CS 188 Spring 2021 | Introduction to Artificial Intelligence | Exam Prep 2

Q1. Search

(a) Rubik's Search
Note: You do not need to know what a Rubik's cube is in order to solve this problem.

A Rubik's cube has about 4.3×10^{19} possible configurations, but any configuration can be solved in 20 moves or less. We pose the problem of solving a Rubik's cube as a search problem, where the states are the possible configurations, and there is an edge between two states if we can get from one state to another in a single move. Thus, we have 4.3×10^{19} states. Each edge has cost 1. Note that the state space graph does contain cycles. Since we can make 27 moves from each state, the branching factor is 27. Since any configuration can be solved in 20 moves or less, we have $h^*(n) \leq 20$.

For each of the following searches, estimate the approximate number of states expanded. Mark the option that is closest to the number of states expanded by the search. Assume that the shortest solution for our start state takes exactly 20 moves. Note that 27^{20} is much larger than 4.3×10^{19}.

(i) DFS Tree Search

Best Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

Worst Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(ii) DFS graph search

Best Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

Worst Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(iii) BFS graph search

Best Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

Worst Case: ○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(iv) A* tree search with a perfect heuristic, $h^*(n)$, Best Case

○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(v) A* tree search with a bad heuristic, $h(n) = 20 - h^*(n)$, Worst Case

○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(vi) A* graph search with a perfect heuristic, $h^*(n)$, Best Case

○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

(vii) A* graph search with a bad heuristic, $h(n) = 20 - h^*(n)$, Worst Case

○ 20 ○ 4.3×10^{19} ○ 27^{20} ○ ∞ (never finishes)

Q2. Searching with Heuristics

Consider the A* searching process on a connected undirected graph, with starting node S and the goal node G. Suppose the cost for each connection edge is **always positive**. We define $h^*(X)$ as the shortest (optimal) distance to G from a node X.

Note: You may want to solve Questions (a) and (b) at the same time.

(a) Suppose h is an **admissible** heuristic, and we conduct A* **tree search** using heuristic h' and finally find a solution. Let C be the cost of the found path (directed by h', defined in part (a)) from S to G.

(i) Choose **one best** answer for each condition below.

1. If $h'(X) = \frac{1}{2}h(X)$ for all Node X, then ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
2. If $h'(X) = \frac{h(X)+h^*(X)}{2}$ for all Node X, then ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
3. If $h'(X) = h(X) + h^*(X)$ for all Node X, then ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
4. If we define the set $K(X)$ for a node X as all its neighbor nodes Y satisfying $h^*(X) > h^*(Y)$, and the following always holds

$$h'(X) \leq \begin{cases} \min_{Y \in K(X)} h'(Y) - h(Y) + h(X) & \text{if } K(X) \neq \emptyset \\ h(X) & \text{if } K(X) = \emptyset \end{cases}$$

then, ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

5. If K is the same as above, we have

$$h'(X) = \begin{cases} \min_{Y \in K(X)} h(Y) + cost(X,Y) & \text{if } K(X) \neq \emptyset \\ h(X) & \text{if } K(X) = \emptyset \end{cases}$$

where $cost(X,Y)$ is the cost of the edge connecting X and Y,
then, ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

6. If $h'(X) = \min_{Y \in K(X)+\{X\}} h(Y)$ (K is the same as above), ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

(ii) In which of the conditions above, h' is still **admissible** and for sure to dominate h? Check all that apply. Remember we say h_1 dominates h_2 when $h_1(X) \geq h_2(X)$ holds for all X. ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

(b) Suppose h is a **consistent** heuristic, and we conduct A* **graph search** using heuristic h' and finally find a solution.

(i) Answer exactly the same questions for each conditions in Question (a)(i).

1. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
2. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
3. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
4. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
5. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$
6. ○ $C = h^*(S)$ ○ $C > h^*(S)$ ○ $C \geq h^*(S)$

(ii) In which of the conditions above, h' is still **consistent** and for sure to dominate h? Check all that apply.

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6