation functions score non-terminals in depth-limited search

Evaluation Functions



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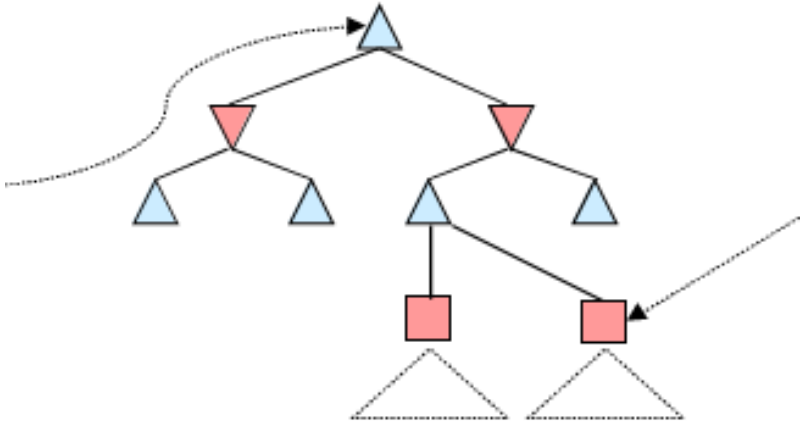

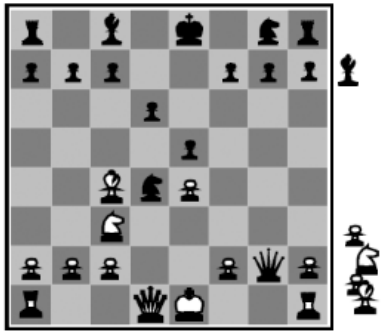

White to move

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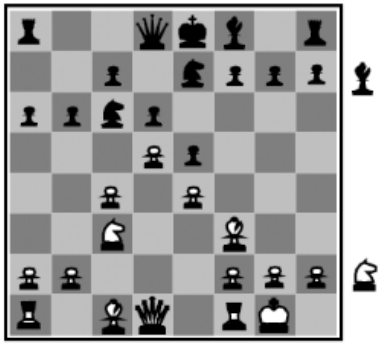
Evaluation functions score non-terminals in depth-limited search

Ideal function: returns the actual minimax value of the position

Evaluation Functions



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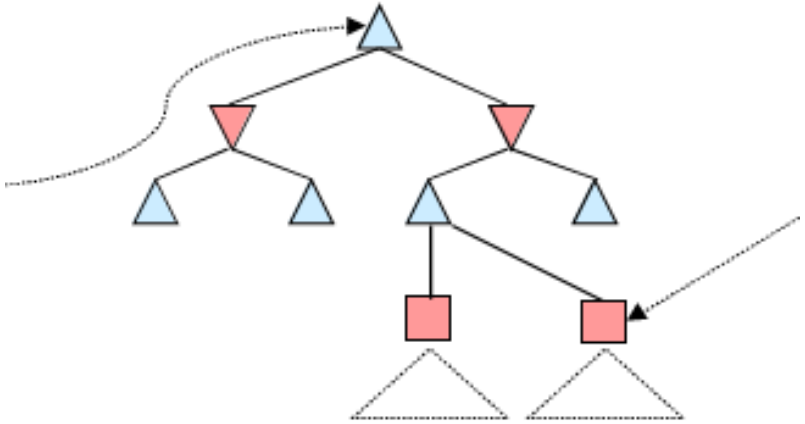

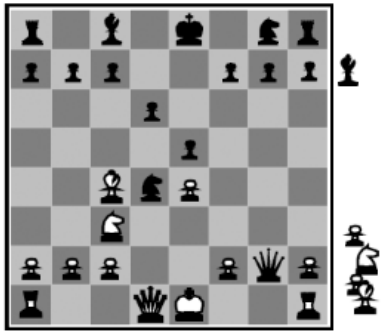

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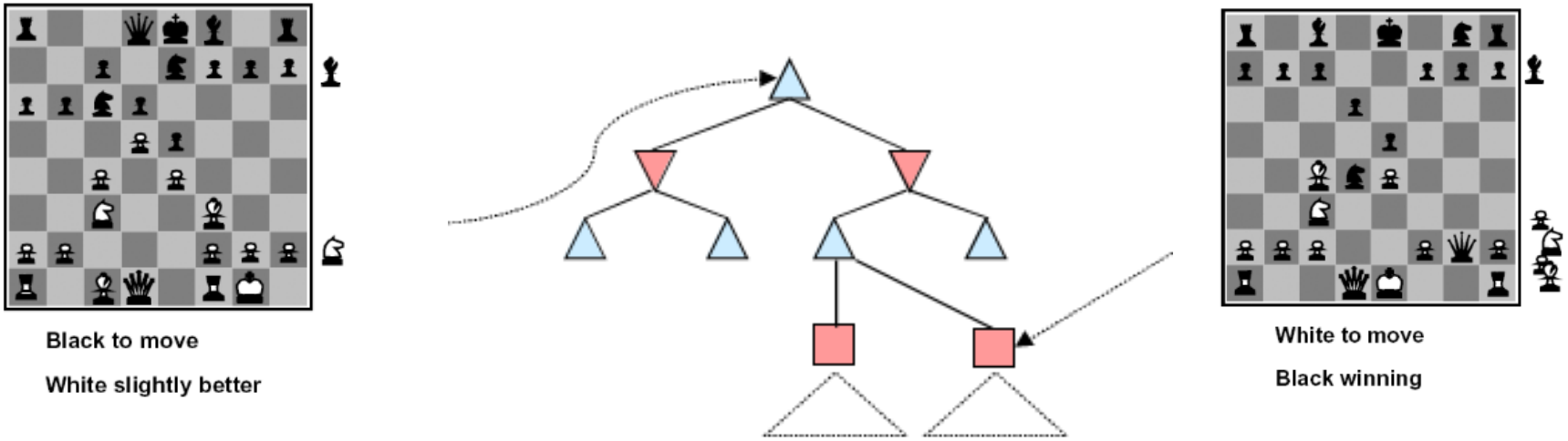
In practice: typically weighted linear sum of features:

Evaluation Functions



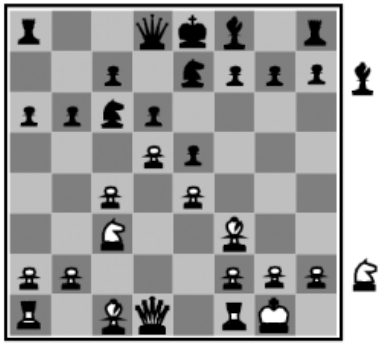
Evaluation functions score non-terminals in depth-limited search

Ideal function: returns the actual minimax value of the position

In practice: typically weighted linear sum of features:

$$w_1 f_1(s) + w_2 f_2(s) + \cdots w_n f_n(s).$$

Evaluation Functions



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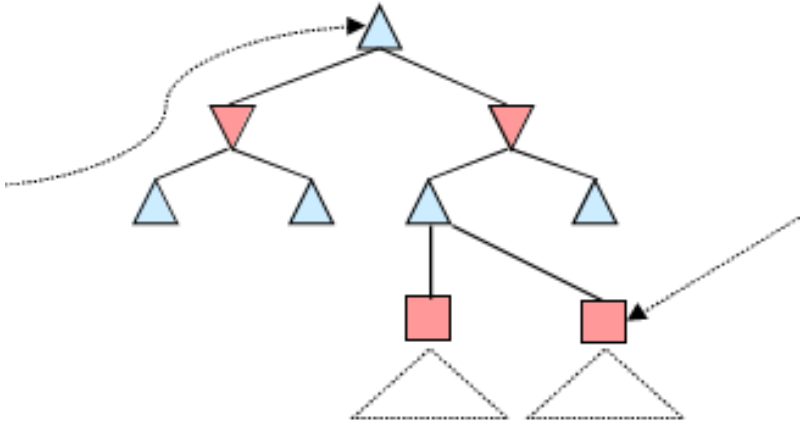
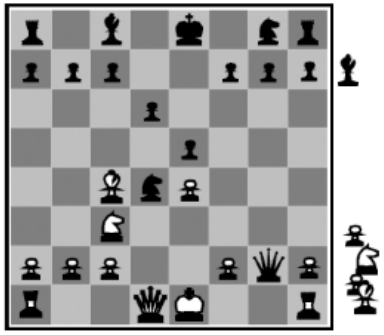




White to move

Black winning

Evaluation functions score non-terminals in depth-limited search

Ideal function: returns the actual minimax value of the position

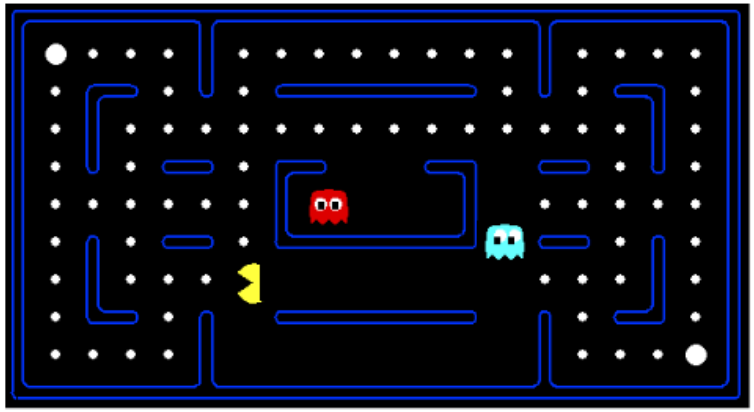
In practice: typically weighted linear sum of features:

$$w_1 f_1(s) + w_2 f_2(s) + \cdots w_n f_n(s).$$

Example: $f1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman

Evaluation for Pacman



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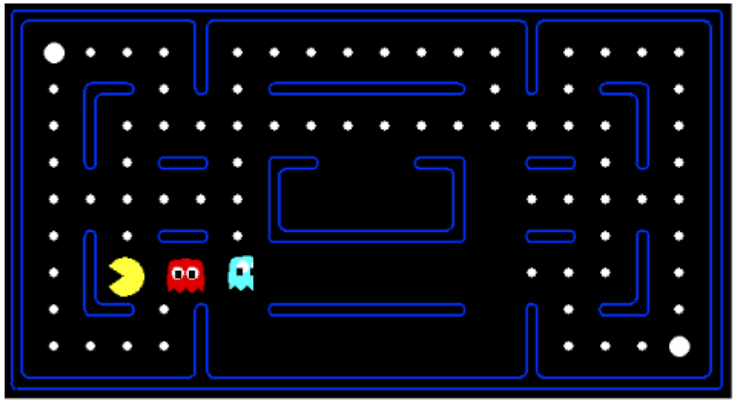

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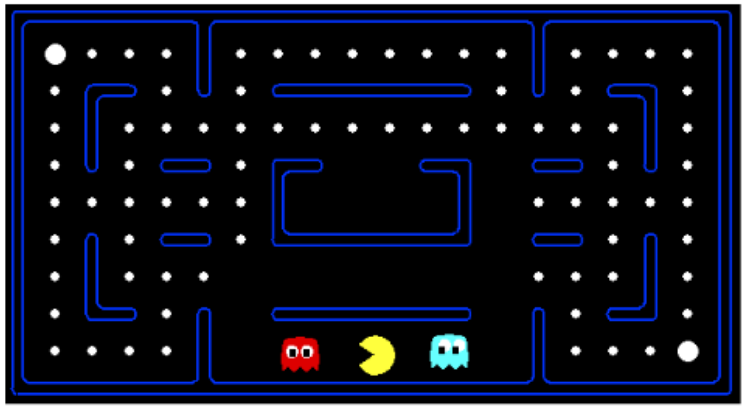

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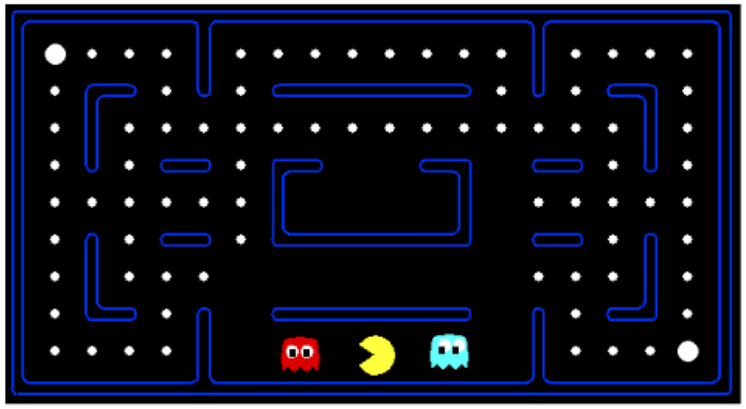
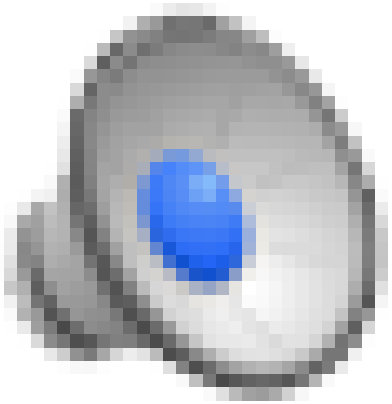


$Demo: thrashingd = 2, thrashingd = 2(fixedevaluationfunction), smartghostsc$

Video of Demo Thrashing (d=2)



Why Pacman Starves

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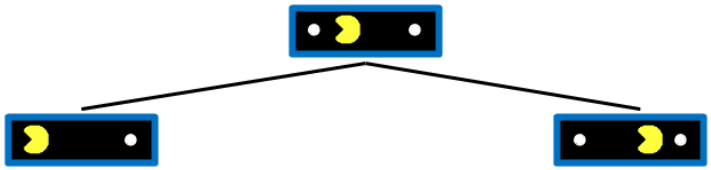

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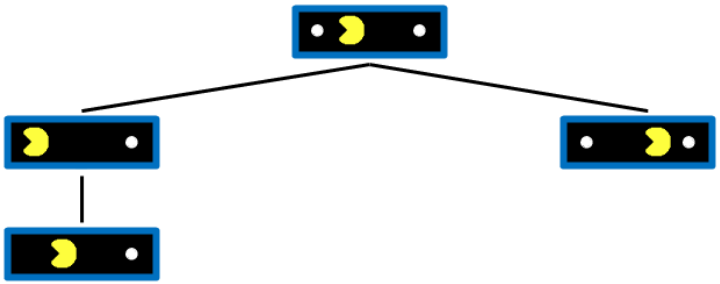

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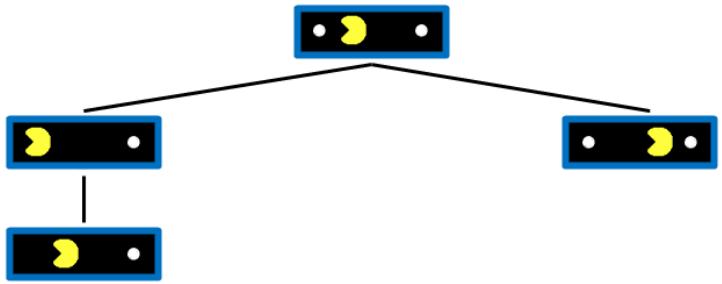
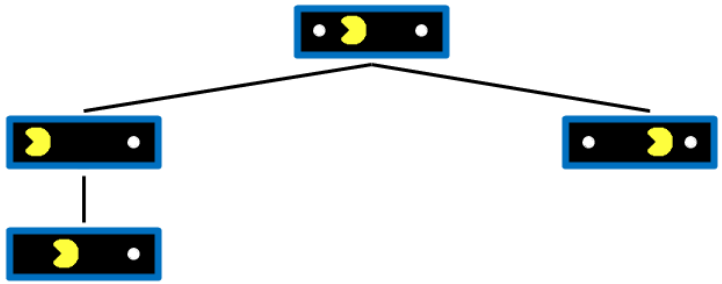


A danger of replanning agents!

- He knows his score will go up by eating the dot now (west, east)

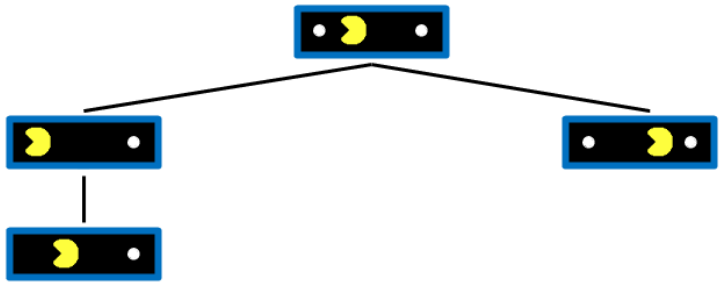
Why Pacman Starves



A danger of replanning agents!

- He knows his score will go up by eating the dot now (west, east)
- He knows his score will go up just as much by eating the dot later (east, west)

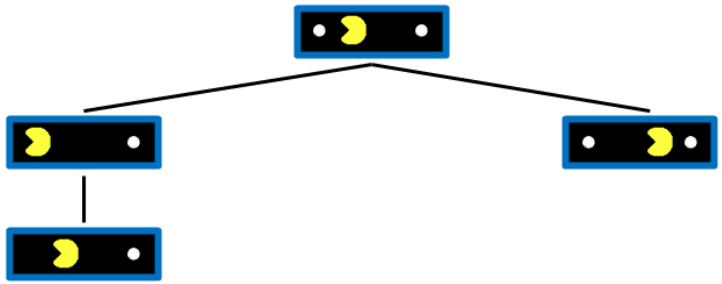
Why Pacman Starves



A danger of replanning agents!

- He knows his score will go up by eating the dot now (west, east)
- He knows his score will go up just as much by eating the dot later (east, west)
- There are no point-scoring opportunities after eating the dot (within the horizon, two here)

Why Pacman Starves



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- He knows his score will go up just as much by eating the dot later (east, west)
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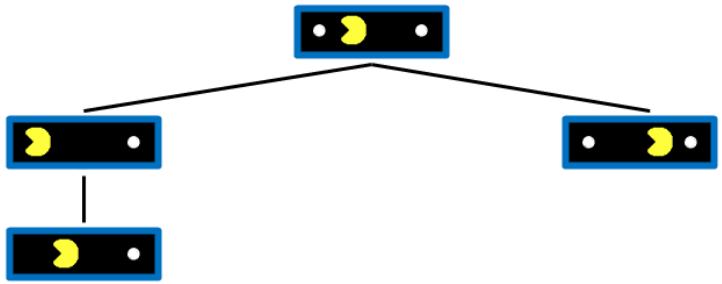
- Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

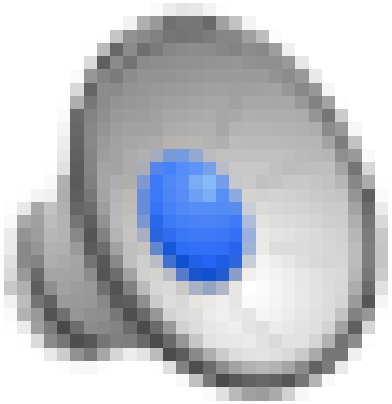
Why Pacman Starves



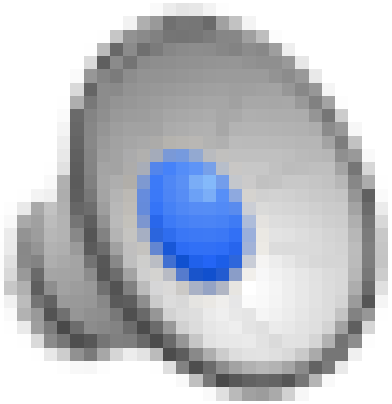
A danger of replanning agents!

- He knows his score will go up by eating the dot now (west, east)
- He knows his score will go up just as much by eating the dot later (east, west)
- There are no point-scoring opportunities after eating the dot (within the horizon, two here)
- Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

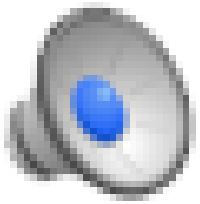
Video of Demo Thrashing – Fixed (d=2)



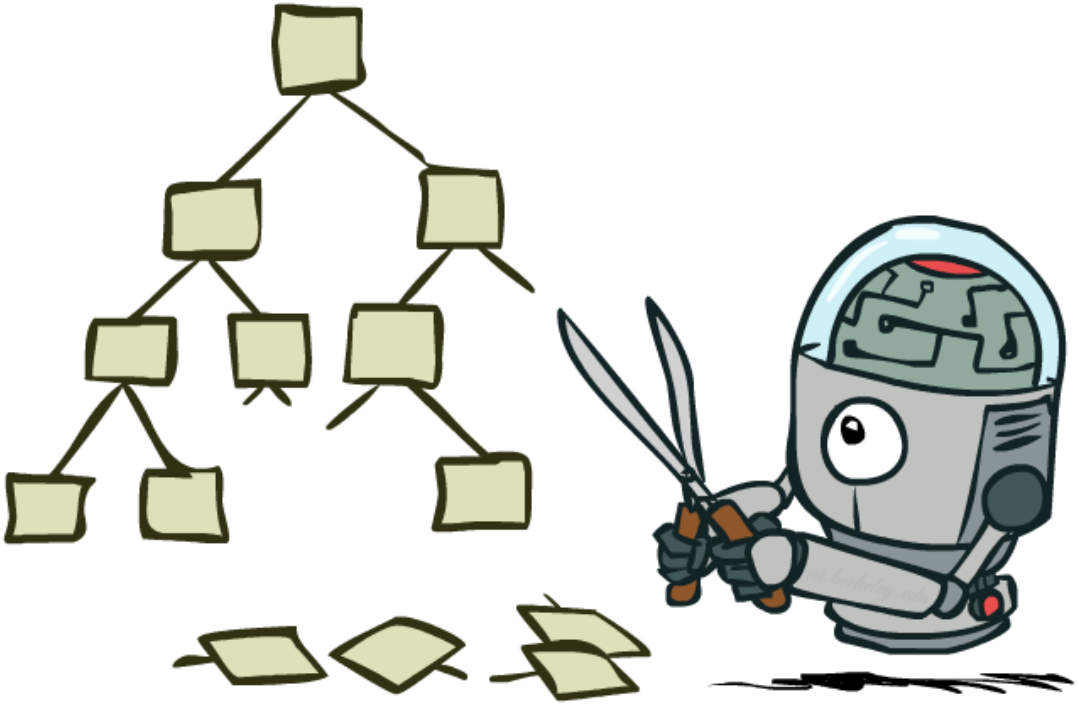
Video of Demo Smart Ghosts (Coordination)



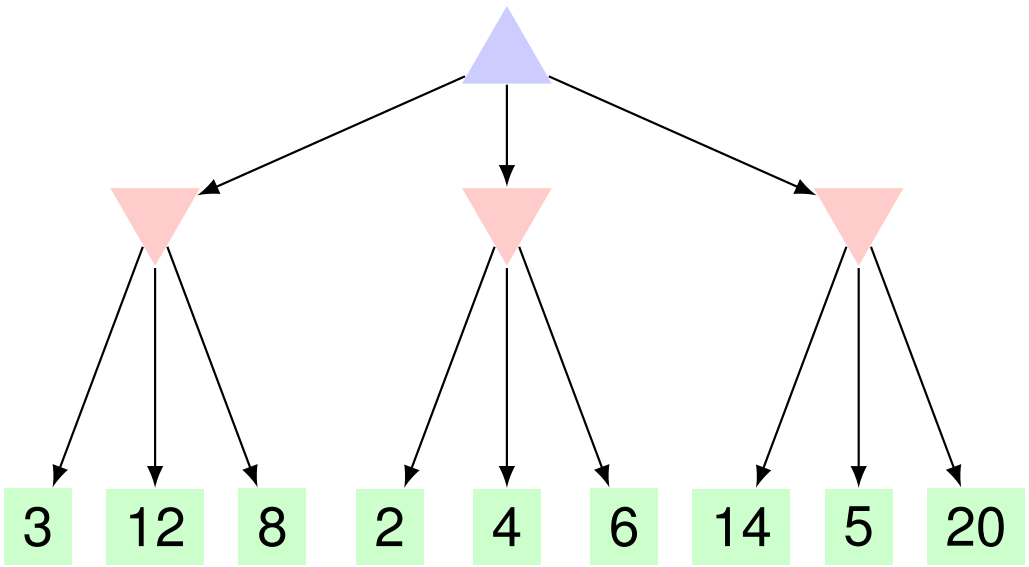
Video of Demo Smart Ghosts (Coordination) – Zoomed In



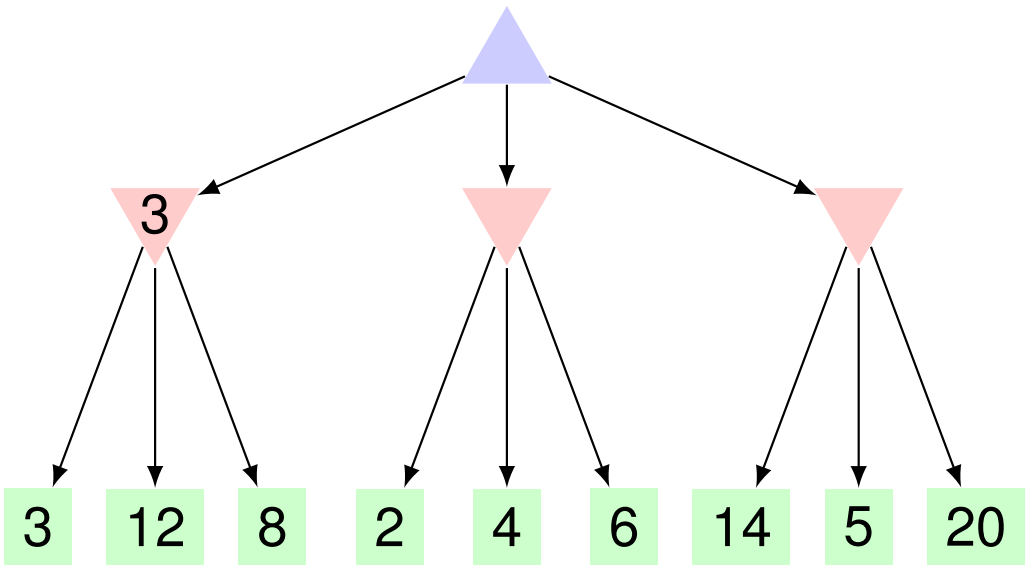
Game Tree Pruning



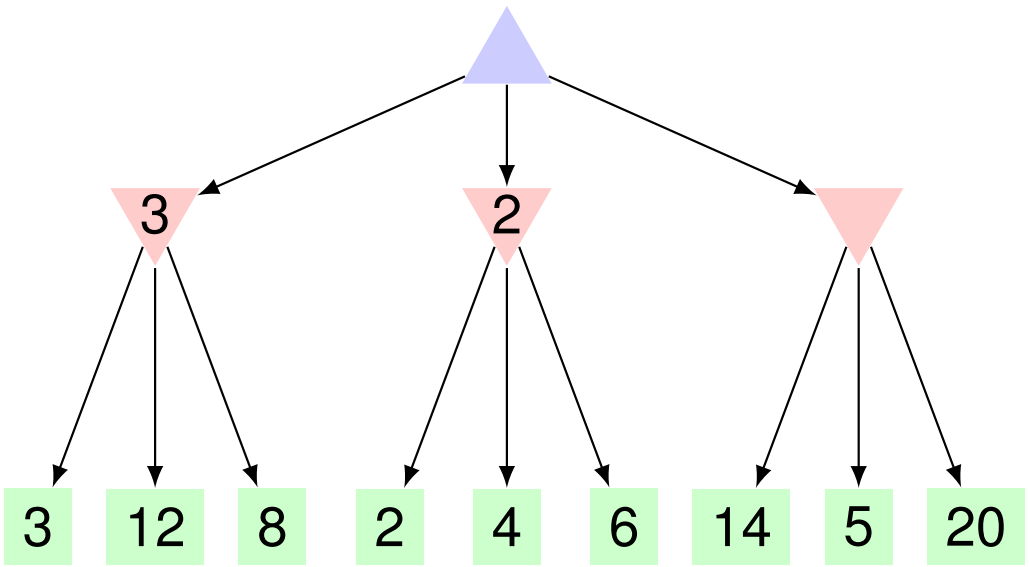
Minimax Example



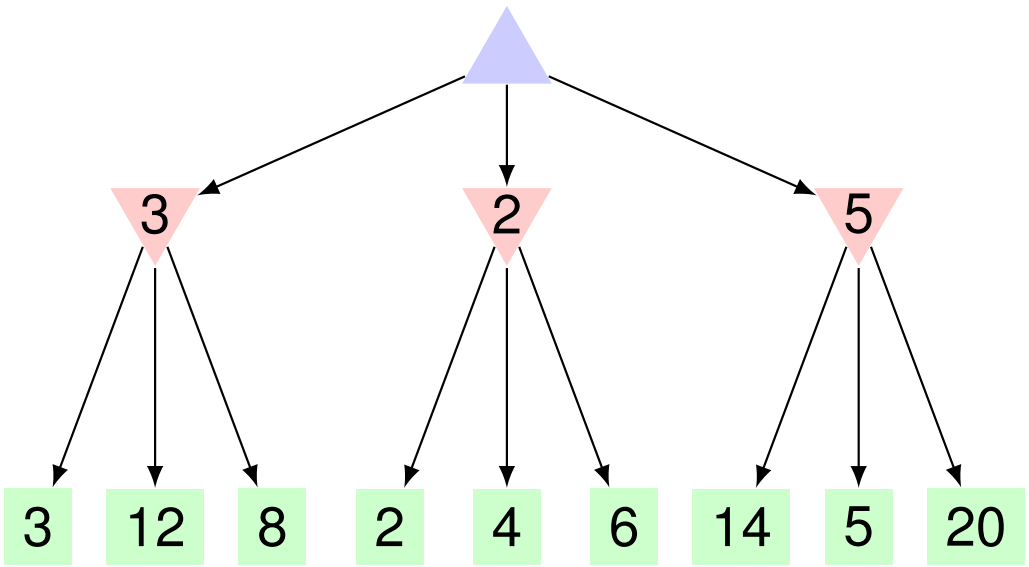
Minimax Example



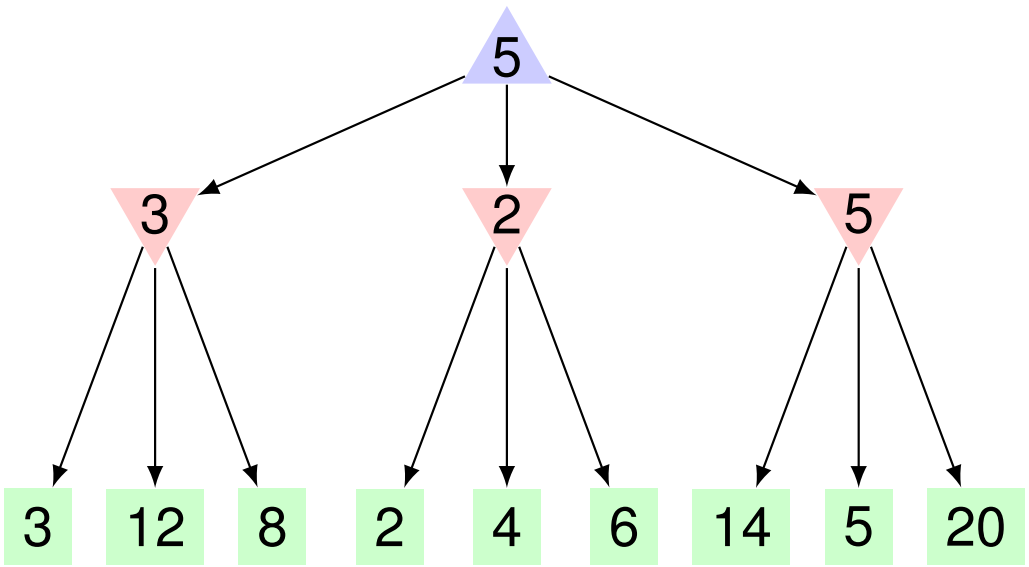
Minimax Example



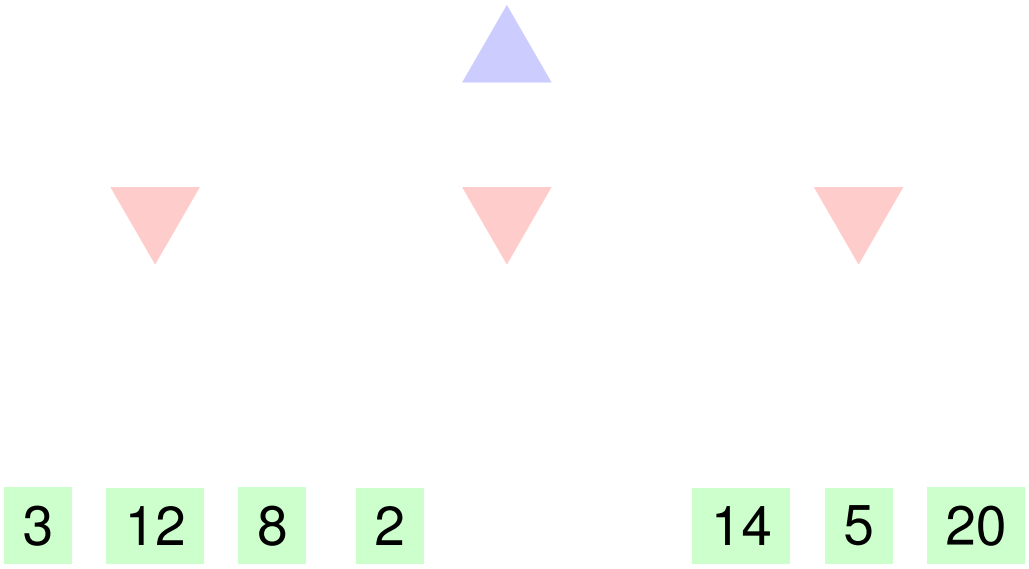
Minimax Example



Minimax Example



Minimax Pruning



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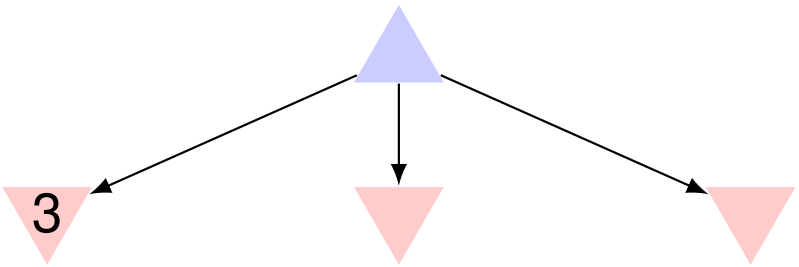
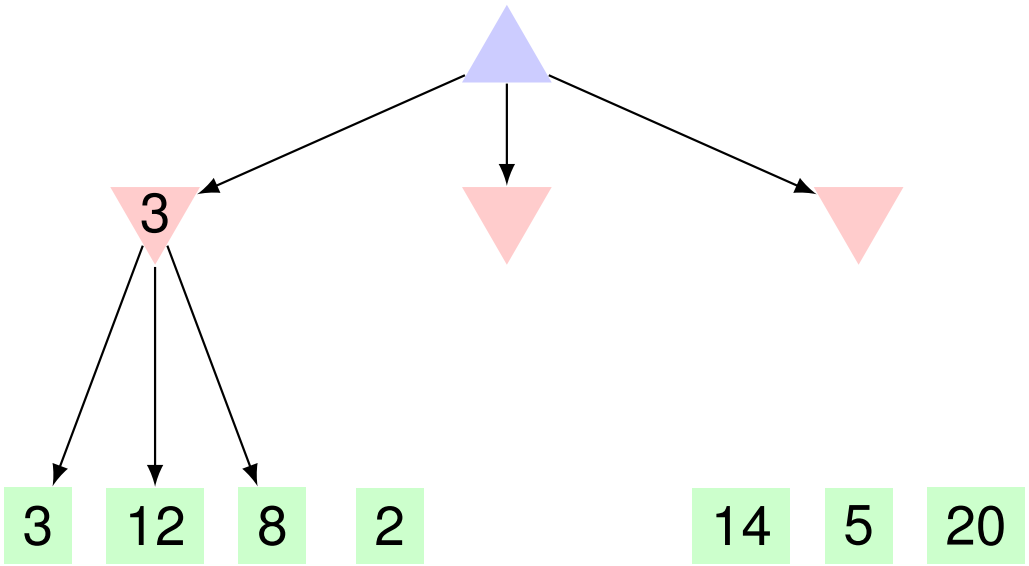
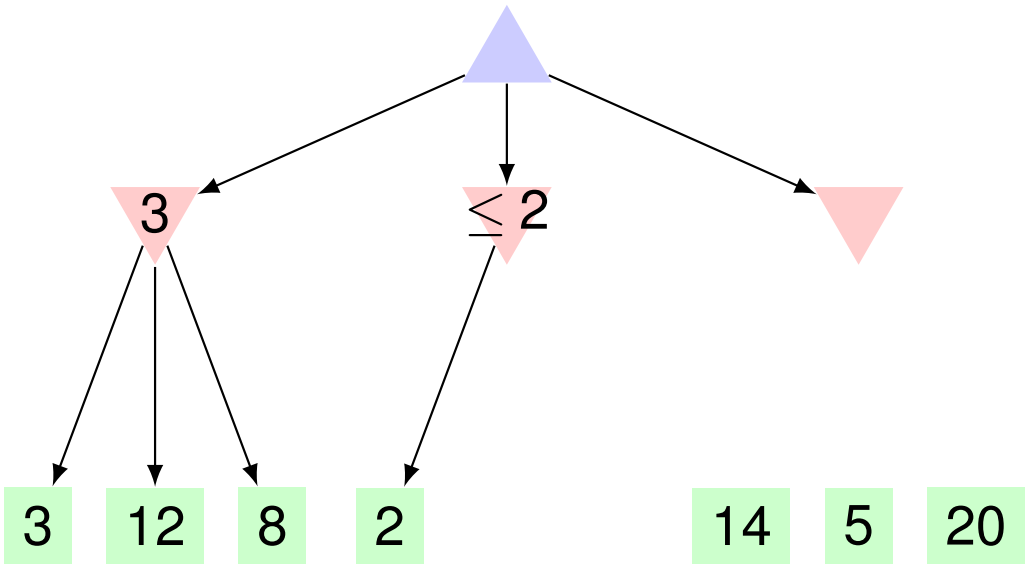


3 12 8 2 14 5 20

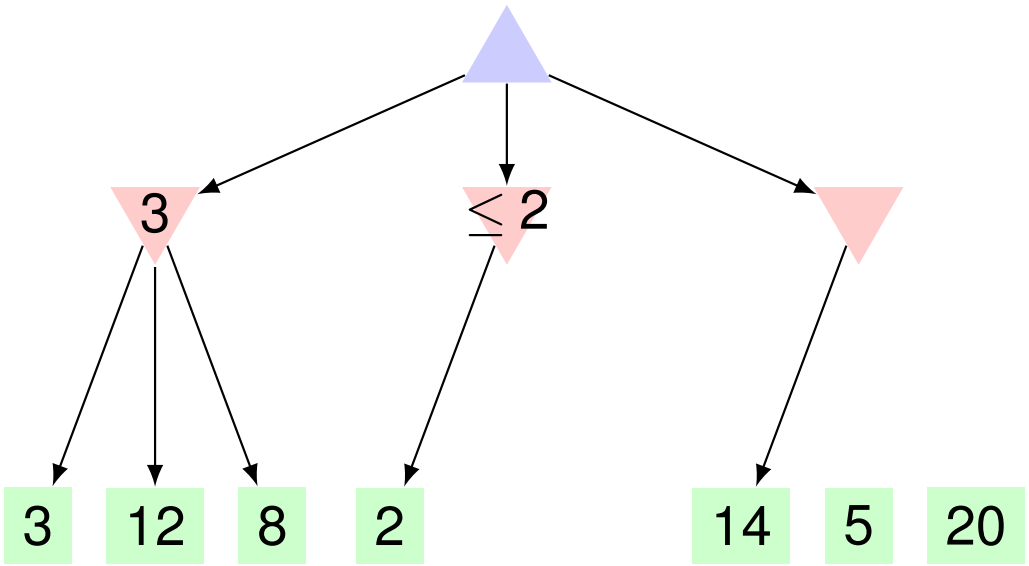
Minimax Pruning



Minimax Pruning



Minimax Pruning



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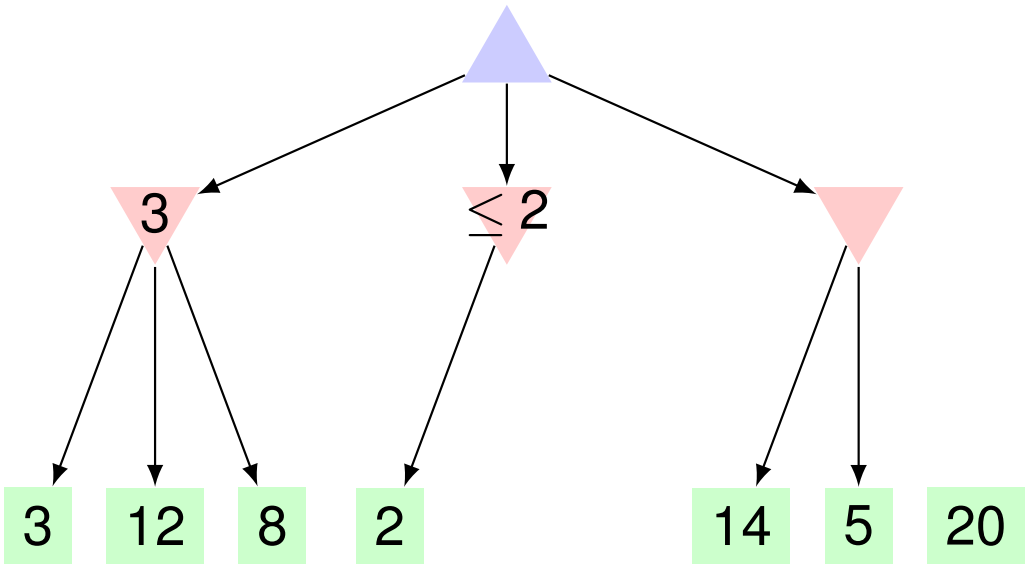

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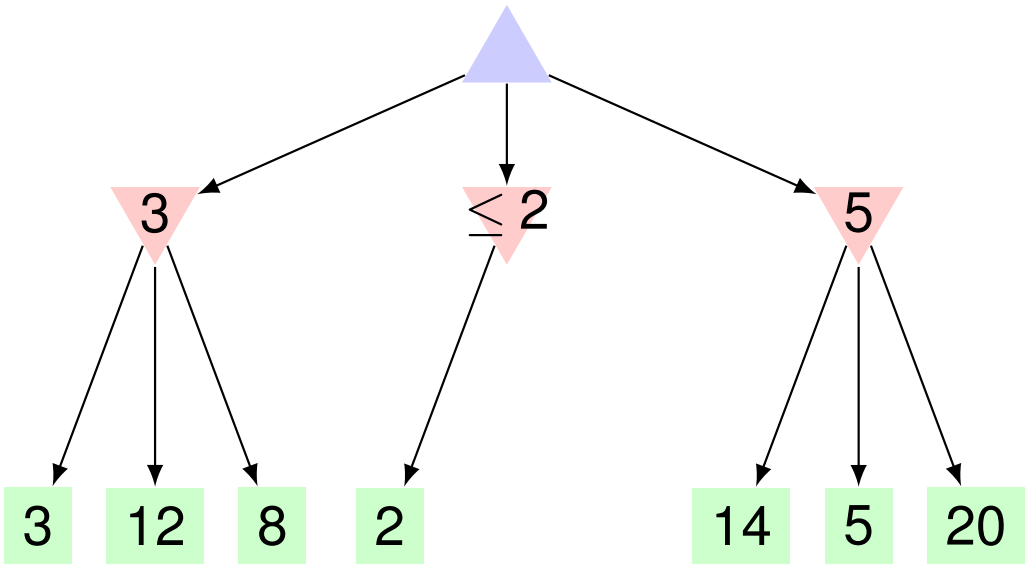
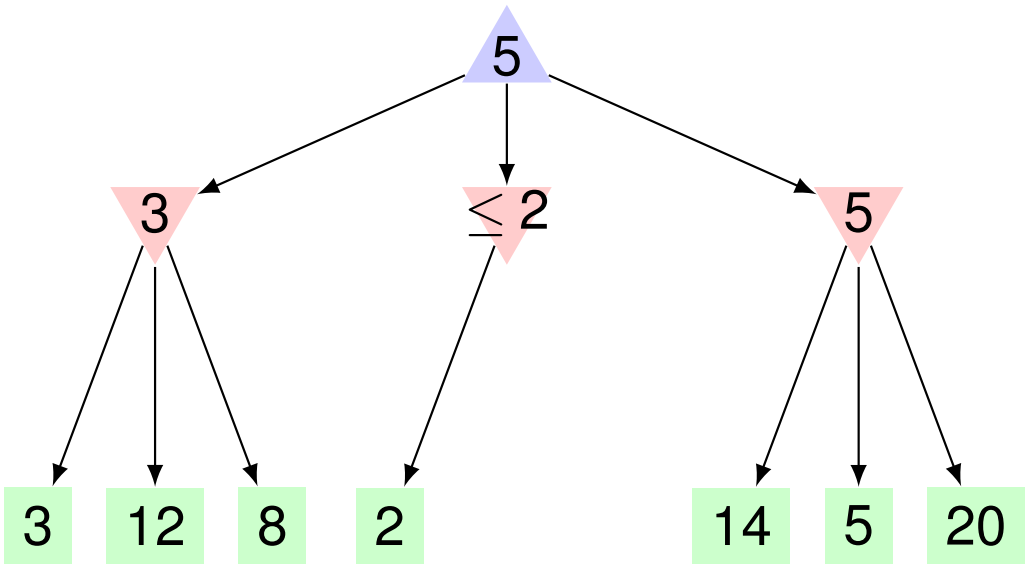
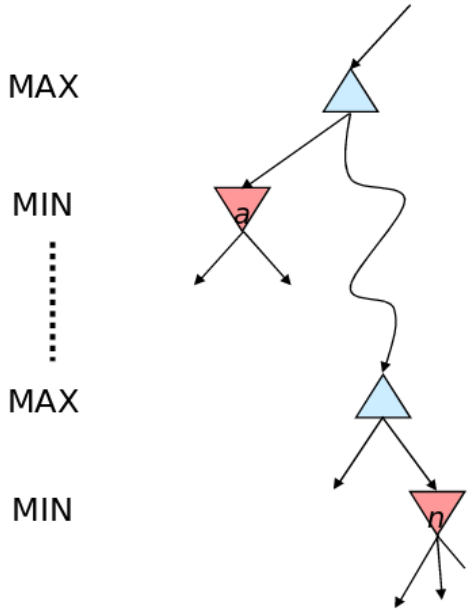


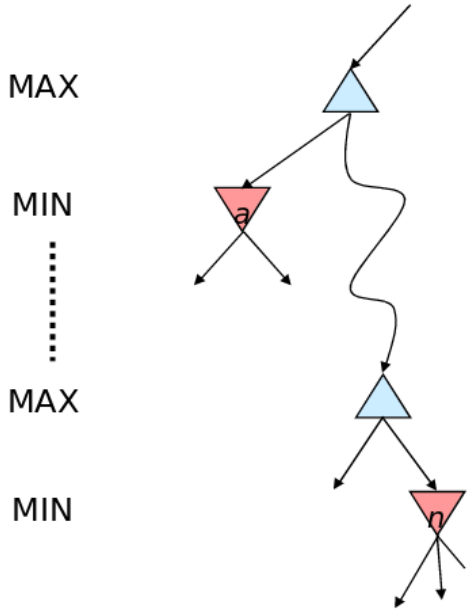
Minimax Pruning



Alpha-Beta Pruning



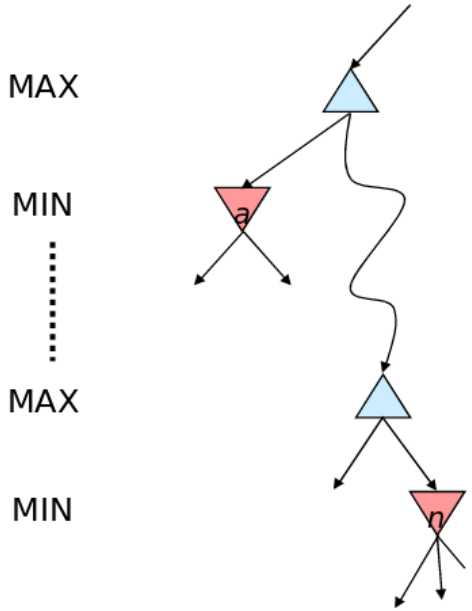
Alpha-Beta Pruning



General configuration (MIN version)

- Computing MIN-VALUE at some node *n*.

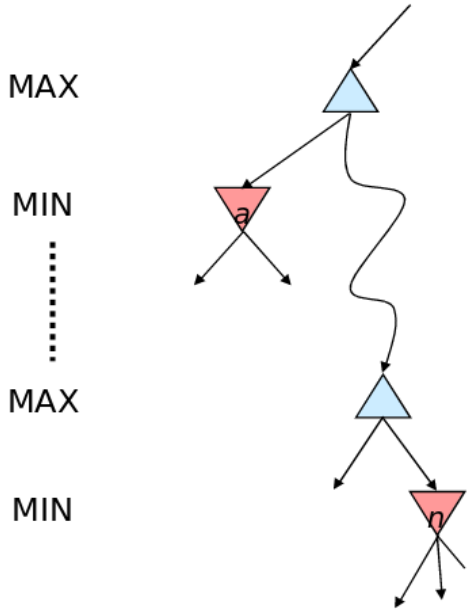
Alpha-Beta Pruning



General configuration (MIN version)

- Computing MIN-VALUE at some node *n*.
- Looping over *n*'s children

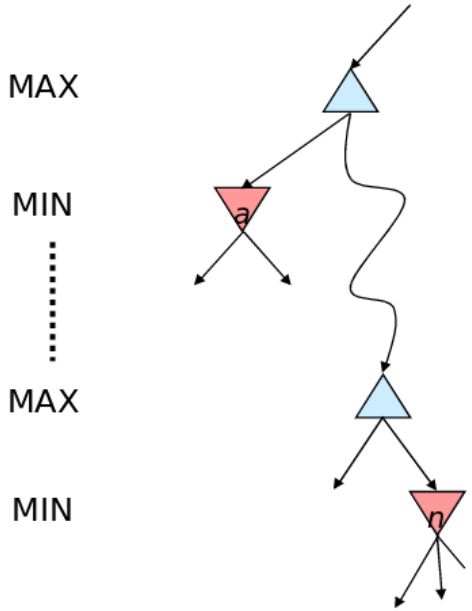
Alpha-Beta Pruning



General configuration (MIN version)

- Computing MIN-VALUE at some node *n*.
- Looping over *n*'s children
- *n*'s estimate of min is dropping

Alpha-Beta Pruning



General configuration (MIN version)

- Computing MIN-VALUE at some node *n*.
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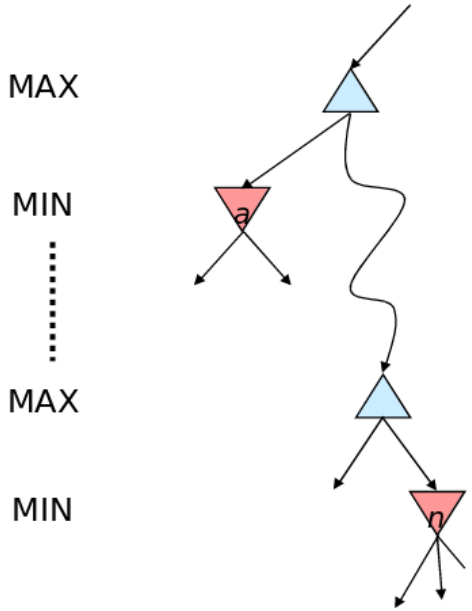
- Who cares about *n*'s value? MAX

Alpha-Beta Pruning



General configuration (MIN version)

- Computing MIN-VALUE at some node *n*.
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- *n*'s estimate of min is dropping
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- $a =$ " best MAX value on path to root."

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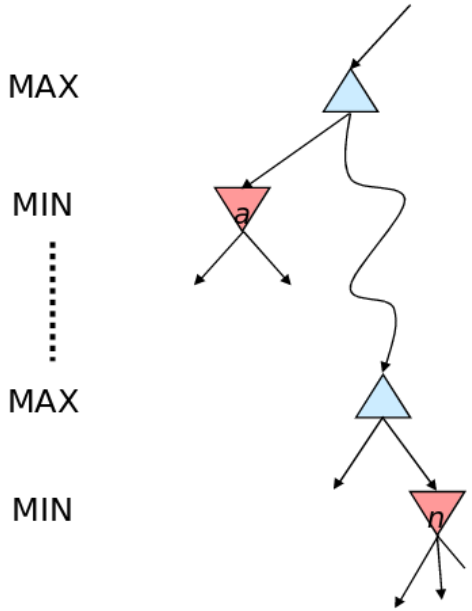

General configuration (MIN version)

- Computing MIN-VALUE at some node n.
- Looping over n's children
- n's estimate of min is dropping
- Who cares about n's value? MAX
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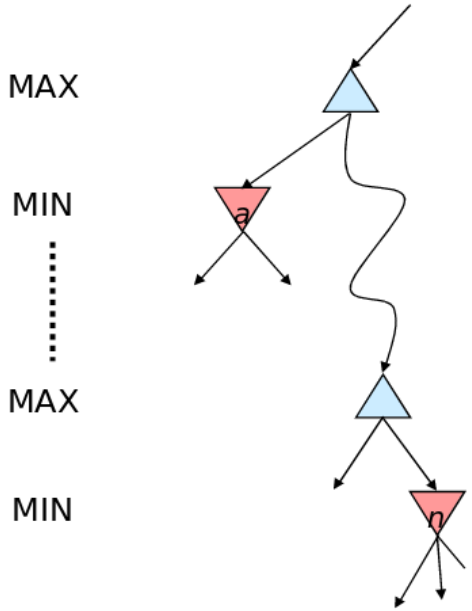
- If $n < a$, than MAX will never choose it.

Alpha-Beta Pruning



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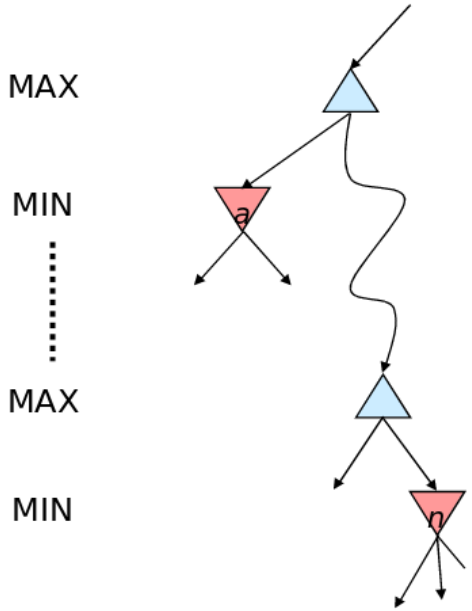
- If $n < a$, than MAX will never choose it.
  So search "prunes" other children.

Alpha-Beta Pruning



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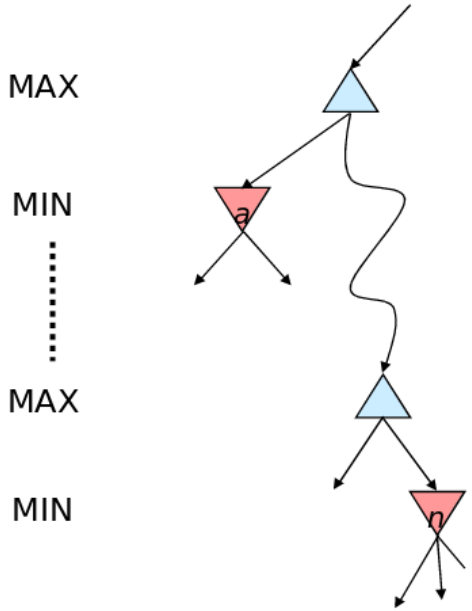

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- $a =$ " best MAX value on path to root."
- If $n < a$, than MAX will never choose it.
  So search "prunes" other children.

Symmetric for MAX version.

Alpha-Beta Implementation

α - MAX's best option on path to root.

Alpha-Beta Implementation

α - MAX's best option on path to root.

β - MIN's best option on path to root.

Alpha-Beta Implementation

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def min-value(state , α, β):
initialize v = $+\infty$.

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def min-value(state , α, β):
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for each successor s of state:

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def min-value(state , α, β):
  initialize v = $+\infty$.
  for each successor *s* of state:
    v = min(v, value(*s*, α, β))

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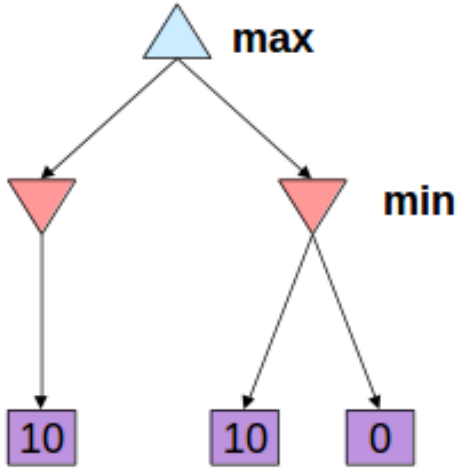
Alpha-Beta Pruning Properties



Alpha-Beta Pruning Properties

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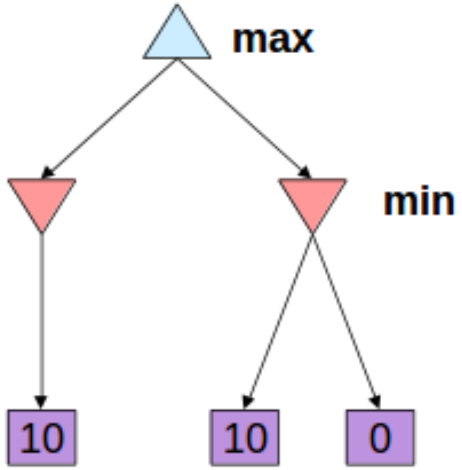

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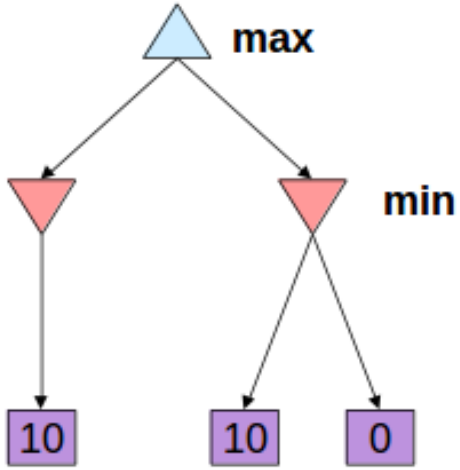

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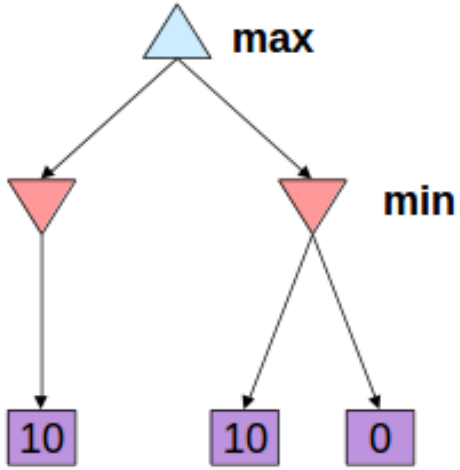

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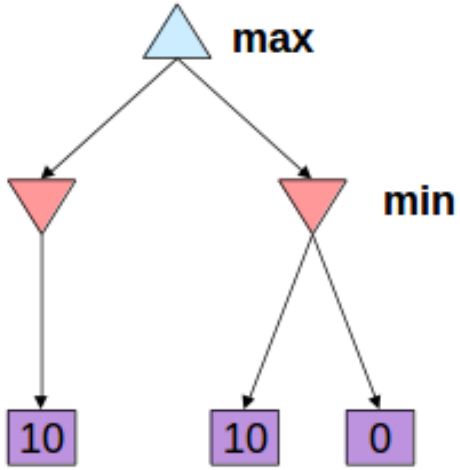

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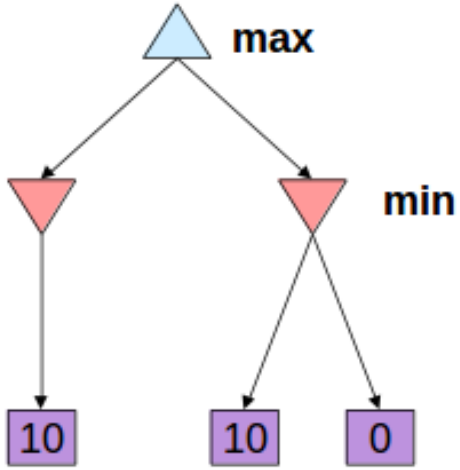

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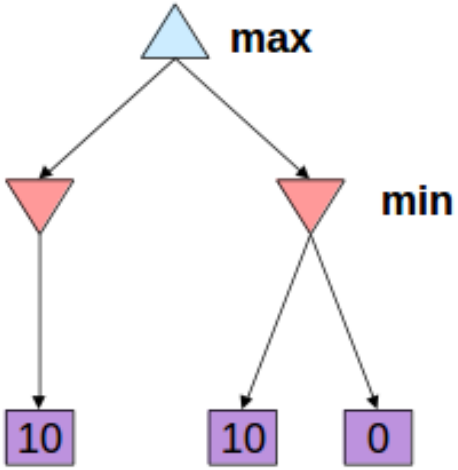

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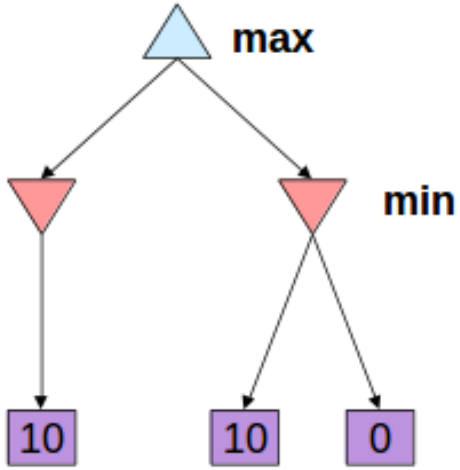

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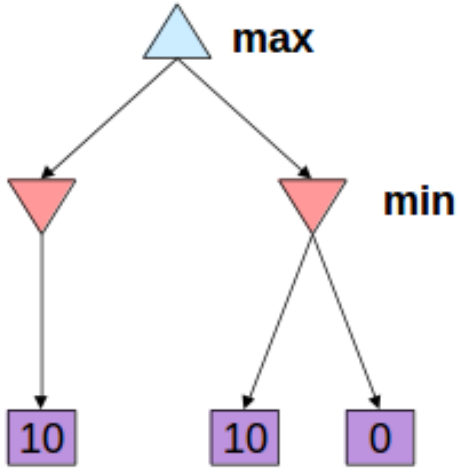

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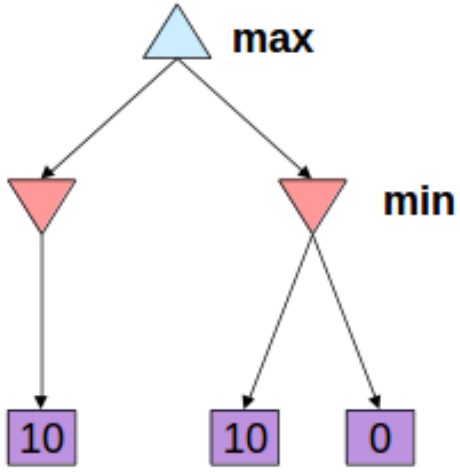

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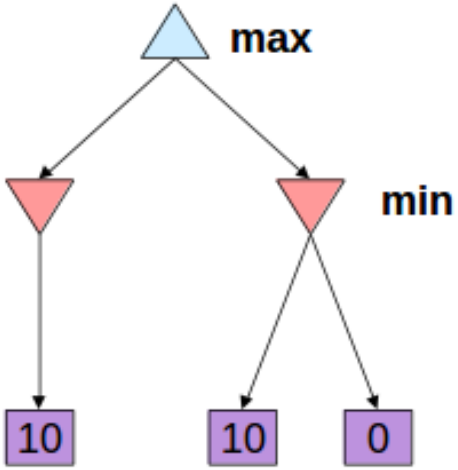

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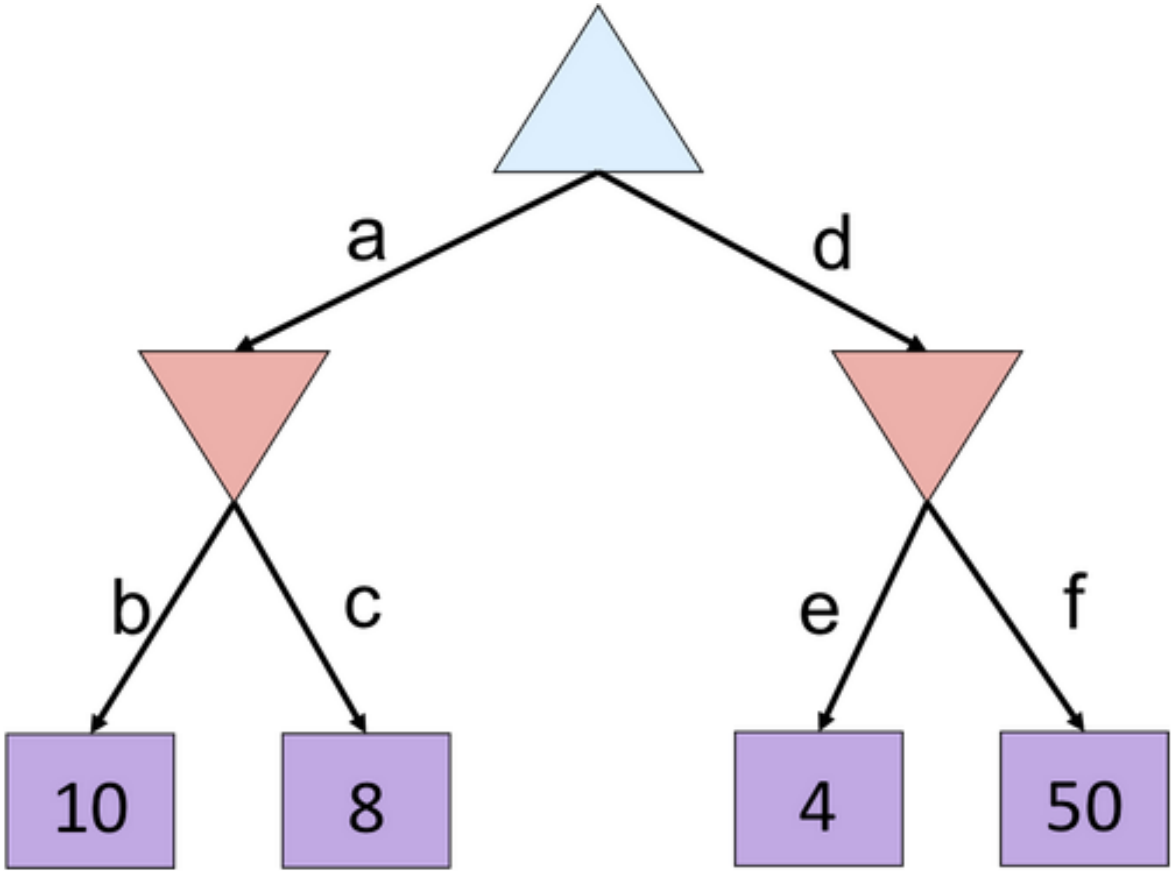
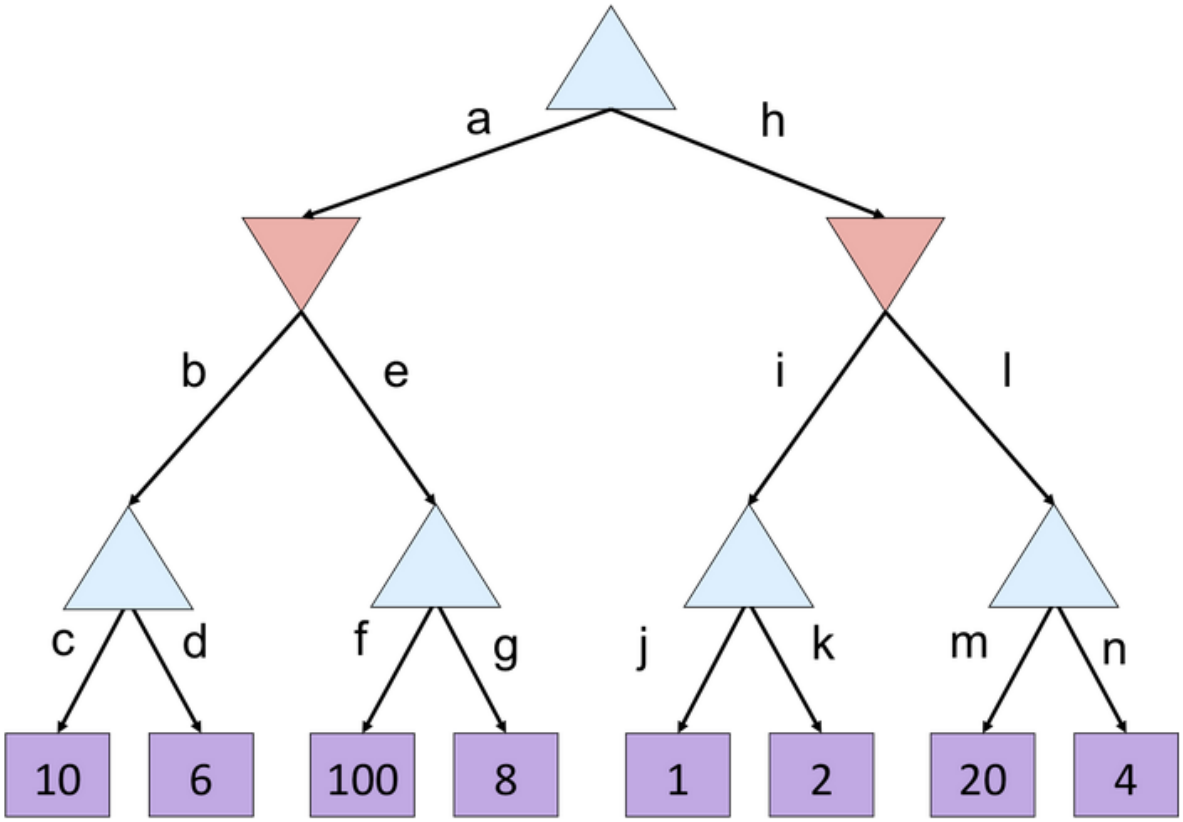
This is a simple example of metareasoning (computing about what to compute)

Alpha-Beta Quiz



Alpha-Beta Quiz 2



Next Time: Uncertainty!