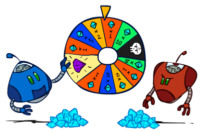
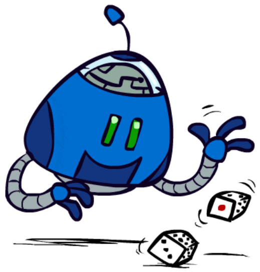
CS 188: Artificial Intelligence

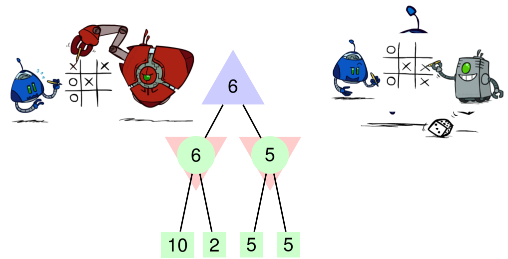


Uncertainty and Utilities

Uncertain Outcomes

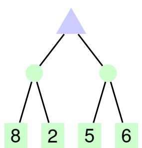


Worst-Case vs. Average Case



Idea: Uncertain outcomes controlled by chance, not an adversary!

Expectimax Search



Why wouldn't we know what the result of an action will be?

- Explicit randomness: rolling dice
- Random opponents: ghosts respond randomly
- Actions can fail: robot wheels might spin

Values reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes

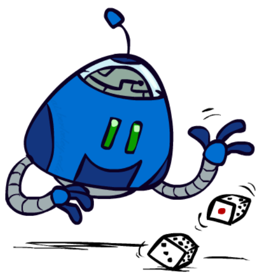
Expectimax search: compute average score under optimal play

- Max nodes as in minimax search
- Chance nodes replace min nodes but the outcome is uncertain
- Calculate their expected utilities
- I.e. take weighted average (expectation) of children

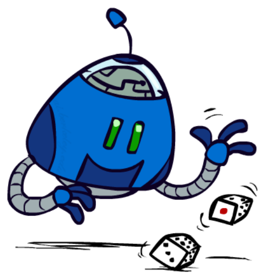
Later: formalize as Markov Decision Processes

[Demo: min vs exp (L7D1,2)]

Video of Demo Minimax vs Expectimax (Min)



Video of Demo Minimax vs Expectimax (Exp)



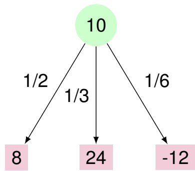
Expectimax Pseudocode

def value(state):
- if the state is a terminal state: return the state's utility
- if the next agent is MAX: return max-value(state)
- if the next agent is EXP: return exp-value(state)

def exp-value(state):
- initialize v = 0
- for each s of succ(state):
-         p = probability(s)
-         v += p * value(s)
- return v

def max-value(state):
- initialize v = -∞
- for each s of succ(state):
-         v = max(v, value(s))
- return v

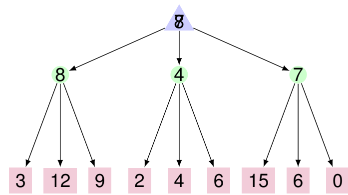
Expectimax Pseudocode



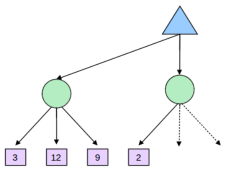
def exp-value(state):
- initialize v = 0
- for each s of succ(state):
-       p = probability(successor)
-       v += p * value(successor)
- return v

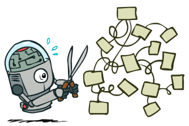
v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10

Expectimax Example

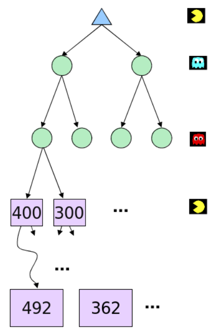


Expectimax Pruning?



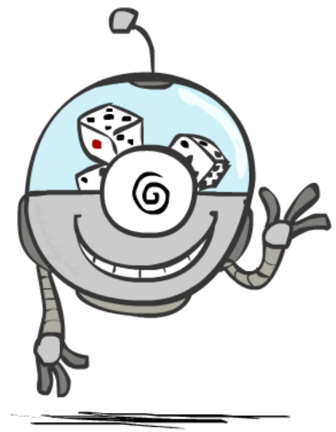


Depth-Limited Expectimax



Estimate true expectimax value (versus lot of work to compute exactly)

Probabilities



Reminder: Probabilities

.25

.50

.25

Random variable picks an outcome

Probability distribution assigns weights to outcomes

Example: Traffic on freeway
- Random variable: T = there's traffic
- Outcomes: T in none, light, heavy
- Distribution:

P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

Some laws of probability (more later):
- Probabilities are always non-negative
- Probabilities of outcomes sum to one

As we get more evidence, probabilities may change:
- P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
- Reasoning and updating probabilities later

Reminder: Expectations

The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes

Example: How long to get to the airport?

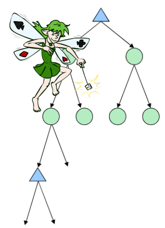
$.25 \times 20\,\mathrm{min}. +$

$.50 \times 60\,\mathrm{min}. +$

$.25 \times 32\,\mathrm{min}. =$

$43\,\mathrm{min}.$

What Probabilities to Use?



Expectimax search: a probabilistic model of opponent (or environment) in any state
- Model: possibly simple uniform distribution (roll die)
- Model: possibly sophisticated and require lots of computation
- Chance node for any outcome out of our control: opponent or environment
- The model might say that adversarial actions are likely!

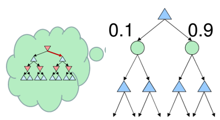
For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes

Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

Quiz: Informed Probabilities

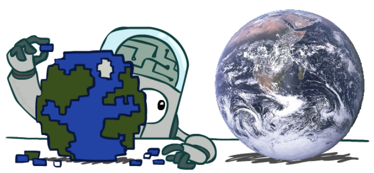
Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise

Question: What tree search should you use?



Answer: Expectimax!
- EACH chance node's probabilities, must run a simulation of your opponent
- Gets very slow very quickly
- Worse if simulate your opponent simulating you
- ... except for minimax, which has the nice property that it all collapses into one game tree

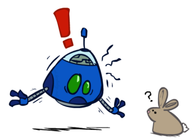
Modeling Assumptions



The Dangers of Optimism and Pessimism

Dangerous Optimism

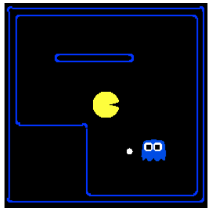
Assuming chance when the world is adversarial



Dangerous Pessimism

Assuming the worst case when it's not likely

Assumptions vs. Reality



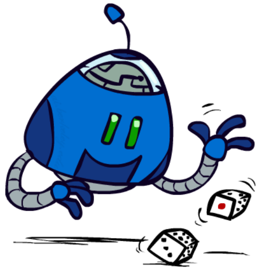
[Demos: world assumptions (L7D3,4,5,6)]

Results from playing 5 games

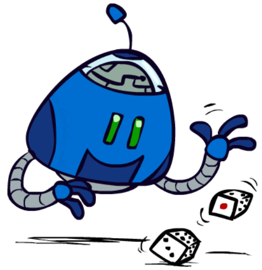
	Advers. Ghost	Random Ghost
Minimax	5/5 Avg:483	5/5 Avg:493
Expectimax	1/5 Avg:-303	5/5 Avg: 503

Pacman used depth 4 search with an eval function that avoids troubleGhost used depth 2 search with an eval function that seeks Pacman

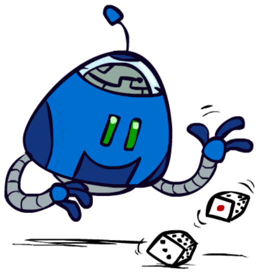
Demo Video:Random Ghost – Expectimax Pacman



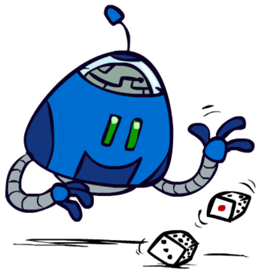
Demo Video – Minimax Pacman



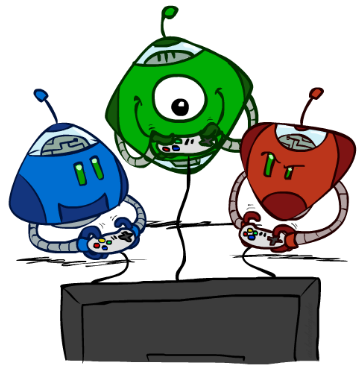
Demo Video: Ghost – Expectimax Pacman



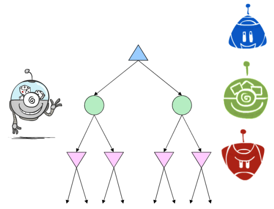
Demo Video: Random Ghost – Minimax Pacman



Other Game Types



Mixed Layer Types



E.g. Backgammon

Expectiminimax

- Environment is an extra "random agent" player that moves after each min/max agent
- Each node computes the appropriate combination of its children

Example: Backgammon

Dice rolls increase b: 21 possible rolls with 2 dice

- Backgammon $\approx$ 20 legal moves
- Depth $2 = 20x(21x20)^3 = 1.2x10^9$

As depth increases, probability of reaching a given search node shrinks

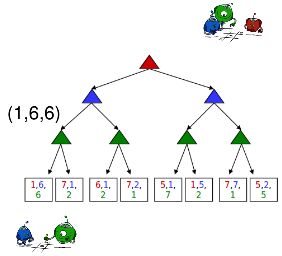
- So usefulness of search is diminished
- So limiting depth is less damaging
- But pruning is trickier. . .

Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play

1st AI world champion in any game!

Image: Wikipedia

Multi-Agent Utilities

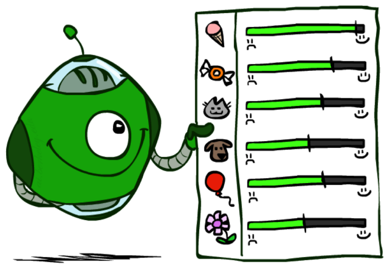


What if the game is not zero-sum, or has multiple players?

Generalization of minimax:

- Terminals have utility tuples
- Node values are also utility tuples
- Each player maximizes its own component
- Can give rise to cooperation and
- competition dynamically. . .

Utilities



Maximum Expected Utility

Why should we average utilities? Why not minimax?

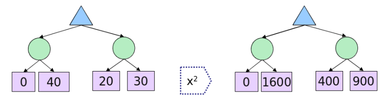
Principle of maximum expected utility:

- A rational agent should chose the action that maximizes its expected utility, given its knowledge

Questions:

- Where do utilities come from?
- How do we know such utilities even exist?
- How do we know that averaging even makes sense?
- What if our behavior (preferences) can't be described by utilities?

What Utilities to Use?



For worst-case minimax reasoning, terminal function scale doesn't matter

- We just want better states to have higher evaluations (get the ordering right)
- We call this insensitivity to monotonic transformations

For average-case expectimax reasoning, we need magnitudes to be meaningful

Utilities

Utilities: functions from outcomes (states of the world) to real numbers that describe agent's preferences

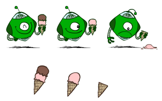
Where do utilities come from?

- In a game, may be simple (+1/-1)
- Utilities summarize the agent's goals

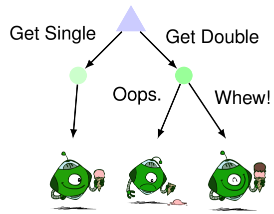
- Theorem: any "rational" preferences can be summarized as a utility function

We hard-wire utilities and let behaviors emerge

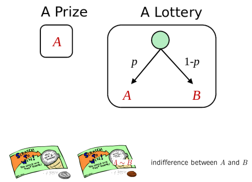
- Why don't we let agents pick utilities?
- Why don't we prescribe behaviors?



Utilities: Uncertain Outcomes



Preferences



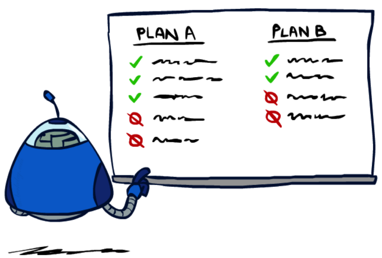
An agent must have preferences among:

- Prizes: A, B, etc.
- Lotteries: uncertain prizes

Notation:

- Preference: $A \succ B$
- Indifference: $A \sim B$

Rationality

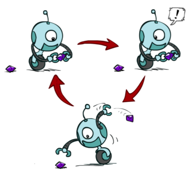


Rational Preferences

We want some constraints on preferences before we call them rational, such as:

Axiom of Transitivity:

$A \succ B \wedge B \succ C \implies A \succ C.$



For example: an agent with intransitive preferences can be induced to give away all of its money

- If $B \succ C$, then an agent with C would pay (say) 1 cent to get B
- If $A \succ B$, then an agent with B would pay (say) 1 cent to get A
- If $C \succ A$, then an agent with A would pay (say) 1 cent to get C

Rational Preferences

Orderability

$(A \succ B) \vee (B \succ A) \vee (A \sim B)$

Transitivity

$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

Continuity

$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$

Substitutability

$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$

Monotonicity

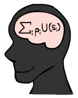
$A \succ B \Rightarrow$

$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

The Axioms of Rationality

MEU Principle



Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]

- Given any preferences satisfying these constraints, there exists a real-valued function U such that:

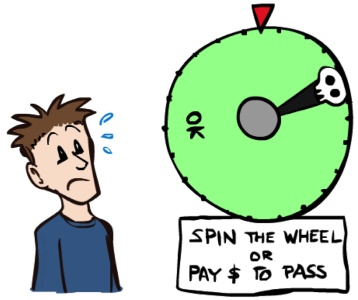
  $U(A) \geq U(B) \leftrightarrow A \succeq B.$
  $U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$

- I.e. values assigned by U preserve preferences of both prizes and lotteries!

Maximum expected utility (MEU) principle:

- Choose the action that maximizes expected utility
- Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
- E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

Human Utilities



Utility Scales

Normalized utilities: $u_+ = 1.0, u_- = 0.0.$

Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.

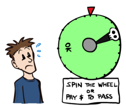
QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk

Note: behavior is invariant under positive linear transformation

$U'(x) = k_1 U(x) + k_2$

With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Human Utilities

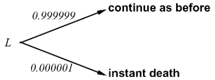


Utilities map states to real numbers. Which numbers?

Standard approach to assessment (elicitation) of human utilities:

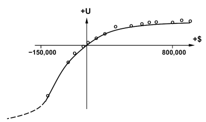
- Compare a prize A to a standard lottery L_p between
- "best possible prize" u_+ with probability p
- "worst possible catastrophe" u_- with probability 1-p

pay $30 ~



- Adjust lottery probability p until indifference: $A \; L_p$
- Resulting p is a utility in [0,1]

Money



Money does not behave as a utility function, but there is utility in having money (or being in debt)

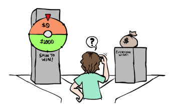
Given a lottery L = [p, $X; (1-p), $Y]

- Expected monetary value EMV(L): $p * X + (1 - p) * Y$
- $U(L) = p * U(\$X) + (1 - p) * U(\$Y)$
- Typically, $U(L) < U(EMV(L))$
- In this sense, people are risk-averse
- When deep in debt, people are risk-prone

Example: Insurance

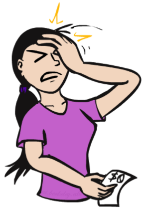
Consider the lottery:

[0.5, $1000; 0.5, $0]



- What is its expected monetary value? ($500)
- What is its certainty equivalent?
- Monetary value acceptable in lieu of lottery
- $400 for most people
- Difference of $100 is the insurance premium
- There's an insurance industry because people will pay to reduce their risk
- If everyone were risk-neutral, no insurance needed!
- It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is flat and they have many lotteries)

Example: Human Rationality?



Famous example of Allais (1953)

- A: [0.8, $4k; 0.2, $0]
- B: [1.0, $3k; 0.0, $0]

- C: [0.2, $4k; 0.8, $0]
- D: [0.25, $3k; 0.75, $0]

Most people prefer $B > A$, $C > D$

But if U($0) = 0, then

- $B > A \implies$ U($3k) $>$ 0.8 U($4k)
- $C > D \implies$ 0.8 U($4k) $>$ U($3k)

What's going on! Doh!

Next Time: MDPs!