

Q1. VPI

You are the latest contestant on Monty Hall's game show, which has undergone a few changes over the years. In the game, there are n closed doors: behind one door is a car ($U(car) = 1000$), while the other $n - 1$ doors each have a goat behind them ($U(goat) = 10$). You are permitted to open exactly one door and claim the prize behind it.

You begin by choosing a door uniformly at random.

(a) What is your expected utility?

(b) After you choose a door but before you open it, Monty offers to open k other doors, each of which are guaranteed to have a goat behind it. If you accept this offer, should you keep your original choice of a door, or switch to a new door? $EU(keep)$:

$EU(switch)$:

Action that achieves MEU :

(c) What is the value of the information that Monty is offering you?

(d) Monty is changing his offer!

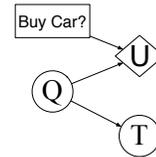
After you choose your initial door, you are given the offer to choose any other door and open this second door. If you do, after you see what is inside the other door, you may switch your initial choice (to the newly opened door) or keep your initial choice.

What is the value of this new offer?

- (e) Monty is generalizing his offer: you can pay $\$d^3$ to open d doors as in the previous part. (Assume that $U(\$x) = x$.) You may now switch your choice to any of the open doors (or keep your initial choice). What is the largest value of d for which it would be rational to accept the offer?

2 Decision Networks and VPI

A buyer is deciding whether to buy a certain used car. The car may be good quality ($Q = +q$) or bad quality ($Q = -q$). A test (T) costs \$50 and can help to figure out the quality of the car. There are only two outcomes for the test: $T = \text{pass}$ or $T = \text{fail}$. The car costs \$1,500, and its market value is \$2,000 if it is good quality; if not, \$700 in repairs will be needed to make it good quality. The buyer's estimate is that the car has 70% chance of being good quality.



- (a) Calculate the expected net gain from buying the car, given no test.
- (b) Tests can be described by the probability that the car will pass or fail the test given that the car is good or bad quality. We know: $P(T = \text{pass}|Q = +q) = 0.9$ and $P(T = \text{pass}|Q = -q) = 0.2$
 Calculate the probability that the car will pass (or fail) its test, and then the probability that it is good (or bad) quality given each possible test outcome.
- (c) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.
- (d) Calculate the value of (perfect) information of the test. Should the buyer pay for a test?