CS 188
Spring 2022

Final Review: Logic

Q1. Disjunctive Normal Form

A sentence is in disjunctive normal form (DNF) if it is the disjunction of conjunctions of literals. For example, the sentence $(A \wedge B \wedge \neg C) \vee (\neg A \wedge C) \vee (B \wedge \neg C)$ is in DNF.

(a) Any propositional logic sentence is logically equivalent to the assertion that some possible world in which it would be true is in fact the case. From this observation, prove that any sentence can be written in DNF.

(b) Construct an algorithm that converts any sentence in propositional logic into DNF. (*Hint*: The algorithm is similar to the algorithm for conversion to CNF.)

(c) Construct a simple algorithm that takes as input a sentence in DNF and returns a satisfying assignment if one exists, or reports that no satisfying assignment exists.

(d) Apply the algorithms in the previous two parts to the following set of sentences:

$$A \implies B$$
$$B \implies C$$
$$C \implies \neg A$$

(e) Since the algorithm in (b) is very similar to the algorithm for conversion to CNF, and since the algorithm in (c) is much simpler than any algorithm for solving a set of sentences in CNF, why is this technique not used in automated reasoning?

Q2. In What Worlds?

(a) We wish to come up with hypotheses that entail the following sentences:

- S_1: $X_1 \wedge X_2 \implies Y$
- S_2: $\neg X_1 \vee X_2 \implies Y$

In this problem, we want to come up with a hypothesis H such that $H \models S_1 \wedge H \models S_2$.

(i) Assume we have the hypothesis H: $Y \iff X_1 \vee X_2$.

Does H entail S_1? ◯ Yes ◯ No

Does H entail S_2? ◯ Yes ◯ No

(ii) Pretend that we have obtained a magical solver, $SAT(s)$ which takes in a sentence s and returns $true$ if s is satisfiable and $false$ otherwise. We wishes to use this solver to determine whether a hypothesis H' entails the two sentences S_1 and S_2. Mark all of the following expressions that correctly return $true$ if and only if $H' \models S_1 \wedge H' \models S_2$. If none of the expressions are correct, select "None of the above".

☐ $SAT(H' \wedge \neg(S_1 \wedge S_2))$ ☐ $SAT(\neg H' \vee (S_1 \wedge S_2))$

☐ $\neg SAT(H' \wedge \neg(S_1 \wedge S_2))$ ☐ $\neg SAT(\neg H' \vee (S_1 \wedge S_2))$ ☐ None of the above

Four people, Alex, Betty, Cathy, and Dan are going to a famliy gathering. They can bring dishes or games. They have the following predicates in their vocabulary:

- $Brought(p, i)$: Person p brought a dish or game i.
- $Cooked(p, d)$: Person p cooked dish d.
- $Played(p, g)$: Person p played game g.

(b) Select which first-order logic sentences are syntactically correct translations for the following English sentences. You must use the syntax shown in class (eg. ∀, ∃, ∧, ⇒, ⇔). **Please select all that apply**.

(i) At least one dish cooked by Alex was brought by Betty.

☐ $\exists d\ Cooked(A, d) \wedge Brought(B, d)$
☐ $[\exists d\ Cooked(A, d)] \wedge [\forall d' \wedge (d' = d)\ Brought(B, d')]$
☐ $\neg[\forall d\ Cooked(A, d) \vee Brought(B, d)]$
☐ $\exists d_1, d_2\ Cooked(A, d_1) \wedge (d_2 = d_1)\ \wedge Brought(B, d_2)$

(ii) At least one game played by Cathy is only played by people who brought dishes.

☐ $\neg[\forall g\ Played(C, g) \vee [\exists p\ Played(p, g) \implies \forall d\ Brought(p, d)]]$
☐ $\forall p\ \exists g\ Played(C, g) \wedge Played(p, g) \implies \exists d\ Brought(p, d)$
☐ $\exists g\ Played(C, g) \implies \forall p \exists d\ Played(p, g) \wedge Brought(p, d)$
☐ $\exists g\ Played(C, g) \wedge [\forall p\ Played(p, g) \Rightarrow \exists d,\ Brought(p, d)]$

(c) Assume we have the following sentence with variables A, B, C, and D, where each variable takes Boolean values:

$$S3 : (A \vee B \vee \neg C) \wedge (A \vee \neg B \vee D) \wedge (\neg B \vee \neg D)$$

(i) For the above sentence $S3$, state how many worlds make the sentence true. [Hint: you can do this and the next part without constructing a truth table!]

(ii) Does $S3 \models (A \wedge B \wedge D)$? ◯ Yes ◯ No