Q1. Propositional Logic

(a) Which of the following are correct?

(i) $(A \land B) \implies C \models (A \implies C) \lor (B \implies C)$.

(ii) $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$.

(iii) $(A \iff B) \land (\neg A \lor B)$ is satisfiable.

(iv) $(A \iff B) \iff C$ has the same number of models as $(A \iff B)$ for any fixed set of proposition symbols that includes A, B, C.

(b) Minesweeper, the well-known computer game, is closely related to the Pacman world. A minesweeper world is a rectangular grid of N squares with M invisible mines scattered among them. Any square may be probed by the agent; instant death follows if a mine is probed. Minesweeper indicates the presence of mines by revealing, in each probed square, the *number* of mines that are directly or diagonally adjacent. The goal is to probe every unmined square.

(i) Let $X_{i,j}$ be true iff square $[i, j]$ contains a mine. Write down the assertion that exactly two mines are adjacent to [1,1] as a sentence involving some logical combination of $X_{i,j}$ propositions.

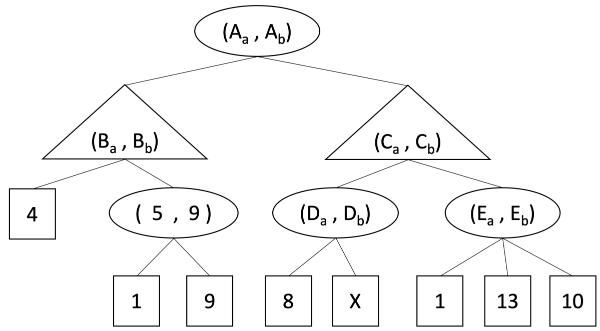
(ii) Generalize your assertion from the previous part by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines. How many disjuncts would we need to use?

(iii) How can an agent use DPLL to prove that a given square does (or does not) contain a mine, ignoring the global constraint that there are exactly M mines in all? Formulate this as precisely as you can.

(iv) Suppose that we are no longer ignoring the global constraint as mentioned in the previous part, and we construct it using your formulation. How does the number of clauses depend on M and N? Suggest a way to modify DPLL so that the global constraint does not need to be represented explicitly.

Q2. [Optional] Games

Alice is playing a two-player game with Bob, in which they move alternately. Alice is a maximizer. Although Bob is also a maximizer, Alice believes Bob is a minimizer with probability 0.5, and a maximizer with probability 0.5. Bob is aware of Alice's assumption.
In the game tree below, square nodes are the outcomes, triangular nodes are Alice's moves, and round nodes are Bob's moves. Each node for Alice/Bob contains a tuple, the left value being Alice's expectation of the outcome, and the right value being Bob's expectation of the outcome.
Tie-breaking: choose the left branch.



(a) In the blanks below, fill in the tuple values for tuples (B_a, B_b) and (E_a, E_b) from the above game tree.

$(B_a, B_b) = ($ ☐ , ☐ $)$

$(E_a, E_b) = ($ ☐ , ☐ $)$

(b) In this part, we will determine the values for tuple (D_a, D_b).

(i) $D_a =$ ○ 8 ○ X ○ 8+X ○ 4+0.5X ○ min(8,X) ○ max(8,X)

(ii) $D_b =$ ○ 8 ○ X ○ 8+X ○ 4+0.5X ○ min(8,X) ○ max(8,X)

(c) Fill in the values for tuple (C_a, C_b) below. For the bounds of X, you may write scalars, ∞ or $-\infty$.
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of $\frac{8}{2}$, and 0.5 instead of $\frac{1}{2}$).

1. If $-\infty <$ X $<$ ☐ , $(C_a, C_b) = ($ ☐ , ☐ $)$
2. Else, $(C_a, C_b) = ($ ☐ , max(☐ , ☐))

(d) Fill in the values for tuple (A_a, A_b) below. For the bounds of X, you may write scalars, ∞ or $-\infty$.
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of $\frac{8}{2}$, and 0.5 instead of $\frac{1}{2}$).

1. If $-\infty <$ X $<$ ☐ , $(A_a, A_b) = ($ ☐ , ☐ $)$
2. Else, $(A_a, A_b) = ($ ☐ , max(☐ , ☐))

(e) When Alice computes the left values in the tree, some branches can be pruned and do not need to be explored. In the game tree graph above, put an 'X' on these branches. If no branches can be pruned, write "Not Possible" below. Assume that the children of a node are visited in left-to-right order and that you should not prune on equality.