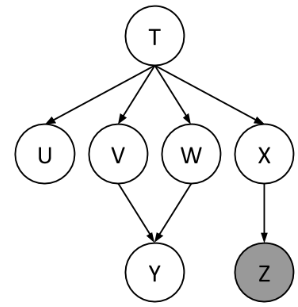
Regular Discussion 7

1 Variable Elimination

Using the Bayes Net shown below, we want to compute $P(Y \mid +z)$. All variables have **binary domains**. We run variable elimination, with the following variable elimination ordering: X, T, U, V, W.

After inserting evidence, we have the following factors to start out with:



$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V,W), P(+z|X)$$

(a) When eliminating X we generate a new factor f_1 as follows,

$$f_1(+z|T) = \sum_x P(x|T)P(+z|x)$$

which leaves us with the factors:

$$P(T), P(U|T), P(V|T), P(W|T), P(Y|V,W), f_1(+z|T)$$

(b) When eliminating T we generate a new factor f_2 as follows, which leaves us with the factors:

(c) When eliminating U we generate a new factor f_3 as follows, which leaves us with the factors:

(d) When eliminating V we generate a new factor f_4 as follows, which leaves us with the factors:

(e) When eliminating W we generate a new factor f_5 as follows, which leaves us with the factors:

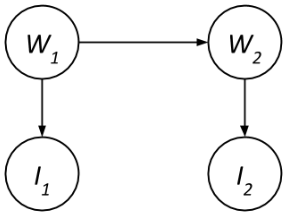
(f) How would you obtain $P(Y \mid +z)$ from the factors left above:

(g) What is the size of the largest factor that gets generated during the above process?

(h) Does there exist a better elimination ordering (one which generates smaller largest factors)?

2 Sampling and Dynamic Bayes Nets

We would like to analyze people's ice cream eating habits on sunny and rainy days Suppose we consider the weather, along with a person's ice-cream eating, over the span of two days. We'll have four random variables: W_1 and W_2 stand for the weather on days 1 and 2, which can either be rainy `R` or sunny `S`, and the variables I_1 and I_2 represent whether or not the person ate ice cream on days 1 and 2, and take values `T` (for truly eating ice cream) or `F`. We can model this as the following Bayes Net with these probabilities.



W_1	$P(W_1)$
S	0.6
R	0.4

W_1	W_2	$P(W_2\|W_1)$
S	S	0.7
S	R	0.3
R	S	0.5
R	R	0.5

W	I	$P(I\|W)$
S	T	0.9
S	F	0.1
R	T	0.2
R	F	0.8

Suppose we produce the following samples of (W_1, I_1, W_2, I_2) from the ice-cream model:

```
R, F, R, F   R, F, R, F   S, F, S, T   S, T, S, T   S, T, R, F
R, F, R, T   S, T, S, T   S, T, S, T   S, T, R, F   R, F, S, T
```

1. What is $\widehat{P}(W_2 = \texttt{R})$, the probability that sampling assigns to the event $W_2 = \texttt{R}$?

2. Cross off samples above which are rejected by rejection sampling if we're computing $P(W_2|I_1 = \texttt{T}, I_2 = \texttt{F})$.

   Rejection sampling seems to be wasting a lot of effort, so we decide to switch to likelihood weighting. Assume we generate the following six samples given the evidence $I_1 = \texttt{T}$ and $I_2 = \texttt{F}$:

$$(W_1, I_1, W_2, I_2) = \Big\{(\texttt{S},\texttt{T},\texttt{R},\texttt{F}), (\texttt{R},\texttt{T},\texttt{R},\texttt{F}), (\texttt{S},\texttt{T},\texttt{R},\texttt{F}), (\texttt{S},\texttt{T},\texttt{S},\texttt{F}), (\texttt{S},\texttt{T},\texttt{S},\texttt{F}), (\texttt{R},\texttt{T},\texttt{S},\texttt{F})\Big\}$$

3. What is the weight of the first sample (`S`, `T`, `R`, `F`) above?

4. Use likelihood weighting to estimate $P(W_2|I_1 = \texttt{T}, I_2 = \texttt{F})$.