CS 188 Spring 2021 | Introduction to Artificial Intelligence | MT Review: Logic

Q1. Propositional Logic

(a) Which of the following are correct?

(i) $(A \wedge B) \implies C \models (A \implies C) \vee (B \implies C)$.

(ii) $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.

(iii) $(A \iff B) \wedge (\neg A \vee B)$ is satisfiable.

(iv) $(A \iff B) \iff C$ has the same number of models as $(A \iff B)$ for any fixed set of proposition symbols that includes A, B, C.

(b) Minesweeper, the well-known computer game, is closely related to the Pacman world. A minesweeper world is a rectangular grid of N squares with M invisible mines scattered among them. Any square may be probed by the agent; instant death follows if a mine is probed. Minesweeper indicates the presence of mines by revealing, in each probed square, the *number* of mines that are directly or diagonally adjacent. The goal is to probe every unmined square.

(i) Let $X_{i,j}$ be true iff square $[i, j]$ contains a mine. Write down the assertion that exactly two mines are adjacent to [1,1] as a sentence involving some logical combination of $X_{i,j}$ propositions.

(ii) Generalize your assertion from the previous part by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines. How many disjuncts would we need to use?

(iii) How can an agent use DPLL to prove that a given square does (or does not) contain a mine, ignoring the global constraint that there are exactly M mines in all? Formulate this as precisely as you can.

(iv) Suppose that we are no longer ignoring the global constraint as mentioned in the previous part, and we construct it using your formulation. How does the number of clauses depend on M and N? Suggest a way to modify DPLL so that the global constraint does not need to be represented explicitly.

Q2. First Order Logic

Consider a vocabulary with the following symbols:

- $Occuption(p, o)$: Predicate. Person p has occuption o.
- $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(v) Emily is either a surgeon or a lawyer.

(vi) Joe is an actor, but he also holds another job.

(vii) All surgeons are doctors.

(viii) Joe does not have a lawyer (i.e., is not a customer of any lawyer).

(ix) Emily has a boss who is a lawyer.

(x) There exists a lawyer all of whose customers are doctors.

(xi) Every surgeon has a lawyer.