CS 188 Spring 2021 | Introduction to Artificial Intelligence | **Exam Prep 5 Solutions**

Q1. Probability

(a) A, B, C, and D are boolean random variables, and E is a random variable whose domain is $\{e_1, e_2, e_3, e_4, e_5\}$.

(i) How many entries are in the following probability tables and what is the sum of the values in each table? Write "?" if there is not enough information given.

Table	Size	Sum
$P(e \mid B)$	2	?
$P(A, B \mid c)$	4	1
$P(A, B \mid C, d, E)$	40	10
$P(a, E \mid B, C)$	20	?
$P(A, c, E)$	10	? OR P(c)

(ii) What is the **minimum** number of parameters needed to fully specify the distribution $P(A, B|C, d, E)$

$(2 \times 2 - 1) \times 2 \times 5 = 30$

(iii) What is the **minimum** number of parameters needed to fully specify the distribution $P(a, E|B, C)$

$5 \times 2 \times 2 = 20$

(b) Given the same set of random variables as defined in part (a). Write each of the following expressions in its simplest form, i.e., a single term. Make no independence assumptions unless otherwise stated.

Write "*Not possible*" if it is not possible to simplify the expression without making further independence assumptions.

(i)

$$\sum_{a'} P(a' \mid B, E) P(c \mid a', B, E)$$

$P(c \mid B, E)$

(ii)

$$\frac{\sum_{a'} P(B \mid a', C) P(a' \mid C) P(C)}{\sum_{d', e'} P(d' \mid e', C) P(e' \mid C) P(C)}$$

$P(B \mid C)$

Q2. More Probability

(a) For the following questions, you will be given a set of probability tables and a set of conditional independence assumptions. Given these tables and independence assumptions, write an expression for the requested probability tables. Keep in mind that your expressions cannot contain any probabilities other than the given probability tables. If it is not possible, mark "Not possible."

(i) Using probability tables $\mathbf{P(A)}, \mathbf{P(A \mid C)}, \mathbf{P(B \mid C)}, \mathbf{P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(A, B \mid C)}$.

$\mathbf{P(A, B \mid C)} =$ ______________________ ● Not possible.

(ii) Using probability tables $\mathbf{P(A)}, \mathbf{P(A \mid C)}, \mathbf{P(B \mid A)}, \mathbf{P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(B \mid A, C)}$.

$\mathbf{P(B \mid A, C)} = \frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_b P(A)\ P(B|A)\ P(C|A,B)}$ ○ Not possible.

(iii) Using probability tables $\mathbf{P(A \mid B)}, \mathbf{P(B)}, \mathbf{P(B \mid A, C)}, \mathbf{P(C \mid A)}$ and conditional independence assumption $\mathbf{A \perp\!\!\!\perp B}$, write an expression to calculate the table $\mathbf{P(C)}$.

$\mathbf{P(C)} = \sum_a P(A \mid B)\ P(C \mid A)$ ○ Not possible.

(iv) Using probability tables $\mathbf{P(A \mid B, C)}, \mathbf{P(B)}, \mathbf{P(B \mid A, C)}, \mathbf{P(C \mid B, A)}$ and conditional independence assumption $\mathbf{A \perp\!\!\!\perp B \mid C}$, write an expression for $\mathbf{P(A, B, C)}$.

$\mathbf{P(A, B, C)} =$ ______________________ ● Not possible.

(b) For each of the following equations, select the *minimal set* of conditional independence assumptions necessary for the equation to be true.

(i) $\mathbf{P(A, C) = P(A \mid B)\ P(C)}$

- ■ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ■ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(ii) $\mathbf{P(A \mid B, C)} = \frac{\mathbf{P(A)\ P(B|A)\ P(C|A)}}{\mathbf{P(B|C)\ P(C)}}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ■ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(iii) $\mathbf{P(A, B) = \sum_c P(A \mid B, c)\ P(B \mid c)\ P(c)}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ■ No independence assumptions needed.

(iv) $\mathbf{P(A, B \mid C, D) = P(A \mid C, D)\, P(B \mid A, C, D)}$

- [ ] $A \perp\!\!\!\perp B$
- [ ] $A \perp\!\!\!\perp B \mid C$
- [ ] $A \perp\!\!\!\perp B \mid D$
- [ ] $C \perp\!\!\!\perp D$
- [ ] $C \perp\!\!\!\perp D \mid A$
- [ ] $C \perp\!\!\!\perp D \mid B$
- [x] No independence assumptions needed.

(c) **(i)** Mark **all** expressions that are equal to $\mathbf{P(A \mid B)}$, **given no independence assumptions**.

- [ ] $\sum_c P(A \mid B, c)$
- [x] $\sum_c P(A, c \mid B)$
- [ ] $\frac{P(B|A)\ P(A|C)}{\sum_c P(B,c)}$
- [x] $\frac{\sum_c P(A,B,c)}{\sum_c P(B,c)}$
- [ ] $\frac{P(A,C|B)}{P(C|B)}$
- [ ] $\frac{P(A|C,B)\ P(C|A,B)}{P(C|B)}$
- [ ] None of the provided options.

(ii) Mark **all** expressions that are equal to $\mathbf{P(A, B, C)}$, given that $\mathbf{A \perp\!\!\!\perp B}$.

- [ ] $P(A \mid C)\ P(C \mid B)\ P(B)$
- [x] $P(A)\ P(B)\ P(C \mid A, B)$
- [ ] $P(C)\ P(A \mid C)\ P(B \mid C)$
- [ ] $P(A)\ P(C \mid A)\ P(B \mid C)$
- [x] $P(A)\ P(B \mid A)\ P(C \mid A, B)$
- [x] $P(A, C)\ P(B \mid A, C)$
- [ ] None of the provided options.

(iii) Mark **all** expressions that are equal to $\mathbf{P(A, B \mid C)}$, given that $\mathbf{A \perp\!\!\!\perp B \mid C}$.

- [x] $P(A \mid C)\ P(B \mid C)$
- [ ] $\frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_c P(A,B,c)}$
- [ ] $P(A \mid B)\ P(B \mid C)$
- [ ] $\frac{P(C)\ P(B|C)\ P(A|C)}{P(C|A,B)}$
- [ ] $\frac{\sum_c P(A,B,c)}{P(C)}$
- [x] $\frac{P(C,A|B)\ P(B)}{P(C)}$
- [ ] None of the provided options.