CS 188 Spring 2023 — Introduction to Artificial Intelligence — Exam Prep 3 Solutions

Q1. Logic

(a) Prove, or find a counterexample to, each of the following assertions:

(i) If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$

True. This follows from monotonicity.

(ii) If $(\alpha \wedge \beta) \models \gamma$ then $\alpha \models \gamma$ or $\beta \models \gamma$ (or both).

False. Consider Consider $\alpha \equiv A$, $\beta \equiv B$, $\gamma \equiv (A \wedge B)$.

(iii) If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

False. Consider $\beta \equiv A$, $\gamma \equiv \neg A$.

(b) Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(i) $Smoke \implies Smoke$

Valid

(ii) $Smoke \implies Fire$

Neither

(iii) $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

Neither

(iv) $Smoke \vee Fire \vee \neg Fire$

Valid

(v) $((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$

Valid

(vi) $(Smoke \implies Fire) \implies ((Smoke \wedge Heat) \implies Fire)$

Valid

(vii) $Big \vee Dumb \vee (Big \implies Dumb)$

Valid

(c) Suppose an agent inhabits a world with two states, S and $\neg S$, and can do exactly one of two actions, a and b. Action a does nothing and action b flips from one state to the other. Let S^t be the proposition that the agent is in state S at time t, and let a^t be the proposition that the agent does action a at time t (similarly for b^t).

(i) Write a successor-state axiom for S^{t+1}.

$S^{t+1} \iff [(S^t \wedge a^t) \vee (\neg S^t \wedge b^t)]$.

(ii) Convert the sentence in the previous part into CNF.

Because the agent can do exactly one action, we know that $b^t \equiv \neg a^t$ so we replace b^t throughout. We obtain four clauses:

1: $(\neg S^{t+1} \vee S^t \vee \neg a^t)$
2: $(\neg S^{t+1} \vee \neg S^t \vee a^t)$
3: $(S^{t+1} \vee \neg S^t \vee \neg a^t)$
4: $(S^{t+1} \vee S^t \vee a^t)$

Q2. First Order Logic

Consider a vocabulary with the following symbols:

- $Occuption(p, o)$: Predicate. Person p has occuption o.
- $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(iii) Emily is either a surgeon or a lawyer.
$O(E, S) \lor O(E, L)$

(iv) Joe is an actor, but he also holds another job.
$O(J, A) \land \exists p\ p \neq A \land O(J, p)$

(v) All surgeons are doctors.
$\forall p\ O(p, S) \Rightarrow O(p, D)$

(vi) Joe does not have a lawyer (i.e., is not a customer of any lawyer).
$\neg \exists p\ C(J, p) \land O(p, L)$

(vii) Emily has a boss who is a lawyer.
$\exists p\ B(p, E) \land O(p, L)$

(viii) There exists a lawyer all of whose customers are doctors.
$\exists\ pO(p, L) \land \forall\ qC(q, p) \Rightarrow O(q, D)$

(ix) Every surgeon has a lawyer.
$\forall p\ O(p, S) \Rightarrow \exists\ qO(q, L) \land C(p, q)$

Q3. [Optional] Local Search

(a) Hill Climbing

(i) Hill-climbing is complete. ☐ True ■ False

Consider hill-climbing for 8-queen.

(ii) Hill-climbing is optimal. ☐ True ■ False

no completeness indicates no optimality.

(b) Simulated Annealing

(i) The higher the temperature T is, the more likely the randomly chosen state will be expanded. ■ True ☐ False

The higher T is, the larger $e^{\Delta E/T}$ is given ΔE is negative.

(ii) In one round of simulated annealing, the temperature is 2 and the current state S has energy 1. It has 3 successors: A with energy 2; B with energy 1; C with energy 1-ln 4. If we assume the temperature does not change, What's the probability that these states will be chosen to expand after S eventually?

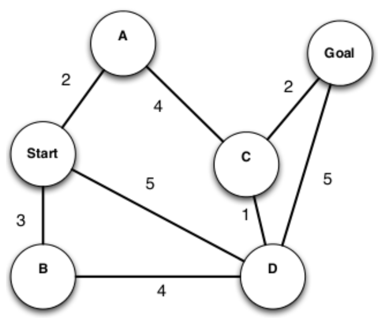
A, B will be expanded with probability $\frac{2}{5}$, C will be expanded with probability $\frac{1}{5}$.

Proof. First, the problem is asking which node will be expanded **next**, not **in this round**. A, B and C are randomly selected for expansion. If A or B is selected, they will surely be expanded. If C is selected, it has probability of 1/2 to be expanded and 1/2 to restart the random selection. Thus the probability ratio between A, B and C is 2:2:1.

(iii) On a undirected graph, If T decreases slowly enough, simulated annealing is guaranteed to converge to the optimal state. ■ True ☐ False

(c) Local Beam Search

The following state graph is being explored with 2-beam graph search. A state's score is its accumulated distance to the start state and lower scores are considered better. Which of the following statements are true?



■ States A and B will be expanded before C and D.

☐ States A and D will be expanded before B and C.

☐ States B and D will be expanded before A and C.

☐ None of above.

(d) Genetic Algorithm

(i) In genetic algorithm, cross-over combine the genetic information of two parents to generate new offspring.

■ True ☐ False

(ii) In genetic algorithm, mutation involves a probability that some arbitrary bits in a genetic sequence will be flipped from its original state.

■ True ☐ False

(e) Gradient Descent

(i) Gradient descent is optimal. ☐ True ■ False

False. Gradient descent can become trapped in a local minimum.

(ii) For a function $f(x)$ with derivative $f'(x)$, write down the gradient descent update to go from x_t to x_{t+1}. Learning rate is α.

$x_{t+1} = x_t - \alpha f'(x_t)$, where α is the learning rate.