#### CS 188 Fall 2021 Introduction to Artificial Intelligence

# Midterm

- You have approximately 110 minutes.
- The exam is open book, open calculator, and open notes.
- In the interest of fairness, we want everyone to have access to the same information. To that end, we will not be answering questions about the content or making clarifications.

#### • For multiple choice questions,

- means mark **all options** that apply
- $\bigcirc$  means mark a single choice

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#### For staff use only:

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## Q1. [16 pts] Potpourri

(a) [3 pts] Q1.1 Let T be the set of all possible game trees with alternating levels of maximizer and expectation nodes. Consider each of the following conditions independently. For each condition, select the condition if and only if there exists a tree in T such that knowing that condition and no others allows us to prune.

When all the leaf node values are bounded by some lower bound

When all the leaf node values are bounded by some upper bound

When all the leaf node values are negative

It is possible to prune a tree in T, but the necessary condition is not in the above list

 $\Box$  We can never prune any tree in T

(b) [3 pts] Q1.2 Jason is on Memorial Glade, and he is searching for Soda Hall. He knows that Soda Hall exists and is reachable in some finite distance. For each action, he can only move one step in the direction North, East, South, or West. Each action incurs a cost of 1. Jason can step off campus so he represents his search problem as a graph with infinite nodes. Which of the following algorithms will allow him to eventually reach Soda?

Breadth First Graph Search
Depth First Graph Search

Uniform Cost Search

A\* tree search with an admissible heuristic

- None of the above
- (c) Consider the following grid. Here, *D*, *E* and *F* are exit states. Pacman starts in state *A*. The reward for entering each state is reflected in the grid. Assume that discount factor  $\gamma = 1$ .



- (i) [3 pts] Write the optimal values  $V^*(s)$  for s = A and s = C and the optimal policy  $\pi^*(s)$  for s = A.
  - **Q1.3**  $V^*(A) =$  \_\_\_\_\_ **Q1.4**  $V^*(C) =$  \_\_\_\_\_
  - **Q1.5**  $\pi^*(A) = \bigcirc$  Up  $\bigcirc$  Down  $\bigcirc$  Left  $\bigcirc$  Right
- (ii) [4 pts] Now, instead of Pacman, Pacbaby is travelling in this grid. Pacbaby has a more limited set of actions than Pacman and can never go left. Hence, Pacbaby has to choose between actions: Up, Down and Right.

Pacman is rational, but Pacbaby is indecisive. If Pacbaby enters state *C*, Pacbaby finds the two best actions and randomly, with equal probability, chooses between the two. Let  $\pi^*(s)$  represent the optimal policy for Pacman. Let V(s) be the values under the policy where Pacbaby acts according to  $\pi^*(s)$  for all  $s \neq C$ , and follows the indecisive policy when at state *C*. What are the values V(s) for s = A and s = C?

**Q1.6** 
$$V(A) =$$
 \_\_\_\_\_ **Q1.7**  $V(C) =$  \_\_\_\_

(iii) [3 pts] Now Pacman knows that Pacbaby is going to be indecisive when at state C and he decides to recompute the optimal policy for Pacbaby at all other states, anticipating his indecisiveness at C. What is Pacbaby's new policy  $\pi(s)$  and new value V(s) for s = A?

**Q1.8** 
$$V(A) =$$
\_\_\_\_

**Q1.9**  $\pi(A) = \bigcirc$  Up  $\bigcirc$  Down  $\bigcirc$  Left  $\bigcirc$  Right

## Q2. [17 pts] CSPs: Secret Santa

The CS 188 staff members are playing Secret Santa! Each person brings a gift to the table, intending for a different person to receive that gift. However, past TA Albert, furious that he was not invited to participate, removes all the name cards on each gift so that none of the staff members know whose gift is whose!

The gifts are: AI textbook (A), Backgammon (B), Chess (C), Dinosaur toy (D), Easter egg (E), Flying drone (F).

The participants are: Angela, Daniel, Lexy, Ryan, Saagar, Yanlai

While the staff members have forgotten which gift is for who, Albert has left them some clues.

- 1. Each person should receive exactly one gift.
- 2. Yanlai should **not** receive the AI textbook (A).
- 3. Backgammon (B) should be gifted to either Lexy, Ryan, or Yanlai.
- 4. The Chess set (C) is not gifted to Daniel, Lexy or Saagar.
- 5. The name of the person who receives Backgammon (B) is alphabetically earlier than the name of the person who receives the AI textbook (A).

We frame this problem as a CSP, with the variables being gifts, and the domain being the six TAs who should receive gifts.

(a) (i) [1 pt] Q2.1 Which of the following constraints are binary constraints?

Constraint 1 Constraint 2 Constraint 3 Constraint 4 Constraint 5

(ii) [2 pts] Q2.2 Given just these clues, we use **local search** to try to find a satisfying assignment. We'll initialize the assignments alphabetically (i.e. assign the  $i^{th}$  person in alphabetical order to the  $i^{th}$  gift in alphabetical order). Which of the variables are conflicted in this assignment?

 $\square A \square B \square C \square D \square E \square F \bigcirc$  None

(iii) [2 pts] Q2.3 Say we randomly choose to swap the value of A with the value of some other variable, and use minconflicts to decide which to swap with. Which of the variables could be selected by the min-conflicts heuristic for A to swap with?

	A	] в	C		D	E	🗌 F		None
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Now Albert gives another clue:

6. The person who should get Backgammon (B) and the person who should get the Dinosaur (D) have names with the same number of letters (Angela/Daniel/Saagar/Yanlai, Lexy/Ryan).

# We have provided a full list of constraints along with a table at the end of this problem. Please feel free to use it as you work through each of the subparts. Note that constraint 7 which is introduced later applies only to questions 2.7 and after.

- (b) The TAs restart with all the variables unassigned, then enforce all unary constraints and perform arc consistency.
  - (i) [1 pt] Q2.4 How many values are left in the domain of C?

 $\bigcirc 1 \bigcirc 2 \bigcirc 3 \bigcirc 4 \bigcirc 5 \bigcirc 6$ 

(ii) [2 pts] Q2.5 Using the MRV heuristic, which variable(s) could be assigned next? Select multiple if there's a tie.



- (iii) [2 pts] Q2.6 Say we choose to assign a value for E. Which value should we assign to E according to the LCV Heuristic? Break ties alphabetically.
  - 🔿 Angela 🔿 Daniel 🔿 Lexy 🔿 Ryan 🔿 Saagar 🔿 Yanlai

Albert now gives one last clue:

- 7. The Easter egg (E) should be given to Daniel, Ryan, or Yanlai.
- (c) We want to use this information to solve our CSP.
  - (i) [1 pt] **Q2.7** Given all the constraints we have so far, do we have enough information to fully deduce the value of **any** of the variables?
    - $\bigcirc$  Yes  $\bigcirc$  No
  - (ii) [3 pts] Complete a full recursive backtracking search and identify a satisfying assignment. Apply the MRV heuristic when needed and break any ties alphabetically. Which gift does each person get?

*Hint:* Try to keep track of assignments by numbering each layer of domain pruning for easier backtracking. It may also help to keep a copy of the variables' domains after only enforcing unary constraints and arc consistency.

- Q2.8 A: O Angela ()Daniel  $\bigcirc$  Lexy O Ryan  $\bigcirc$ Saagar  $\bigcirc$ Yanlai Q2.9 B: O Angela  $\bigcirc$ Daniel  $\bigcirc$ Lexy Ryan  $\bigcirc$ Saagar  $\bigcirc$ Yanlai ()O Angela  $\bigcirc$ Q2.10 0  $\bigcirc$  $\bigcirc$ Ryan Yanlai C: Daniel Lexy Saagar 02.11 D: O Angela  $\bigcirc$  Lexy Ryan Saagar Yanlai  $\bigcirc$ Daniel () $\bigcirc$ ()Q2.12 E: () Angela () Daniel () Lexy Ryan ()Saagar Yanlai Q2.13 F: ()Angela ()Daniel ()Lexy ()Ryan ()Saagar Yanlai
- (iii) [1 pt] Q2.14 Which variable is assigned last in the backtracking search?

 $\bigcirc$  A  $\bigcirc$  B  $\bigcirc$  C  $\bigcirc$  D  $\bigcirc$  E  $\bigcirc$  F

(iv) [2 pts] Q2.15 How many different solutions does this CSP have?

(d) [0 pts] Q2.16 Optional: Which TA do you think would be most happy with their gift?

🔘 Angela 🔘 Daniel 🔘 Lexy 🔘 Ryan 🔘 Saagar 🔾 Yanlai

Please feel free to use the table below to help you solve this problem (your work on this will not be graded)

A:	Angela	Daniel	Lexy	Ryan	Saagar	Yanlai
B:	Angela	Daniel	Lexy	Ryan	Saagar	Yanlai
C:	Angela	Daniel	Lexy	Ryan	Saagar	Yanlai
D:	Angela	Daniel	Lexy	Ryan	Saagar	Yanlai
E:	Angela	Daniel	Lexy	Ryan	Saagar	Yanlai
F:	Angela	Daniel	Lexy	Ryan	Saagar	Yanlai

Constraints repeated:

- 1. Each person should receive exactly one gift.
- 2. Yanlai should **not** receive the AI textbook (A).
- 3. Backgammon (B) should be gifted to either Lexy, Ryan, or Yanlai.
- 4. The Chess set (C) is not gifted to Daniel, Lexy or Saagar.
- 5. The name of the person who receives Backgammon (B) is alphabetically earlier than the name of the person who receives the AI textbook (A).
- 6. The person who should get Backgammon (B) and the person who should get the Dinosaur (D) have names with the same number of letters (Angela/Daniel/Saagar/Yanlai, Lexy/Ryan).
- 7. The Easter egg (E) should be given to Daniel, Ryan, or Yanlai. (Q2.7 and after)

## Q3. [14 pts] Collaborative Search

We have a grid of *m* by *n* squares where some edges between adjacent squares are blocked by a wall. An Explorer wants to move from a starting square *E* to a goal square *G*. At each time step, the Explorer can move Up (*U*), Down (*D*), Left (*L*) or Right (*R*) to the adjacent square unless there is a wall on the edge that the Explorer wants to cross, and each move incurs a cost of 1. The locations of the walls are fixed. In addition, some squares are occupied by a booster (*B*). If the Explorer moves to a square with a booster at time step *t*, then at time step *t* + 1 the booster disappears from the grid, and the Explorer can choose to *teleport* in a certain direction (*U*, *D*, *L*, or *R*) for 1, 2, 3 or 4 squares (bypassing all the walls along the way), but the move incurs a cost of *k* the  $k^{th}$  time a booster is used.

- (a) (i) [3 pts] Q3.1 Assume for now that there are b boosters in the whole m by n grid, numbered 1, 2, ..., b, and the locations of these boosters are fixed and known. The Explorer wants to search for a *minimum-cost* path from E to G. Which of the following quantities should be included in a **minimal but sufficient** state space for this search problem?
  - The current position of the Explorer
  - An array of booleans indicating whether each square contains a booster
  - The number of boosters used so far
  - An array of booleans of length b, indicating whether each of the b boosters has been used
  - The number of total time steps traveled so far
  - An array of booleans indicating whether each edge is occupied by a wall
  - (ii) [1 pt] Q3.2 What is the maximum branching factor?
- (b) Suppose that we introduce a second agent called a Transporter and we have an unknown number of boosters on the grid to start. The Transporter starts at square T and its moves follow all the same rules as the Explorer, except that when the Transporter moves to a square with a booster at time step t, then at time step t + 1 it can choose to either move normally to an adjacent square without moving the booster, or carry the booster and move to an adjacent square along with the booster together, and put the booster down at any time. The Explorer and the Transporter can occupy the same square. For each time step, we first control the Explorer to make a move, and then we control the Transporter to make a move.
  - (i) [3 pts] Q3.3 We want to search for a *minimum-cost* sequence of actions for both agents which enables the Explorer to reach the goal *G*. Which of the following quantities should be included in a **minimal but sufficient** state space for this search problem?
- The current position of the Transporter
- An array of booleans indicating whether each square contains a booster
- The number of boosters used so far
- (ii) [1 pt] Q3.4 What is the maximum branching factor? Represent your answer in terms of A, the solution to Q3.2.
- (iii) [3 pts] Q3.5 Under which of the following modifications to the state space or to the rules of moving can DFS Tree Search always find a sequence of moves which enables the Explorer to move from E to G, if one such sequence exists (in the modified problem)? These modifications are applied individually and independently.

Include in the state space an array of numbers indicating the number of times each square has been visited by Explorer.

The Transporter is not allowed to visit a previously visited square.

The Explorer is not allowed to visit a previously visited square.

The Explorer is not allowed to visit a previously visited square, but after each usage of a booster, the set of previously visited squares is reset to empty.

(c) [3 pts] Q3.6 Which of the following statements are correct regarding the existence of solutions to the search problems in parts (a) and (b)?

When there is at least one booster, the search problem in part (a) always has a solution.

When there is at least one booster, the search problem in part (b) always has a solution.

For a given configuration of booster locations, if the search problem in part (a) has a solution, then the search problem in part (b) also has a solution.

For a given configuration of booster locations, it is possible that the search problem in part (a) does not have a solution, but the search problem in part (b) has a solution.

### Q4. [15 pts] Morning Commute

Pacman's apartment is really far from campus, so he decides to model his commute as a search problem for fun. He models Berkeley as an M by N square grid and starts out in the top left square; his goal is to find his way to campus C in the bottom right square in the least number of timesteps. He can move *left, right, up*, or *down*, as long as he doesn't leave the grid.

There is a single fixed bus route on the grid where a bus travels in a cycle at a constant rate of three spaces per timestep. You may assume that any square along this route is a valid bus stop. Pacman knows the route of the bus and if he is on the same square as the bus, **he can get on the bus and travel at a rate of three spaces per timestep along the bus route.** He can travel on the route for any number of timesteps and get off at any square adjacent to the path. Pacman's Clipper card is only good for one use, so he can only enter and exit the bus route at most once per commute.

Below is an example start state on a 6 by 6 grid. The arrows represent the bus's route. Note that this is just an example route, and the bus route does not necessarily have to be in this pattern. For the following questions, consider the general case, **not** the specific example below. **Also consider each part separately (i.e.: do not carry over assumptions).** 



- (a) [3 pts] Q4.1 Which of the following are admissible heuristics? Select all that apply.
  - The Manhattan distance between Pacman and campus.
  - The Manhattan distance between Pacman and campus divided by 4.
  - The Manhattan distance between Pacman and the bus.
  - The Manhattan distance between the closest bus stop to Pacman and campus.
  - None of the above.
- (b) [3 pts] Q4.2 Suppose Pacman knows that there is at least one bus stop adjacent to campus. Which of the following are admissible heuristics? Select all that apply.

The Manhattan distance between Pacman and the bus.

- The Manhattan distance between the bus and the stop next to campus divided by 3.
- $\frac{1}{3}$  for all states.

The minimum of the above heuristics.

- None of the above.
- (c) [3 pts] Q4.3 Suppose that there are *B* different buses with their own routes. Pacman can now ride the bus multiple times, but he must also ride a bus at least once. Which of the following are admissible heuristics? Select all that apply.

The minimum of the Manhattan distances between every bus stop and campus, and Pacman and campus.

The number of times Pacman has ridden the bus so far.

The Manhattan distance from Pacman to the closest bus stop plus the distance from that bus stop to campus divided by three.

None of the above.

(d) [3 pts] Suppose Pacman now wants to work out at the RSF (campus gym), located in the bottom left square of the grid. It doesn't matter if he goes before or after he gets to campus but he must visit both locations. Below is an example start state with the RSF represented by a dumbbell in the bottom left corner:



Q4.4 Which of the following are admissible heuristics? Select all that apply.

The minimum of the Manhattan distances between Pacman and the RSF, the RSF and campus, and Pacman and campus, divided by 3.

The sum of the Manhattan distances between Pacman and the RSF, the RSF and campus, and Pacman and campus, divided by 3.

The maximum of the above heuristics.

None of the above.

(e) [3 pts] Suppose that Pacman and his friend Albert want to go to campus together today. Albert lives in an apartment in the top right corner of the grid and wants to meet up with Pacman somewhere before moving together to campus. Assume that Albert moves independently from Pacman, one space per timestep, and that they must meet in another square before either is able to go to campus. Albert cannot take the bus with or without Pacman (although Pacman can still take it alone). In this updated search problem, you can control both Pacman and Albert.

Below is an example start state with Albert's apartment in the top right corner. Albert is currently in his apartment:

C				
				₽
		•	4	-
	Ŧ	+	+	•

Q4.5 Which of the following are admissible heuristics? Select all that apply.

The Manhattan distance between Pacman and Albert divided by 2.

The Manhattan distance from Albert to campus.

The Manhattan distance from Pacman to campus divided by 3.

None of the above.

### Q5. [19 pts] Games

Alice, Eve, and Bob are playing a multiplayer game. Each game state consists of three numbers where the left value represents Alice's score, the middle value represents Eve's score, and the right value represents Bob's score. Alice makes the first move, followed by Eve, and finally Bob. All scores for a single player are **between 1 and 9 inclusive**. In all pruning scenarios, **remember that we do not prune on equality.** 

Rather than trying to maximize their individual scores, Alice and Bob decide to work together to maximize their combined score, hoping that this will allow them to score higher. At each of Alice's and Bob's nodes, they will choose the option that maximizes **left value + right value**.

(a) Eve overhears their plan and decides that instead of maximizing her own score, she will try to minimize Alice and Bob's combined score. Alice and Bob are aware of Eve's strategy. Let the value of a node be the sum of the left and right scores of the node. Answer the following questions based on the game tree shown below.



Figure 1: Game tree where Alice is the root maximizer, Eve is the minimizer, and Bob is the bottom maximizer. Eve's score at each node (center cell) is not shown for simplicity.

(i) [1 pt] Q5.1 Solve the game tree shown in figure 1. What is the value of the root node?

- $\bigcirc 6$
- 12
- 0 13
- $\bigcirc$  Depends on the value of X only.
- $\bigcirc$  Depends on the value of Y only.
- $\bigcirc$  Depends on the values of both X and Y.
- $\bigcirc$  None of the above.
- (ii) [2 pts] Q5.2 Without pruning, which of the following are possible values for the right minimizer?



(iii) [2 pts] Q5.3 Which of the following nodes are **guaranteed** to be pruned when running alpha beta pruning on the game tree above? If there are nodes that may or may not be pruned depending on the values of X and Y, do not select them.



(iv) [3 pts] Q5.4 Which of the following nodes may or may not be pruned depending on the values of X and Y? Do not select any nodes from the previous part which are guaranteed to be pruned regardless of the values for X and Y.



- (b) Eve now decides that in addition to minimizing Alice and Bob's scores, she also wants to maximize her own score. Her new strategy is to choose the option that maximizes her own score minus Alice and Bob's combined score. That is, at Eve's turn, she will choose the option that maximizes middle value (left value + right value). Alice and Bob are aware of this new strategy. Using the same game tree shown above, assume that we can choose any number between 1 and 9 (inclusive) for X, Y, and Eve's score at each leaf node.
  - (i) [1 pt] **Q5.5** *True/False*: Compared to Eve's strategy in part (a), Eve's new strategy will result in an equal or higher final score for Eve in any leaf node configuration.
    - True False
  - (ii) [1 pt] **Q5.6** *True/False*: Compared to Eve's strategy in part (a), Eve's new strategy will result in an equal or higher final combined score for Alice and Bob in any leaf node configuration.
    - $\bigcirc$  True  $\bigcirc$  False
  - (iii) [3 pts] Q5.7 Which of the following leaf nodes could possibly be the game outcome if all players play optimally according to their strategy?



- (iv) [2 pts] Q5.8 Is it possible to prune in this scenario?
  - Yes because scores in each cell are bounded between 1 and 9.
  - $\bigcirc$  Yes but not for the reason above.
  - $\bigcirc$  No because Alice, Bob, and Eve are all acting as maximizers.
  - $\bigcirc$  No but not for the reason above.

(c) Eve is fed up with Alice and Bob teaming up and quits the game. Alice and Bob continue playing and decide to use brand new strategies that incorporate Eve's score for fun. This new game setup can be represented in the diagram below. In each of the following scenarios, Alice and Bob are aware of each other's strategies.



Figure 2: Game tree where Alice is the root and Bob controls the nodes in the middle level. Black triangles above a cell indicate that the cell's value contains the current player's score. (As a reminder, Bob's score is the right value and Alice's score is the left value.)

- (i) [2 pts] Q5.9 Alice and Bob agree to use the following strategy: each player maximizes their own score **plus** the average of the remaining two scores at each node. Assume that you can assign any value between 1 to 9 (inclusive) to all the leaf node scores. Is it possible to prune in this scenario?
  - O Yes.
  - No because Alice and Bob are both acting as maximizers.
  - No because Alice and Bob are both acting as expectimax nodes.
  - $\bigcirc$  No but not for the above reasons.
- (ii) [2 pts] Q5.10 Alice and Bob decide to follow a new strategy: each player maximizes their own score minus the average of the remaining two scores at each node. Assume that you can assign any value between 1 to 9 (inclusive) to all the leaf node scores. Is it possible to prune in this scenario?
  - O Yes.
  - No because Alice and Bob are both acting as maximizers.
  - $\bigcirc$  No because Alice and Bob are both acting as expectimax nodes.
  - No because Alice and Bob are maximizing different values which are not directly adversarial.
  - $\bigcirc$  No but not for the above reasons.

### Q6. [19 pts] Reinforcement Learning

- (a) The first part of this problem includes a number of conceptual short questions. Unless otherwise specified, for every subpart and answer choice, assume a discount factor  $\gamma < 1$  and rewards are bounded.
  - (i) [4 pts] Q6.1 Select all of the following statements about MDP and RL that are true.

Let  $\pi^*$  be the optimal policy. Then value iteration starting from random values will converge to  $V^{\pi^*}(s)$  for all states.

Approximate Q-learning is guaranteed to return the optimal policy upon convergence.

In environments with deterministic transitions, no exploration is required for Q-learning to converge to the optimal policy.

A large discount factor  $\gamma$  (approaching 1) on an MDP means that the agent emphasizes long-term rewards.

(ii) [2 pts] Q6.2 In which of the following scenarios is Q-learning necessarily preferable compared to vanilla Value Iteration if we want to extract an optimal policy?

Transition function known and reward function unknown.

Transition function unknown and reward function known.

Transition function unknown and reward function unknown.

None of the choices.

(iii) [2 pts] Q6.3 Assume that we run  $\epsilon$ -greedy Q-learning until convergence. What is the optimal policy  $\pi^*$  we obtain for an arbitrary state *s*?

$$\Box \pi^* (s) = \arg \max_s V(s) \qquad \Box \pi^* (s) = \arg \max_a Q(s, a)$$
$$\Box \pi^* (s) = \begin{cases} \arg \max_a Q(s, a), \text{ w.p. } 1 - \epsilon \\ \text{random action w.p. } \epsilon \end{cases} \qquad \Box \pi^* (s) = \arg \max_s Q(s, a)$$

- (iv) [2 pts] Q6.4 Following a Boltzmann policy  $\pi$  at each state *s* an agent is selecting action *a* with probability  $\pi$  (*a*) =  $\frac{e^{Q^*(s,a)/T}}{\sum_{a'} e^{Q^*(s,a')/T}}$ . If we increase *T* the agent will:
  - $\bigcirc$  Tend to follow a more greedy policy.
  - $\bigcirc$  Tend to randomize more among the actions.
  - $\bigcirc$  There is no effect on the agent's policy.
- (v) [3 pts] Q6.5 We are running Q-learning with exploration on an agent to determine an optimal policy. At each iteration, the agent will take the argmax over actions of the current Q-value at each state. Which of the following exploration functions will make the agent explore more actions that are rarely sampled initially and then later act greedily as the number of iterations increases? The update rule of Q-learning using an exploration function f is  $Q(s, a) = (1 \alpha)Q(s, a) + \alpha (R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a')))$ , with N(s', a') being the visitation count of the state-action pair  $s', a', \alpha$  the learning rate and  $\gamma$  the discount factor.

$$\begin{array}{c} \square \ f\left(Q\left(s',a'\right),N\left(s',a'\right)\right) = Q\left(s',a'\right) + N\left(s',a'\right)^{2} \\ \square \ f\left(Q\left(s',a'\right),N\left(s',a'\right)\right) = Q\left(s',a'\right) + \log\left(N\left(s',a'\right)\right)/N\left(s',a'\right) \\ \square \ f\left(Q\left(s',a'\right),N\left(s',a'\right)\right) = Q\left(s',a'\right) + 1/N\left(s',a'\right)^{2} \\ \square \ f\left(Q\left(s',a'\right),N\left(s',a'\right)\right) = Q\left(s',a'\right) + 1/N\left(s',a'\right) \\ \square \ f\left(Q\left(s',a'\right),N\left(s',a'\right)\right) = Q\left(s',a'\right) + N\left(s',a'\right)^{1/3} \\ \square \ f\left(Q\left(s',a'\right),N\left(s',a'\right)\right) = Q\left(s',a'\right) + 1/\sqrt[3]{N\left(s',a'\right)} \end{array}$$

(b) Now let's look more closely at a Q-learning problem. This is more of a conceptual question and should not require too many calculations. Assume we have a simple MDP with three states A, B, and C, and one action  $\rightarrow$ . We are given access to the following transitions:

state	action	reward	next state
А	$\rightarrow$	-1	В
В	$\rightarrow$	-1	C
С	$\rightarrow$	-1	А

In what follows, we run Q-learning using the data on the table for an infinite number of iterations.

(i) [1 pt] **Q6.6** If the discount factor  $\gamma$  is 1 and the learning rate  $\alpha = 1$  what will the value of  $Q(A, \rightarrow)$  be?

 $\bigcirc -1 \\ \bigcirc -2 \\ \bigcirc -\infty$ 

- (ii) [2 pts] Q6.7 If we decrease the learning rate  $\alpha$  to 0.5 in the previous question what will the value of  $Q(A, \rightarrow)$  now be after an infinite number of iterations?
  - $\bigcirc -1 \\ \bigcirc -2 \\ \bigcirc -\infty$

Let's add another state D in the MDP and an extra transition  $\leftarrow$  in our dataset:

state	action	reward	next state
А	$\rightarrow$	-1	В
В	$\rightarrow$	-1	C
С	$\rightarrow$	-1	А
С	←	-1	D
D	$\rightarrow$	0	D

We still run Q-learning on the new dataset with a learning rate  $\alpha \in (0, 1)$  for an infinite number of iterations.

- (iii) [1 pt] Q6.8 If we still keep  $\gamma = 1$  then what is the value for  $Q(B, \rightarrow)$  after Q-learning has converged?
  - $\bigcirc$  -1  $\bigcirc$  -2
  - \_\_\_\_∞
- (iv) [2 pts] Q6.9 Is there a value for the discount factor  $\gamma > 0$  such that the optimal policy at state *C* is action  $\rightarrow$  instead of  $\leftarrow$ ?
  - O Yes
  - O No