

0/18 Questions Answered

HW 4 (Electronic Component)

STUDENT NAME

Q1 First Order Logic

25 Points

This exercise uses the function $\text{MapColor}(x)$ and predicates $\text{In}(x, y)$, $\text{Borders}(x, y)$, and $\text{Country}(x)$, whose arguments are geographical regions, along with constant symbols for various regions. Note that $\text{Borders}(x, x)$ is always False.

In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it is

- (1) correctly expresses the English sentence;
- (2) syntactically invalid and therefore meaningless;
- (3) syntactically valid but does not express the meaning of the English sentence.

Q1.1

5 Points

Paris and Marseilles are both in France.

- (i) $\text{In}(\text{Paris} \wedge \text{Marseilles}, \text{France})$.

Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

(ii) $In(Paris, France) \wedge In(Marseilles, France)$.

 Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

(iii) $In(Paris, France) \vee In(Marseilles, France)$.

 Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

Q1.2

5 Points

There is a country that borders both Iraq and Pakistan.

(i) $\exists c \text{Country}(c) \wedge \text{Borders}(c, \text{Iraq}) \wedge \text{Borders}(c, \text{Pakistan})$.

Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

(ii) $\exists c \text{ Country}(c) \Rightarrow [Borders(c, Iraq) \wedge Borders(c, Pakistan)]$.

 Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

(iii) $[\exists c \text{ Country}(c)] \Rightarrow [Borders(c, Iraq) \wedge Borders(c, Pakistan)]$.

 Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

(iv) $\exists c \text{ Borders}(\text{Country}(c), Iraq \wedge Pakistan)$.

 Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning**Q1.3**

5 Points

All countries that border Ecuador are in South America.

(i) $\forall c \text{ Country}(c) \wedge \text{Borders}(c, \text{Ecuador}) \Rightarrow \text{In}(c, \text{SouthAmerica})$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

(ii) $\forall c \text{ Country}(c) \Rightarrow [\text{Borders}(c, \text{Ecuador}) \Rightarrow \text{In}(c, \text{SouthAmerica})]$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

(iii) $\forall c [\text{Country}(c) \Rightarrow \text{Borders}(c, \text{Ecuador})] \Rightarrow \text{In}(c, \text{SouthAmerica})$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

(iv) $\forall c \text{ Country}(c) \wedge \text{Borders}(c, \text{Ecuador}) \wedge \text{In}(c, \text{SouthAmerica})$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

Save Answer

Q1.4

5 Points

No region in South America borders any region in Europe.

(i) $\neg[\exists c, d \text{ In}(c, \text{SouthAmerica}) \wedge \text{In}(d, \text{Europe}) \wedge \text{Borders}(c, d)]$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

(ii) $\forall c, d [\text{In}(c, \text{SouthAmerica}) \wedge \text{In}(d, \text{Europe})] \Rightarrow \neg \text{Borders}(c, d)$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

(iii) $\neg \forall c [\text{In}(c, \text{SouthAmerica}) \Rightarrow \exists d [\text{In}(d, \text{Europe}) \wedge \neg \text{Borders}(c, d)]]$.

Correct

Syntactically Invalid

Syntactically Valid but Doesn't Match Meaning

(iv) $\forall c \text{ In}(c, \text{SouthAmerica}) \Rightarrow \forall d \text{ In}(d, \text{Europe}) \Rightarrow \neg \text{Borders}(c, d)$.

Correct

 Syntactically Invalid

 Syntactically Valid but Doesn't Match Meaning

Q1.5

5 Points

No two adjacent countries have the same map color.

(i) $\forall x, y \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee \neg (MapColor(x) = MapColor(y))$.

 Correct

 Syntactically Invalid

 Syntactically Valid but Doesn't Match Meaning

(ii) $\forall x, y (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow \neg (MapColor(x) = MapColor(y))$.

 Correct

 Syntactically Invalid

 Syntactically Valid but Doesn't Match Meaning

(iii) $\forall x, y Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg (MapColor(x) = MapColor(y))$.

Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

(iv) $\forall x, y (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow MapColor(x \neq y)$.

 Correct Syntactically Invalid Syntactically Valid but Doesn't Match Meaning

Q2 Numerical Answer Formatting

0 Points

Many of the questions below about probability calculation have answers that are decimal numbers. Due to current limitations of Gradescope, your answers must be an exact string match to ours. In order to ensure an exact match, please carefully follow the following formatting for your numerical answers.

- If the number is an integer, do not include a decimal.
- Otherwise, please round the number to at most 4 places after the decimal.
- Do not include any trailing 0s for decimals.

Examples:

should be rounded to , while should be rounded to .

Note: If you use the Python interpreter to do your math, floating point error may lead to inexact decimal numbers. It is probably best to use another calculator, but if you do use Python you may need to adjust its output to get the actual exact answer.

Q3 Probability, Part I

14 Points

Below is a table listing the probabilities of three binary random variables.

Fill in the correct values for each marginal or conditional probability below.

X_0	X_1	X_2	$P(X_0, X_1, X_2)$
0	0	0	0.160
1	0	0	0.100
0	1	0	0.120
1	1	0	0.040
0	0	1	0.180
1	0	1	0.200
0	1	1	0.120
1	1	1	0.080

Q3.1

7 Points

$$P(X_0 = 1, X_1 = 0, X_2 = 1)$$

$$P(X_0 = 0, X_1 = 1)$$

$$P(X_2 = 0)$$

Q3.2

7 Points

$$P(X_1 = 0 \mid X_0 = 1)$$

$$P(X_0 = 1, X_1 = 0 \mid X_2 = 1)$$

$$P(X_0 = 1 \mid X_1 = 0, X_2 = 1)$$

Q4 Probability, Part II

14 Points

You are given the prior distribution $P(X)$, and two conditional distributions $P(Y \mid X)$ and $P(Z \mid Y)$ as below (you are also given

the fact that Z is independent from X given Y).

All variables are binary variables.

Compute the following joint distributions based on the chain rule.

X	$P(X)$	Y	X	$P(Y X)$	Z	Y	$P(Z Y)$
0	0.500	0	0	0.600	0	0	0.100
1	0.500	1	0	0.400	1	0	0.900
		0	1	0.900	0	1	0.700
		1	1	0.100	1	1	0.300

Q4.1

7 Points

$$P(X = 0, Y = 0)$$

$$P(X = 1, Y = 0)$$

$$P(X = 0, Y = 1)$$

$$P(X = 1, Y = 1)$$

Q4.2

7 Points

$$P(X = 0, Y = 0, Z = 0)$$

$$P(X = 1, Y = 1, Z = 0)$$

$$P(X = 1, Y = 0, Z = 1)$$

$$P(X = 1, Y = 1, Z = 1)$$

Q5 Probability, Part III

16 Points

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false.

For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

Q5.1

4 Points

X	Y	$P(X, Y)$
0	0	0.240
1	0	0.160
0	1	0.360
1	1	0.240

X	$P(X)$
0	0.600
1	0.400

Y	$P(Y)$
0	0.400
1	0.600

X is independent from Y .

- True
- False

Save Answer

Q5.2

4 Points

X	Y	$P(X, Y)$
0	0	0.540
1	0	0.360
0	1	0.060
1	1	0.040

X	$P(X)$
0	0.600
1	0.400

X	Y	$P(X Y)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

X is independent from Y .

- True
- False

Save Answer

Q5.3

4 Points

X	Y	Z	$P(X, Y, Z)$
0	0	0	0.280
1	0	0	0.070
0	1	0	0.210
1	1	0	0.140
0	0	1	0.060
1	0	1	0.060
0	1	1	0.030
1	1	1	0.150

X	Z	$P(X Z)$
0	0	0.700
1	0	0.300
0	1	0.300
1	1	0.700

Y	Z	$P(Y Z)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

X	Y	Z	$P(X, Y Z)$
0	0	0	0.400
1	0	0	0.100
0	1	0	0.300
1	1	0	0.200
0	0	1	0.200
1	0	1	0.200
0	1	1	0.100
1	1	1	0.500

X is independent from Y given Z .

True

False

Save Answer

Q5.4

4 Points

X	Y	Z	$P(X, Y, Z)$
0	0	0	0.140
1	0	0	0.140
0	1	0	0.060
1	1	0	0.060
0	0	1	0.048
1	0	1	0.192
0	1	1	0.072
1	1	1	0.288

X	Z	$P(X Z)$
0	0	0.500
1	0	0.500
0	1	0.200
1	1	0.800

Y	Z	$P(Y Z)$
0	0	0.700
1	0	0.300
0	1	0.400
1	1	0.600

X	Y	Z	$P(X, Y Z)$
0	0	0	0.350
1	0	0	0.350
0	1	0	0.150
1	1	0	0.150
0	0	1	0.080
1	0	1	0.320
0	1	1	0.120
1	1	1	0.480

X is independent from Y given Z .

True

False

Save Answer

Q6 Chain Rule

16 Points

Select all expressions that are equivalent to the specified probability using the given independence assumptions.

Q6.1

4 Points

Given no independence assumptions, $P(A, B | C) =$

$\frac{P(C|A)P(A|B)P(B)}{P(C)}$

$\frac{P(B,C|A)P(A)}{P(B,C)}$

$P(A | B, C)P(B | C)$

$\frac{P(A|C)P(B,C)}{P(C)}$

Save Answer

Q6.2

4 Points

Given that A is independent of B given C, $P(A, B | C) =$

$\frac{P(C|A)P(A|B)P(B)}{P(C)}$

$\frac{P(B,C|A)P(A)}{P(B,C)}$

$P(A | B, C)P(B | C)$

$\frac{P(A|C)P(B,C)}{P(C)}$

Save Answer

Q6.3

4 Points

Given no independence assumptions, $P(A | B, C) =$

$\frac{P(C|A)P(A|B)P(B)}{P(C)}$

$\frac{P(B,C|A)P(A)}{P(B,C)}$

$\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$

$\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

Save Answer

Q6.4

4 Points

Given that A is independent of B given C, $P(A | B, C) =$

$$\frac{P(C|A)P(A|B)P(B)}{P(C)}$$

$$\frac{P(B,C|A)P(A)}{P(B,C)}$$

$$\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$$

$$\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$$

Save Answer

Save All Answers

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