CS 188 Spring 2022 | Introduction to Artificial Intelligence | Exam Prep 5

Q1. First Order Logic

Consider a vocabulary with the following symbols:

- $Occuption(p, o)$: Predicate. Person p has occuption o.
- $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(i) Emily is either a surgeon or a lawyer.

(ii) Joe is an actor, but he also holds another job.

(iii) All surgeons are doctors.

(iv) Joe does not have a lawyer (i.e., is not a customer of any lawyer).

(v) Emily has a boss who is a lawyer.

(vi) There exists a lawyer all of whose customers are doctors.

(vii) Every surgeon has a lawyer.

Q2. [Optional] Logic

(a) Prove, or find a counterexample to, each of the following assertions:

(i) If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$

(ii) If $(\alpha \land \beta) \models \gamma$ then $\alpha \models \gamma$ or $\beta \models \gamma$ (or both).

(iii) If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

(b) Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(i) $Smoke \implies Smoke$

(ii) $Smoke \implies Fire$

(iii) $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

(iv) $Smoke \lor Fire \lor \neg Fire$

(v) $((Smoke \land Heat) \implies Fire) \Longleftrightarrow ((Smoke \implies Fire) \lor (Heat \implies Fire))$

(vi) $(Smoke \implies Fire) \implies ((Smoke \land Heat) \implies Fire)$

(vii) $Big \lor Dumb \lor (Big \implies Dumb)$

(c) Suppose an agent inhabits a world with two states, S and $\neg S$, and can do exactly one of two actions, a and b. Action a does nothing and action b flips from one state to the other. Let S^t be the proposition that the agent is in state S at time t, and let a^t be the proposition that the agent does action a at time t (similarly for b^t).

(i) Write a successor-state axiom for S^{t+1}.

(ii) Convert the sentence in the previous part into CNF.