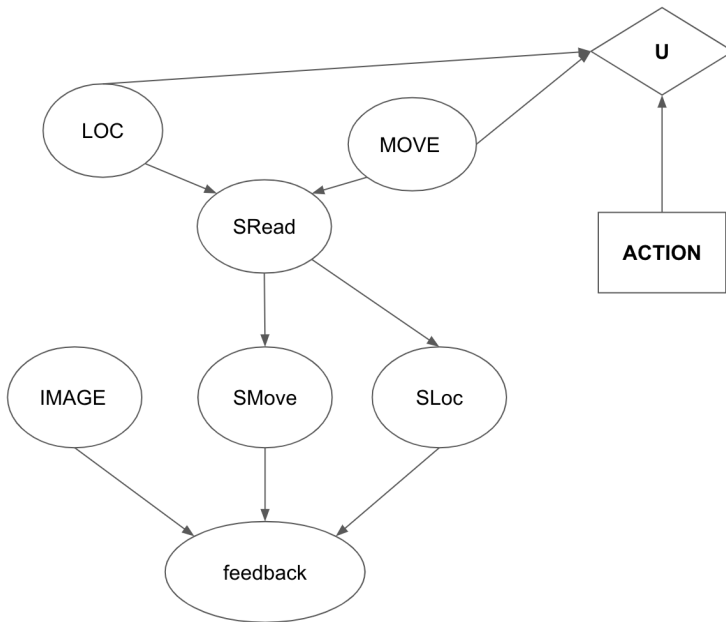


Q1. Vehicle Perception Indication

A vehicle is trying to identify the situation of the world around it using a set of sensors located around the vehicle.

Each sensor reading (SRead) is based off of an object's location (LOC) and an object's movement (MOVE). The sensor reading will then produce various values for its predicted location (SLoc) and predicted movement (SMove). The user will receive these readings, as well as the the image (IMAGE) as feedback.

- (a) The vehicle takes an action, and we assign some utility to the action based on the object's location and movement. Possible actions are MOVE TOWARDS, MOVE AWAY, and STOP. Suppose the decision network faced by the vehicle is the following.



- (i) Based on the diagram above, which of the following **could possibly be true**?

- VPI (Image) = 0
- VPI (SRead) < 0
- VPI (SMove, SRead) > VPI (SRead)
- VPI (Feedback) = 0
- None of the above

VPI(Image) = 0 because there is not active path connecting Image and U

VPI cannot be negative, so option 2 is not selected.

$VPI(SMove, SRead) = VPI(SMove | SRead) + VPI(SRead)$, therefore we can cancel $VPI(SRead)$ from both side, and it becomes asking if $VPI(SMove | SRead) > 0$. And we can see that cannot be true, because shading in SRead, there is no active path connecting SMove and U.

There is an active path connecting Feedback and U, therefore $VPI(\text{Feedback}) \geq 0$. It could still be 0 because active path only gives the possibility of > 0 .

(ii) Based on the diagram above, which of the following **must necessarily be true**?

- $VPI(\text{Image}) = 0$
- $VPI(\text{SRead}) = 0$
- $VPI(\text{SMove, SRead}) = VPI(\text{SRead})$
- $VPI(\text{Feedback}) = 0$
- None of the above

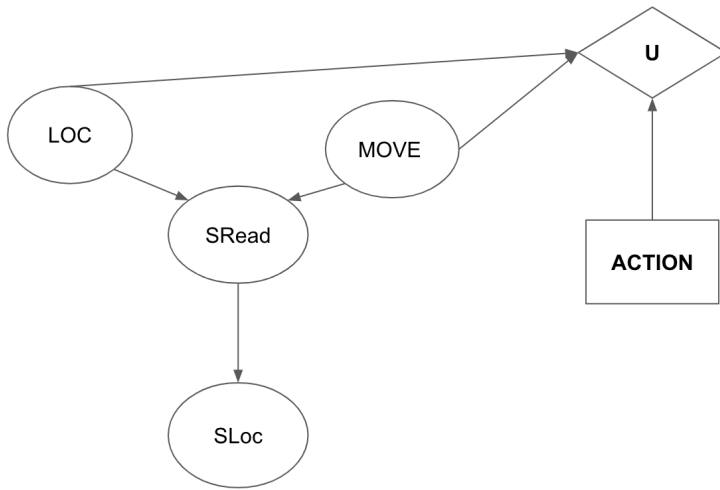
$VPI(\text{Image}) = 0$ because there is not active path connecting Image and U

$VPI(\text{SRead})$ could be > 0 because SRead-MOVE-U is an active path between SRead and U

$VPI(\text{SMove, SRead}) = VPI(\text{SMove} \mid \text{SRead}) + VPI(\text{SRead})$, therefore we can cancel $VPI(\text{SRead})$ from both side, and it becomes asking if $VPI(\text{SMove} \mid \text{SRead}) = 0$. And we can see that must true, because shading in SRead, there is no active path connecting SMove and U.

$VPI(\text{Feedback})$ could be > 0 because $\text{feedback-SLoc-SRead-MOVE-U}$ is an active path

Let's assume that your startup has less money, so we use a simpler sensor network. One possible sensor network can be represented as follows.



You have distributions of $P(\text{LOC})$, $P(\text{MOVE})$, $P(\text{SRead}|\text{LOC}, \text{MOVE})$, $P(\text{SLoc}|\text{SRead})$ and utility values $U(a, l, m)$.

(b) Complete the equation for determining the expected utility for some ACTION a .

$$EU(a) = \left(\text{(i)} \quad \text{(ii)} \quad \text{(iii)} \right) U(a, l, m)$$

- (i) $\sum_l P(l)$ $\sum_{sloc} P(sloc|l)$ $\sum_l \sum_{sloc} P(sloc|l)$ 1
- (ii) $\sum_m P(m)$ $\sum_m P(sloc|m)$ $\sum_l \sum_m \sum_{sloc} P(sloc|l)P(sloc|m)$ 1
- (iii) $* \sum_l \sum_m \sum_{sloc} P(sloc|l)P(sloc|m)$ $+ \sum_l \sum_m \sum_{sloc} P(sloc|l)P(sloc|m)$
- $+ \sum_l \sum_m \sum_{sloc} P(sloc|l, m)P(l)P(m)$ $*1$

$$EU(a) = \sum_l P(l) \sum_m P(m) U(a, l, m)$$

We can eliminate SRead and SLoc via marginalization, so they don't need to be included the expression

(c) Your colleague Bob invented a new sensor to observe values of $SLoc$.

(i) Suppose that your company had no sensors till this point. Which of the following expression is equivalent to $VPI(SLoc)$?

- $VPI(SLoc) = (\sum_{sloc} P(sloc) MEU(SLoc = sloc)) - \max_a EU(a)$
- $VPI(SLoc) = MEU(SLoc) - MEU(\emptyset)$
- $VPI(SLoc) = \max_{sloc} MEU(SLoc = sloc) - MEU(\emptyset)$
- None of the above

Option 2 is correct by definition, and option 1 is the expanded version of option 2

(ii) Gaagle, an established company, wants to sell your startup a device that gives you $SRead$. Given that you already have Bob's device (that gives you $SLoc$), what is the maximum amount of money you should pay for Gaagle's device? Suppose you value \$1 at 1 utility.

- $VPI(SRead)$
- $VPI(SRead) - VPI(SLoc)$
- $VPI(SRead, SLoc)$
- $VPI(SRead, SLoc) - VPI(SLoc)$
- None of the above

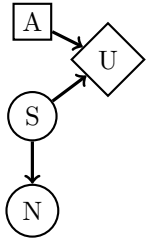
Choice 4 is correct by definition

Choice 2 is true because $VPI(SLoc | SRead) = 0$, and thus $VPI(SRead) = VPI(SRead) + 0 = VPI(SRead) + VPI(SLoc | SRead) = VPI(SRead, SLoc)$, which makes choice 2 the same as choice 4

Q2. [Optional] Dressed to Impress

- (a) Alice is invited to a party tonight which is said to be once-in-a-lifetime. However, this mysterious party doesn't publicize who is going and thus Alice has no idea whether the size S of the party will be large ($S = +s$) or tiny ($S = -s$). The size can affect the noise N outside the party, and it will either be noisy ($N = +n$) or quiet ($N = -n$). Alice has three dresses: blue, red and yellow. Each dress will have a different utility for her depending on the size of the party. Let's help her decide which will be best!

We have the following decision network, where circles are chance nodes, squares are decision nodes, and diamonds are utility nodes:



S	$P(S)$	S	N	$P(N S)$	S	A	U
+s	0.5	+s	+n	0.7	+s	blue	80
-s	0.5	+s	-n	0.3	-s	blue	60
		-s	+n	0.1	+s	red	40
		-s	-n	0.9	-s	red	100
					+s	yellow	60
					-s	yellow	40

- (i) What is the expected utility of wearing each dress, with both S and N unobserved?

- $EU(A=\text{blue}) = \underline{80 * 0.5 + 60 * 0.5 = 70}$
- $EU(A=\text{red}) = \underline{40 * 0.5 + 100 * 0.5 = 70}$
- $EU(A=\text{yellow}) = \underline{60 * 0.5 + 40 * 0.5 = 50}$

What is Alice's maximum expected utility?

- $MEU(\{\}) = \underline{70}$

- (ii) Suppose Alice observes that the party is quiet, $N = -n$. Compute the following conditional probabilities with this observation:

- $P(+s | -n) = \underline{\frac{P(-n|+s)P(+s)}{P(-n|+s)P(+s)+P(-n|-s)P(-s)} = \frac{0.3 \cdot 0.5}{0.3 \cdot 0.5 + 0.9 \cdot 0.5} = 0.25}$
- $P(-s | -n) = \underline{P(-s | -n) = 1 - P(+s | -n) = 0.75}$

What is the expected utility of wearing each dress?

- $EU(A=\text{blue} | N = -n) = \underline{80 * 0.25 + 60 * 0.75 = \frac{145}{2} = 65}$
- $EU(A=\text{red} | N = -n) = \underline{40 * 0.25 + 100 * 0.75 = \frac{155}{2} = 85}$
- $EU(A=\text{yellow} | N = -n) = \underline{60 * 0.25 + 40 * 0.75 = 45}$

What is Alice's maximum expected utility given that $N = -n$?

- $MEU(\{N=-n\}) = \underline{85}$

- (iii) Construct a formula for $VPI(N)$ for the given network. To decouple this problem from your work above, use any of the symbolic terms from the following list (rather than plugging in numeric values):

$P(+n | +s)$, $P(+n | -s)$, $P(-n | +s)$, $P(-n | -s)$, $P(+n)$, $P(-n)$, $P(+s)$, $P(-s)$,
 $MEU(\{\})$, $MEU(\{N = +n\})$, $MEU(\{N = -n\})$

- $VPI(N) = \underline{P(+n)MEU(\{N = +n\}) + P(-n)MEU(\{N = -n\}) - MEU(\{\})}$