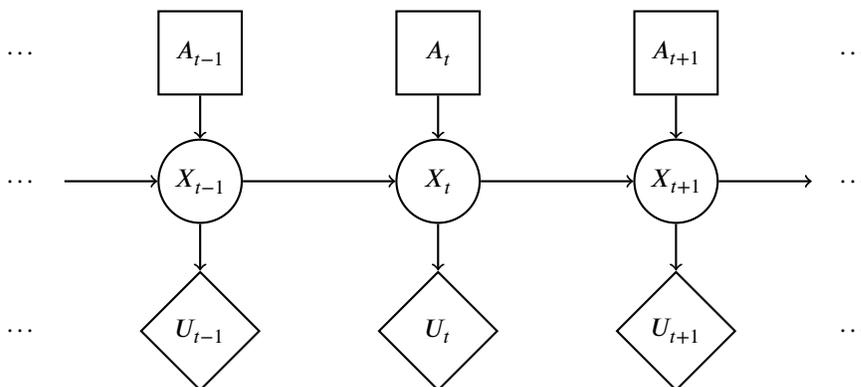


### Q1. Planning ahead with HMMs

Pacman is tired of using HMMs to estimate the location of ghosts. He wants to use HMMs to plan what actions to take in order to maximize his utility. Pacman uses the HMM (drawn to the right) of length  $T$  to model the planning problem. In the HMM,  $X_{1:T}$  is the sequence of hidden states of Pacman's world,  $A_{1:T}$  are actions Pacman can take, and  $U_t$  is the utility Pacman receives at the particular hidden state  $X_t$ . Notice that there are no evidence variables, and utilities are not discounted.



(a) The belief at time  $t$  is defined as  $B_t(X_t) = p(X_t|a_{1:t})$ . The forward algorithm update has the following form:

$$B_t(X_t) = \underline{\hspace{2cm} \text{(i)} \hspace{2cm}} \underline{\hspace{2cm} \text{(ii)} \hspace{2cm}} B_{t-1}(x_{t-1}).$$

Complete the expression by choosing the option that fills in each blank.

- (i)        $\max_{x_{t-1}}$         $\sum_{x_{t-1}}$         $\max_{x_t}$         $\sum_{x_t}$        1  
 (ii)       $p(X_t|x_{t-1})$         $p(X_t|x_{t-1})p(X_t|a_t)$         $p(X_t)$         $p(X_t|x_{t-1}, a_t)$        1

None of the above combinations is correct

(b) Pacman would like to take actions  $A_{1:T}$  that maximizes the expected sum of utilities, which has the following form:

$$MEU_{1:T} = \underline{\hspace{2cm} \text{(i)} \hspace{2cm}} \underline{\hspace{2cm} \text{(ii)} \hspace{2cm}} \underline{\hspace{2cm} \text{(iii)} \hspace{2cm}} \underline{\hspace{2cm} \text{(iv)} \hspace{2cm}} \underline{\hspace{2cm} \text{(v)} \hspace{2cm}}$$

Complete the expression by choosing the option that fills in each blank.

- (i)        $\max_{a_{1:T}}$         $\max_{a_T}$         $\sum_{a_{1:T}}$         $\sum_{a_T}$        1  
 (ii)       $\max_t$         $\prod_{t=1}^T$         $\sum_{t=1}^T$         $\min_t$        1  
 (iii)      $\sum_{x_t, a_t}$         $\sum_{x_t}$         $\sum_{a_t}$         $\sum_{x_T}$        1  
 (iv)      $p(x_t|x_{t-1}, a_t)$         $p(x_t)$         $B_t(x_t)$         $B_T(x_T)$        1  
 (v)       $U_T$         $\frac{1}{U_t}$         $\frac{1}{U_T}$         $U_t$        1

None of the above combinations is correct

(c) A greedy ghost now offers to tell Pacman the values of some of the hidden states. Pacman needs your help to figure out if the ghost's information is useful. Assume that the transition function  $p(x_t|x_{t-1}, a_t)$  is not deterministic. **With respect to the utility  $U_t$** , mark all that can be True:

- $VPI(X_{t-1}|X_{t-2}) > 0$         $VPI(X_{t-2}|X_{t-1}) > 0$         $VPI(X_{t-1}|X_{t-2}) = 0$         $VPI(X_{t-2}|X_{t-1}) = 0$         
 None of the above

(d) Pacman notices that calculating the beliefs under this model is very slow using exact inference. He therefore decides to try out various particle filter methods to speed up inference. Order the following methods by how accurate their estimate of  $B_T(X_T)$  is? If different methods give an equivalently accurate estimate, mark them as the same number.

	Most accurate			Least accurate
Exact inference	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4
Particle filtering with no resampling	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4
Particle filtering with resampling before every time elapse	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4
Particle filtering with resampling before every other time elapse	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4



