CS 188 Spring 2023 Regular Discussion 11 Solutions

1 Utilities

1. Consider a utility function of $U(x) = 2x$. What is the utility for each of the following outcomes?

   (a) 3
   U(3) = 2(3) = 6

   (b) L($\frac{2}{3}$, 3; $\frac{1}{3}$, 6)
   U(L($\frac{2}{3}$, 3; $\frac{1}{3}$, 6)) = $\frac{2}{3}$U(3) + $\frac{1}{3}$U(6) = 8

   (c) -2
   U(-2) = 2(-2) = -4

   (d) L(0.5, 2; 0.5, L(0.5, 4; 0.5, 6)) U(L(0.5, 2; 0.5, L(0.5, 4; 0.5, 6))) = 0.5U(2) + 0.5(0.5U(4) + 0.5U(6)) = 2 + 0.5(4 + 6) = 7

2. Consider a utility function of $U(x) = x^2$. What is the utility for each of the following outcomes?

   (a) 3
   U(3) = 3^2 = 9

   (b) L($\frac{2}{3}$, 3; $\frac{1}{3}$, 6)
   U(L($\frac{2}{3}$, 3; $\frac{1}{3}$, 6)) = $\frac{2}{3}$U(3) + $\frac{1}{3}$U(6) = 6 + 12 = 18

   (c) -2
   U(-2) = $(-2)^2$ = 4

   (d) L(0.5, 2; 0.5, L(0.5, 4; 0.5, 6))
   U(L(0.5, 2; 0.5, L(0.5, 4; 0.5, 6))) = 0.5U(2) + 0.5(0.5U(4) + 0.5U(6)) = 2 + 0.5(8 + 18) = 15

3. What is the expected monetary value (EMV) of the lottery $L(\frac{2}{3}, \$3; \frac{1}{3}, \$6)$?
   $\frac{2}{3} \cdot \$3 + \frac{1}{3} \cdot \$6 = \$4$

4. For each of the following types of utility function, state how the utility of the lottery $U(L)$ compares to the utility of the amount of money equal to the EMV of the lottery, $U(EMV(L))$. Write $<, >, =$, or ? for can't tell.

   (a) U is an arbitrary function.
   $U(L)$? $U(EMV(L))$

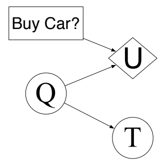
   (b) U is monotonically increasing and its rate of increase is increasing (its second derivative is positive).
   $U(L) > U(EMV(L))$.
   As an example, consider $U = x^2$ from Q2. Then $U(L) = 18$ and $U(EMV(L)) = 4^2 = 16$.

   (c) U is monotonically increasing and linear (its second derivative is zero).
   $U(L) = U(EMV(L))$

   (d) U is monotonically increasing and its rate of increase is decreasing (its second derivative is negative).
   $U(L) < U(EMV(L))$.
   Consider $U = \sqrt{x}$. Then $U(L) = \frac{2}{3} \cdot \sqrt{3} + \frac{1}{3} \cdot \sqrt{6} \approx 1.97$, and $U(EMV(L)) = \sqrt{4} = 2$.

2 Decision Networks and VPI

A buyer is deciding whether to buy a certain used car. The car may be good quality ($Q = q$) or bad quality ($Q = q$). A test (T) costs \$50 and can help to figure out the quality of the car. There are only two outcomes for the test: T = pass or T = fail. The car costs \$1,500, and its market value is \$2,000 if it is good quality; if not, \$700 in repairs will be needed to make it good quality. The buyer's estimate is that the car has 70% chance of being good quality.



1. Calculate the expected net gain from buying the car, given no test.

$$\begin{aligned} EU(\text{buy}) &= P(Q = +q) \cdot U(+q, \text{buy}) + P(Q = \neg q) \cdot U(\neg q, buy) \\ &= .7 \cdot 500 + 0.3 \cdot -200 = 290 \end{aligned}$$

2. Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:

$$P(T = \text{pass}|Q = +q) = 0.9$$
$$P(T = \text{pass}|Q = \neg q) = 0.2$$

Calculate the probability that the car will pass (or fail) its test, and then the probability that it is in good (or bad) shape given each possible test outcome.

$$\begin{aligned} P(T = \text{pass}) &= \sum_q P(T = \text{pass}, Q = q) \\ &= P(T = \text{pass}|Q = +q)P(Q = +q) + P(T = \text{pass}|Q = \neg q)P(Q = \neg q) \\ &= 0.69 \\ P(T = fail) &= 0.31 \\ P(Q = +q|T = \text{pass}) &= \frac{P(T = \text{pass}|Q = +q)P(Q = +q)}{P(T = \text{pass})} \\ &= \frac{0.9 \cdot 0.7}{0.69} = \frac{21}{23} \approx 0.91 \\ P(Q = +q|T = \text{fail}) &= \frac{P(T = \text{fail}|Q = +q)P(Q = +q)}{P(T = \text{fail})} \\ &= \frac{0.1 \cdot 0.7}{0.31} = \frac{7}{31} \approx 0.22 \end{aligned}$$

3. Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

$$\begin{aligned} EU(\text{buy}|T = pass) &= P(Q = +q|T = \text{pass})U(+q, \text{buy}) + P(Q = \neg q|T = \text{pass})U(\neg q, \text{buy}) \\ &\approx 0.91 \cdot 500 + 0.09 \cdot (-200) \approx 437 \\ EU(\text{buy}|T = \text{fail}) &= P(Q = +q|T = \text{fail})U(+q, \text{buy}) + P(Q = \neg q|T = \text{fail})U(\neg q, \text{buy}) \\ &\approx 0.22 \cdot 500 + 0.78 \cdot (-200) = -46 \\ EU(\neg\text{buy}|T = pass) &= 0 \\ EU(\neg\text{buy}|T = fail) &= 0 \end{aligned}$$

Therefore: $MEU(T = \text{pass}) = 437$ (with buy) and $MEU(T = \text{fail}) = 0$ (using $\neg$buy)

4. Calculate the value of (perfect) information of the test. Should the buyer pay for a test?

$$VPI(T) = (\sum_t P(T=t)MEU(T=t)) - MEU(\phi)$$
$$= 0.69 \cdot 437 + 0.31 \cdot 0 - 290 \approx 11.53$$

You shouldn't pay for it, since the cost is $50.