1 Variable Elimination

Using the Bayes Net shown below, we want to compute $P(Y \mid +z)$. All variables have binary domains. We run variable elimination, with the following variable elimination ordering: $X, T, U, V, W$. After inserting evidence, we have the following factors to start out with:

$$P(T), P(U\mid T), P(V\mid T), P(W\mid T), P(X\mid T), P(Y\mid V, W), P(+z\mid X)$$

(a) When eliminating $X$ we generate a new factor $f_1$ as follows,

$$f_1(+z\mid T) = \sum_x P(x\mid T)P(+z\mid x)$$

which leaves us with the factors:

$$P(T), P(U\mid T), P(V\mid T), P(W\mid T), P(Y\mid V, W), f_1(+z\mid T)$$

(b) When eliminating $T$ we generate a new factor $f_2$ as follows, which leaves us with the factors:

(c) When eliminating $U$ we generate a new factor $f_3$ as follows, which leaves us with the factors:

(d) When eliminating $V$ we generate a new factor $f_4$ as follows, which leaves us with the factors:

(e) When eliminating $W$ we generate a new factor $f_5$ as follows, which leaves us with the factors:

(f) How would you obtain $P(Y \mid +z)$ from the factors left above:

(g) What is the size of the largest factor that gets generated during the above process?

(h) Does there exist a better elimination ordering (one which generates smaller largest factors)?
2 Sampling and Dynamic Bayes Nets

We would like to analyze people’s ice cream eating habits on sunny and rainy days. Suppose we consider the weather, along with a person’s ice-cream eating, over the span of two days. We’ll have four random variables: \( W_1 \) and \( W_2 \) stand for the weather on days 1 and 2, which can either be rainy \( R \) or sunny \( S \), and the variables \( I_1 \) and \( I_2 \) represent whether or not the person ate ice cream on days 1 and 2, and take values \( T \) (for truly eating ice cream) or \( F \). We can model this as the following Bayes Net with these probabilities.

\[
\begin{array}{c|c|c|c|}
W_1 & P(W_1) & W_1 & P(W_2|W_1) \\
\hline
S & 0.6 & S & 0.7 \\
R & 0.4 & R & 0.3 \\
\end{array}
\]

1. What is \( P(W_2 = R) \), the probability that sampling assigns to the event \( W_2 = R \)?

2. Cross off samples above which are rejected by rejection sampling if we’re computing \( P(W_2|I_1 = T, I_2 = F) \).

Rejection sampling seems to be wasting a lot of effort, so we decide to switch to likelihood weighting. Assume we generate the following six samples given the evidence \( I_1 = T \) and \( I_2 = F \):

\[
(W_1, I_1, W_2, I_2) = \{ (S, T, R, F), (R, T, R, F), (S, T, S, F), (S, T, R, F), (S, T, S, F), (R, T, S, F) \}.
\]

3. What is the weight of the first sample \( (S, T, R, F) \) above?

4. Use likelihood weighting to estimate \( P(W_2|I_1 = T, I_2 = F) \).