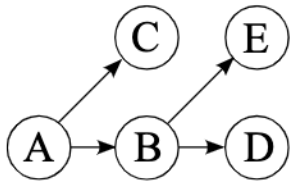


Q1. Bayes' Nets



	$P(A)$
$+a$	0.25
$-a$	0.75

	$P(B A)$	$+b$	$-b$
$+a$		0.5	0.5
$-a$		0.25	0.75

	$P(C A)$	$+c$	$-c$
$+a$		0.2	0.8
$-a$		0.6	0.4

	$P(D B)$	$+d$	$-d$
$+b$		0.6	0.4
$-b$		0.8	0.2

	$P(E B)$	$+e$	$-e$
$+b$		0.25	0.75
$-b$		0.1	0.9

(a) Using the Bayes' Net and conditional probability tables above, calculate the following quantities:

(i)  $P(+a, +b) = 0.25 * 0.5 = 0.125 = \frac{1}{8}$

(ii)  $P(+a|+b) = \frac{0.25*0.5}{0.25*0.5+0.25*0.75} = 0.4 = \frac{2}{5}$

(iii)  $P(+b|+a) = 0.5$

(b) Now we are going to consider variable elimination in the Bayes' Net above.

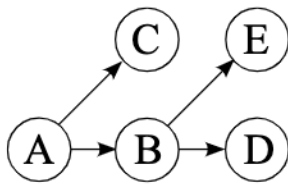
(i) Assume we have the evidence  $+c$  and wish to calculate  $P(E | +c)$ . What factors do we have initially?

$P(A), P(B | A), P(+c | A), P(D | B), P(E | B)$

(ii) If we eliminate variable B, we create a new factor. What probability does that factor correspond to?

$P(D, E | A)$

This is the same figure as the previous page, repeated here for your convenience:



	$P(A)$
$+a$	0.25
$-a$	0.75

	$P(B A)$	$+b$	$-b$
$+a$	0.5	0.5	
$-a$	0.25	0.75	

	$P(C A)$	$+c$	$-c$
$+a$	0.2	0.8	
$-a$	0.6	0.4	

	$P(D B)$	$+d$	$-d$
$+b$	0.6	0.4	
$-b$	0.8	0.2	

	$P(E B)$	$+e$	$-e$
$+b$	0.25	0.75	
$-b$	0.1	0.9	

(iii) What is the equation to calculate the factor we create when eliminating variable B?

$$f(A, D, E) = \sum_B P(B | A) * P(D | B) * P(E | B)$$

(iv) After eliminating variable B, what are the new set of factors? As in (ii), write the probabilities that the factors represent. For each factor, also provide its size. Use only as many rows as you need to.

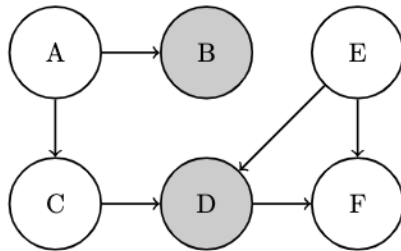
Factor	Size after elimination
$P(A)$	2
$P(+c   A)$	2
$P(D, E   A)$	$2^3$

(v) Now assume we have the evidence  $-c$  and are trying to calculate  $P(A|-c)$ . What is the most efficient elimination ordering? If more than one ordering is most efficient, provide any one of them. **E, D, B** or **D, E, B**

(vi) Once we have run variable elimination and have  $f(A, -c)$  how do we calculate  $P(+a | -c)$ ? (give an equation)  $\frac{f(+a, -c)}{f(+a, -c) + f(-a, -c)}$  or note that elimination is unnecessary - just use Bayes' rule

## Q2. Bayes Nets and Sampling

You are given a bayes net with the following probability tables:



E	D	F	$P(F E, D)$
0	0	0	0.6
0	0	1	0.4
0	1	0	0.7
0	1	1	0.3
1	0	0	0.2
1	0	1	0.8
1	1	0	0.7
1	1	1	0.3

A	$P(A)$	A	B	$P(B A)$	A	C	$P(C A)$
0	0.75	0	0	0.1	0	0	0.3
0	0.75	0	1	0.9	0	1	0.7
1	0.25	1	0	0.5	1	0	0.7
1	0.25	1	1	0.5	1	1	0.3

E	$P(E)$	E	C	D	$P(D E, C)$
0	0.1	0	0	0	0.5
0	0.1	0	0	1	0.5
0	0.1	0	1	0	0.2
0	0.1	0	1	1	0.8
1	0.9	1	0	0	0.5
1	0.9	1	0	1	0.5
1	0.9	1	1	0	0.2
1	0.9	1	1	1	0.8

You want to know  $P(C = 0|B = 1, D = 0)$  and decide to use sampling to approximate it.

(a) With prior sampling, what would be the likelihood of obtaining the sample  $[A=1, B=0, C=0, D=0, E=1, F=0]$ ?

- $0.25*0.1*0.3*0.9*0.8*0.7$ 
  $0.25*0.5*0.3*0.2*0.9*0.2$   
  $0.75*0.1*0.3*0.9*0.5*0.8$   
  $0.25*0.9*0.7*0.1*0.5*0.6$ 
  $0.75*0.1*0.3*0.9*0.5*0.2 + 0.25*0.5*0.7*0.5*0.9*0.2$   
  $0.25*0.5*0.7*0.5*0.9*0.2$

Other \_\_\_\_\_ Prior sampling samples without taking the evidence into account, so the probability of the sample is  $P(A)P(B|A)P(C|A)P(D|C,E)P(E)P(F|E,D)$

(b) Assume you obtained the sample  $[A = 1, B=1, C=0, D=0, E=1, F=1]$  through likelihood weighting. What is its weight?

- $0.25*0.5*0.7*0.5*0.9*0.8$ 
 0  
  $0.25*0.7*0.9*0.8 + 0.75*0.3*0.9*0.8$ 
  $0.5*0.5$   
  $0.25*0.5*0.7*0.5*0.8$ 
  $0.9*0.5 + 0.1*0.5$

Other \_\_\_\_\_ The weight of a sample in gibbs sampling is the probability of the evidence given their parents:  $P(D=0|E=1,C=0)*P(B=1|A=1)$

(c) You decide to use Gibb's sampling instead. Starting with the initialization  $[A = 1, B=1, C=0, D=0, E=0, F=0]$ , suppose you resample F first, what is the probability that the next sample drawn is  $[A = 1, B=1, C=0, D=0, E=0, F=1]$ ?

0.4

0.6

$0.6 \cdot 0.1 \cdot 0.5$

0

$0.25 \cdot 0.5 \cdot 0.7 \cdot 0.5 \cdot 0.1 \cdot 0.3$

$0.9 \cdot 0.5 + 0.1 \cdot 0.5$

Other \_\_\_\_\_ In Gibb's sampling, you resample individual vairables conditioned on the rest of the sample. The distribution of F given the rest of the sample is 0.4 for F=1 and 0.6 for F=0.