

## Q1. Propositional Logic

(a) Provide justification for whether each of the following are correct or incorrect.

(i)  $(X \vee Y) \models Y$

(ii)  $\neg X \vee (Y \wedge Z) \models (X \implies Y)$

(iii)  $(X \vee Y) \wedge (Z \vee \neg Y) \models (X \vee Z)$

(b) Consider the following sentence:

$$[(Food \implies Party) \vee (Drinks \implies Party)] \implies [(Food \wedge Drinks) \implies Party].$$

(i) Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

(ii) Convert the left-hand and right-hand sides of the main implication into CNF.

(iii) What do you observe about the LHS and RHS after converting to CNF? Explain how your results prove the answer to part b.i.

## Q2. Encrypted Knowledge Base

We have a propositional logic knowledge base as shown below, and we are trying to find a satisfying assignment for the variables  $A, B, C, D$ , and  $E$ . Each line corresponds to a valid propositional logic sentence:

$$\begin{aligned} &\neg A \\ &B \Rightarrow A \\ &D \\ &C \vee B \\ &D \vee E \end{aligned}$$

(a) Your buddy Albert runs his solver, and hands you the model  $M = \{A = \text{False}, B = \text{False}, C = \text{True}, D = \text{True}, E = \text{True}\}$  that causes all of the knowledge base sentences to be true. We have a query sentence  $\alpha$  specified as  $(A \vee C) \Rightarrow E$ . Our model  $M$  also causes  $\alpha$  to be true. Can we say that the knowledge base entails  $\alpha$ ? Explain briefly (in one sentence) why or why not.

(b) Now we attempt to use theorem-proving methods to see whether our knowledge base entails a query sentence. To use these methods, it is useful to convert our knowledge base to conjunctive normal form (CNF), which satisfies:

- The sentence is a conjunction of (one or more) clauses.
- Each clause is a disjunction of literals.
- Each literal is a symbol or a negated symbol.

(i) Which sentences in the knowledge base are not already in conjunctive normal form? Convert them to CNF.

(ii) Write the entire knowledge base as a single sentence in CNF.

(iii) Describe the steps necessary for converting  $(A \wedge B) \vee (C \wedge D)$  to CNF.