## Q1. Linear Separability

(a) For each of the datasets represented by the graphs below, please select the feature maps for which the perceptron algorithm can perfectly classify the data.
Each data point is in the form $\left(x_{1}, x_{2}\right)$, and has some label $Y$, which is either a 1 (dot) or -1 (cross).


## Q2. Deep Learning

(a) Perform forward propagation on the neural network below for $x=1$ by filling in the values in the table. Note that (i), ..., (vii) are outputs after performing the appropriate operation as indicated in the node.

| (i) | (ii) | (iii) | (iv) | (v) | (vi) | (vii) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |


(b) [Optional] Below is a neural network with weights $a, b, c, d, e, f$. The inputs are $x_{1}$ and $x_{2}$. The first hidden layer computes $r_{1}=\max \left(c \cdot x_{1}+e \cdot x_{2}, 0\right)$ and $r_{2}=\max \left(d \cdot x_{1}+f \cdot x_{2}, 0\right)$.
The second hidden layer computes $s_{1}=\frac{1}{1+\exp \left(-a \cdot r_{1}\right)}$ and $s_{2}=\frac{1}{1+\exp \left(-b \cdot r_{2}\right)}$.
The output layer computes $y=s_{1}+s_{2}$. Note that the weights $a, b, c, d, e, f$ are indicated along the edges of the neural network here.

Suppose the network has inputs $x_{1}=1, x_{2}=-1$.
The weight values are $a=1, b=1, c=4, d=1, e=2, f=2$.
Forward propagation then computes $r_{1}=2, r_{2}=0, s_{1}=0.9, s_{2}=0.5, y=1.4$. Note: some values are rounded.


Using the values computed from forward propagation, use backpropagation to numerically calculate the following partial derivatives. Write your answers as a single number (not an expression). You do not need a calculator. Use scratch paper if needed.
Hint: For $g(z)=\frac{1}{1+\exp (-z)}$, the derivative is $\frac{\partial g}{\partial z}=g(z)(1-g(z))$.

| $\frac{\partial y}{\partial a}$ | $\frac{\partial y}{\partial b}$ | $\frac{\partial y}{\partial c}$ | $\frac{\partial y}{\partial d}$ | $\frac{\partial y}{\partial e}$ | $\frac{\partial y}{\partial f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |

(c) Below are two plots with horizontal axis $x_{1}$ and vertical axis $x_{2}$ containing data labelled $\times$ and $\bullet$. For each plot, we wish to find a function $f\left(x_{1}, x_{2}\right)$ such that $f\left(x_{1}, x_{2}\right) \geq 0$ for all data labelled $\times$ and $f\left(x_{1}, x_{2}\right)<0$ for all data labelled $\bullet$.
Below each plot is the function $f\left(x_{1}, x_{2}\right)$ for that specific plot. Complete the expressions such that all the data is labelled correctly. If not possible, mark "No valid combination".

$f\left(x_{1}, x_{2}\right)=\max (\underline{(i)}+(\underline{\text { (ii })},(\underline{\text { (iii) }}+\underline{(\mathbf{i v})})+(\underline{(v)}$


No valid combination
$f\left(x_{1}, x_{2}\right)=(\underline{(\mathbf{i})}-\max (\underline{(\mathbf{v i i})}+(\underline{(\mathbf{v i i})}, \underline{(\mathbf{i x})}+\underline{(\mathbf{x})})$


No valid combination

