Q1. Logic

(a) Prove, or find a counterexample to, each of the following assertions:

(i) If \( \alpha \vdash \gamma \) or \( \beta \vdash \gamma \) (or both) then \( (\alpha \land \beta) \vdash \gamma \)
   
   True. This follows from monotonicity.

(ii) If \( (\alpha \land \beta) \vdash \gamma \) then \( \alpha \vdash \gamma \) or \( \beta \vdash \gamma \) (or both).
   
   False. Consider \( \alpha \equiv A \), \( \beta \equiv B \), \( \gamma \equiv (A \land B) \).

(iii) If \( \alpha \vdash (\beta \lor \gamma) \) then \( \alpha \vdash \beta \) or \( \alpha \vdash \gamma \) (or both).
   
   False. Consider \( \beta \equiv A \), \( \gamma \equiv \neg A \).

(b) Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(i) \( Smoke \implies Smoke \)
   
   Valid

(ii) \( Smoke \implies Fire \)
   
   Neither

(iii) \( (Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire) \)
   
   Neither

(iv) \( Smoke \lor Fire \lor \neg Fire \)

   Valid

(v) \( ((Smoke \land Heat) \implies Fire) \iff ((Smoke \implies Fire) \lor (Heat \implies Fire)) \)
   
   Valid

(vi) \( (Smoke \implies Fire) \implies ((Smoke \land Heat) \implies Fire) \)
   
   Valid

(vii) \( Big \lor Dumb \lor (Big \implies Dumb) \)

   Valid

(c) Suppose an agent inhabits a world with two states, \( S \) and \( \neg S \), and can do exactly one of two actions, \( a \) and \( b \). Action \( a \) does nothing and action \( b \) flips from one state to the other. Let \( S^t \) be the proposition that the agent is in state \( S \) at time \( t \), and let \( a' \) be the proposition that the agent does action \( a \) at time \( t \) (similarly for \( b' \)).

(i) Write a successor-state axiom for \( S^{t+1} \).
   
   \( S^{t+1} \iff [(S^t \land a') \lor (\neg S^t \land b')] \).

(ii) Convert the sentence in the previous part into CNF.
   
   Because the agent can do exactly one action, we know that \( b' \equiv \neg a' \) so we replace \( b' \) throughout. We obtain four clauses:
   
   1: \( (\neg S^{t+1} \lor S^t \lor \neg a') \)
   
   2: \( (\neg S^{t+1} \lor \neg S^t \lor a') \)
   
   3: \( (S^{t+1} \lor \neg S^t \lor \neg a') \)
   
   4: \( (S^{t+1} \lor S^t \lor a') \)
Q2. First Order Logic

Consider a vocabulary with the following symbols:

- \textit{Occupation}(p, o): Predicate. Person \(p\) has occupation \(o\).
- \textit{Customer}(p1, p2): Predicate. Person \(p1\) is a customer of person \(p2\).
- \textit{Boss}(p1, p2): Predicate. Person \(p1\) is a boss of person \(p2\).
- \textit{Doctor, Surgeon, Lawyer, Actor}: Constants denoting occupations.
- \textit{Emily, Joe}: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(iii) Emily is either a surgeon or a lawyer.
\[ O(E, S) \lor O(E, L) \]

(iv) Joe is an actor, but he also holds another job.
\[ O(J, A) \land \exists p \ p \neq A \land O(J, p) \]

(v) All surgeons are doctors.
\[ \forall p \ O(p, S) \Rightarrow O(p, D) \]

(vi) Joe does not have a lawyer (i.e., is not a customer of any lawyer).
\[ \neg \exists p \ C(J, p) \land O(p, L) \]

(vii) Emily has a boss who is a lawyer.
\[ \exists p \ B(p, E) \land O(p, L) \]

(viii) There exists a lawyer all of whose customers are doctors.
\[ \exists p \ O(p, L) \land \forall q C(q, p) \Rightarrow O(q, D) \]

(ix) Every surgeon has a lawyer.
\[ \forall p \ O(p, S) \Rightarrow \exists q O(q, L) \land C(p, q) \]
Q3. [Optional] Local Search

(a) Hill Climbing

(i) Hill-climbing is complete. □ True ■ False
Consider hill-climbing for 8-queen.

(ii) Hill-climbing is optimal. □ True ■ False
no completeness indicates no optimality.

(b) Simulated Annealing

(i) The higher the temperature T is, the more likely the randomly chosen state will be expanded. ■ True □ False
The higher T is, the larger $e^{\Delta E/T}$ is given $\Delta E$ is negative.

(ii) In one round of simulated annealing, the temperature is 2 and the current state S has energy 1. It has 3 successors:
A with energy 2; B with energy 1; C with energy 1-ln 4. If we assume the temperature does not change, What’s the probability that these states will be chosen to expand after S eventually?
A, B will be expanded with probability $\frac{2}{5}$. C will be expanded with probability $\frac{1}{5}$.
Proof. First, the problem is asking which node will be expanded next, not in this round. A, B and C are randomly selected for expansion. If A or B is selected, they will surely be expanded. If C is selected, it has probability of 1/2 to be expanded and 1/2 to restart the random selection. Thus the probability ratio between A, B and C is 2:2:1.

(iii) On a undirected graph, If T decreases slowly enough, simulated annealing is guaranteed to converge to the optimal state. ■ True □ False

(c) Local Beam Search

The following state graph is being explored with 2-beam graph search. A state’s score is its accumulated distance to the start state and lower scores are considered better. Which of the following statements are true?

- States A and B will be expanded before C and D.
- States A and D will be expanded before B and C.
- States B and D will be expanded before A and C.
- None of above.

(d) Genetic Algorithm

(i) In genetic algorithm, cross-over combine the genetic information of two parents to generate new offspring. ■ True □ False

(ii) In genetic algorithm, mutation involves a probability that some arbitrary bits in a genetic sequence will be flipped from its original state. ■ True □ False

(e) Gradient Descent

(i) Gradient descent is optimal. □ True ■ False
False. Gradient descent can become trapped in a local minimum.
(ii) For a function $f(x)$ with derivative $f'(x)$, write down the gradient descent update to go from $x_t$ to $x_{t+1}$. Learning rate is $\alpha$.

$$x_{t+1} = x_t - \alpha f'(x_t),$$
where $\alpha$ is the learning rate.