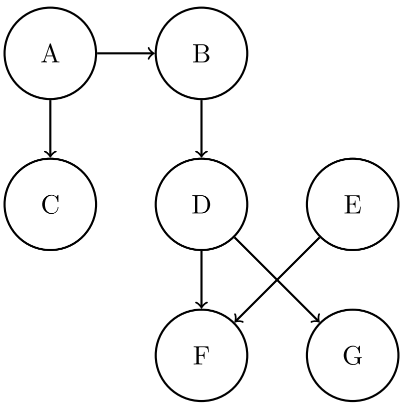
Regular Discussion 9

1 Bayes' Nets: Representation and Independence

Parts (a) and (b) pertain to the following Bayes' Net.



(a) Express the joint probability distribution as a product of terms representing individual conditional probabilities tables associated with the Bayes Net.

(b) Assume each node can take on 4 values. How many entries do the factors at A, D, and F have?

A: ________________________________

D: ________________________________

F: ________________________________

Consider the following probability distribution tables. The joint distribution $P(A,B,C,D)$ is equal to the product of these probability distribution tables.

A	$P(A)$
+a	0.8
−a	0.2

A	B	$P(B \mid A)$
+a	+b	0.9
+a	−b	0.1
−a	+b	0.6
−a	−b	0.4

B	C	$P(C \mid B)$
+b	+c	0.8
+b	−c	0.2
−b	+c	0.8
−b	−c	0.2

C	D	$P(D \mid C)$
+c	+d	0.25
+c	−d	0.75
−c	+d	0.5
−c	−d	0.5

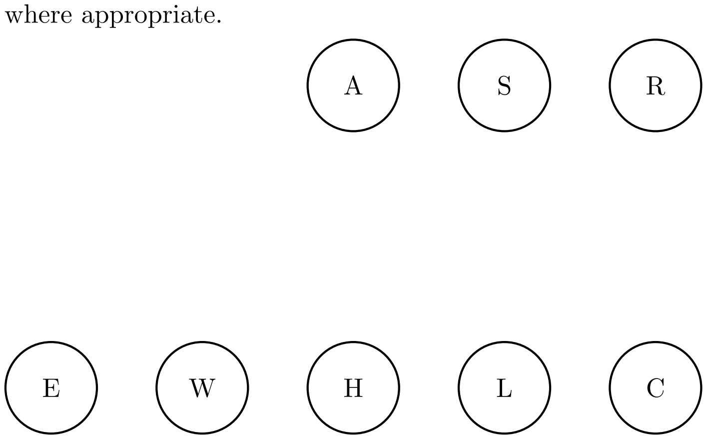
(c) State all non-conditional independence assumptions that are implied by the probability distribution tables.

You are building advanced safety features for cars that can warn a driver if they are falling asleep (A) and also calculate the probability of a crash (C) in real time. You have at your disposal 6 sensors (random variables):

- E: whether the driver's eyes are open or closed
- W: whether the steering wheel is being touched or not
- L: whether the car is in the lane or not
- S: whether the car is speeding or not
- H: whether the driver's heart rate is somewhat elevated or resting
- R: whether the car radar detects a close object or not

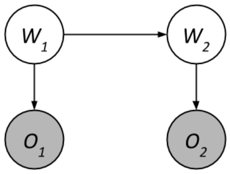
A influences $\{E, W, H, L, C\}$. C is influenced by $\{A, S, L, R\}$.

(d) Draw the Bayes Net associated with the description above by adding edges between the provided nodes where appropriate.



2 HMMs

Consider the following Hidden Markov Model. O_1 and O_2 are supposed to be shaded.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}\|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

Suppose that we observe $O_1 = a$ and $O_2 = b$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = a, O_2 = b)$ one step at a time.

(a) Compute $P(W_1, O_1 = a)$.

(b) Using the previous calculation, compute $P(W_2, O_1 = a)$.

(c) Using the previous calculation, compute $P(W_2, O_1 = a, O_2 = b)$.

(d) Finally, compute $P(W_2|O_1 = a, O_2 = b)$.