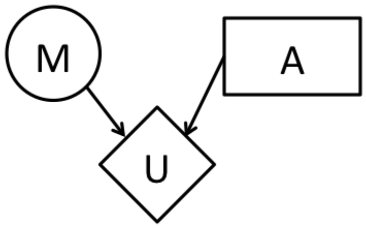
CS 188
Spring 2023

Introduction to
Artificial Intelligence

HW8 Part 2 Solutions

SP23 HW 8 Part 2 Solutions. [27 pts]

Q1) (4 pts) Assume that you want to watch a film M that can either be great $+m$ or pretty bad $-m$. You can either watch the film in a theater or at home by renting it. This is controlled by your choice A. Consider the following decision network and tables:



M	P(M)
+m	0.5
-m	0.5

M	A	U(M,A)
+m	*theater*	100
-m	*theater*	10
+m	*rent*	80
-m	*rent*	40

Figure 1: Decision network and tables.

Compute the following quantities. EU and MEU stand for expected and maximum expected utility respectively.

Q1.1) $EU(theater) = P(+m)U(+m, theater) + P(-m)U(-m, theater) = 0.5 \cdot 100 + 0.5 \cdot 10 = 55$

Q1.2) $EU(rent) = P(+m)U(+m, rent) + P(-m)U(-m, rent) = 0.5 \cdot 80 + 0.5 \cdot 40 = 60$

Q1.3) $MEU(\emptyset) = 60$

Q1.4) $\text{argmax}_A EU(A) = rent$

Q2) (7 pts) You would like obtain more information about whether the film is good or not. For that, we introduce another variable F which designates the "fullness" (how sold-out the tickets are) in the theaters. This variable is affected by another variable S which designates possible Covid-19 restrictions. The prior of M and the utilities are the same as before. Assuming that both F and S are binary, consider the following network and tables:

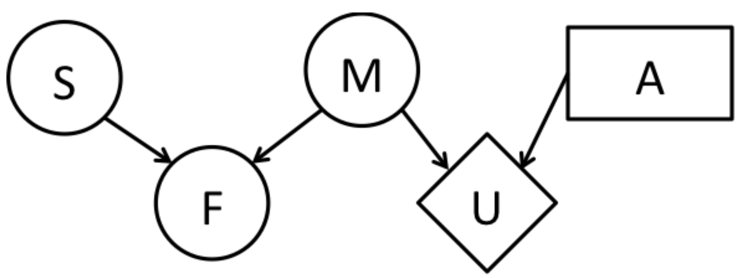


Figure 2: Decision network.

S	M	F	$P(F\|S,M)$
+s	+m	+f	0.6
+s	+m	-f	0.4
+s	-m	+f	0.0
+s	-m	-f	1.0

S	M	F	$P(F\|S,M)$
-s	+m	+f	1.0
-s	+m	-f	0.0
-s	-m	+f	0.3
-s	-m	-f	0.7

S	$P(S)$
+s	0.2
-s	0.8

Figure 3: Tables.

We want to figure out the value of revealing the Covid-19 restrictions S. Compute the values of the following quantities.

Q2.1) $EU(theater| + s) = 55$

The shortage variable is independent of the parents of the utility node when no additional evidence is present; thus, the same values hold: $EU(theater|+s) = EU(theater) = 55$

Q2.2) $EU(rent|+s) = EU(rent) = 60$

Q2.3) $MEU(\{+s\}) = 60$

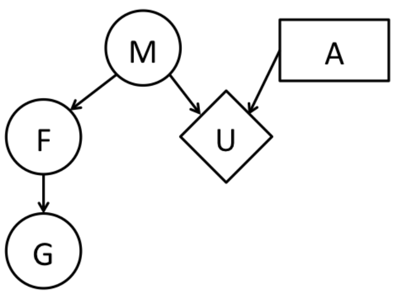
Q2.4) Optimal action for $+s =$ $rent$

Q2.5) $MEU(\{-s\}) = 60$

Q2.6) Optimal action for $-s =$ $rent$

Q2.7) $VPI(S) = 0$, since the Value of Perfect Information is the expected difference in MEU given the evidence vs. without the evidence and here the evidence is uninformative.

Q3) (6 pts) Now let's assume that we want to determine the "fullness" of the theaters F without using information about the Covid-19 restrictions but using information about the garbage disposal G outside the theaters. This new variable is also binary and the new decision network and tables are as follows:



G	F	$P(G\|F)$
+g	+f	0.8
-g	+f	0.2
+g	-f	0.3
-g	-f	0.7

M	P(M)
+m	0.5
-m	0.5

F	M	$P(F\|M)$
+f	+m	0.92
-f	+m	0.08
+f	-m	0.24
-f	-m	0.76

M	A	U(M,A)
+m	$theater$	100
-m	$theater$	10
+m	$rent$	80
-m	$rent$	40

Figure 4: Decision network and tables.

We also provide you with the following extra tables that might help you to answer the question.

F	$P(F)$
+f	0.58
-f	0.42

G	$P(G)$
+g	0.59
-g	0.41

M	G	$P(M\|G)$
+m	+g	0.644
-m	+g	0.356
+m	-g	0.293
-m	-g	0.707

G	M	$P(G\|M)$
+g	+m	0.760
-g	+m	0.240
+g	-m	0.420
-g	-m	0.580

M	F	$P(M\|F)$
+m	+f	0.793
-m	+f	0.207
+m	-f	0.095
-m	-f	0.905

Figure 5: Extra tables.

Fill in the following values:

Q3.1) $MEU(+g) = max(EU(theater|+g), EU(rent|+g)) = 67.96$

$EU(theater|+g) = P(+m|+g) \cdot U(+m, theater) + P(-m|+g) \cdot U(-m, theater) = 0.644 \cdot 100 + 0.356 \cdot 10 = 67.96$

$EU(rent|+g) = P(+m|+g) \cdot U(+m, rent) + P(-m|+g) \cdot U(-m, rent) = 0.644 \cdot 80 + 0.356 \cdot 40 = 65.76$

Q3.2) $MEU(-g) = max(EU(theater|-g), EU(rent|-g)) = 51.72$

$EU(theater|-g) = P(+m|-g) \cdot U(+m, theater) + P(-m|-g) \cdot U(-m, theater) = 0.293 \cdot 100 + 0.707 \cdot 10 = 36.37$

$EU(rent|-g) = P(+m|-g) \cdot U(+m, rent) + P(-m|-g) \cdot U(-m, rent) = 0.293 \cdot 80 + 0.707 \cdot 40 = 51.72$

Q3.3) $VPI(G) = P(+g) \cdot MEU(+g) + P(-g)MEU(-g) - MEU(\emptyset) = 0.59 \cdot 67.96 + 0.41 \cdot 51.72 - MEU(\emptyset) = 61.3 - 60 = 1.3$

Q4) (1 pt) Now we change the theme and we switch to a HMM smoothing problem which has state variables X_t and observation variables E_t. The joint distribution is written as follows:

$$P(X_{1:T}, E_{1:T} = e_{1:T}) = P(X_1) \prod_{t=1}^{T-1} P(X_{t+1}|X_t) \prod_{t=1}^{T} P(E_t = e_t|X_t).$$

Recall that the notation $1 : T$ is used to refer to indices $1, \ldots, T$ and $E_{1:T} = e_{1:T}$ stands for $E_1 = e_1, \ldots, E_T = e_T$. The forward algorithm we covered in class can be used to calculate $P(X_t|E_{1:t} = e_{1:t})$ (*filtering*). The *smoothing* problem on the other hand calculates $P(X_t|E_{1:T} = e_{1:T})$ for $1 \le t < T$, aiming to obtain more accurate estimates given the observed sequence of evidence from $1 : T$. We define for convenience two probability vectors $\alpha(X_t) := P(E_{1:t} = e_{1:t}, X_t)$ and $\beta(X_t) := P(E_{t+1:T} = e_{t+1:T}|X_t)$.

Which of the following expressions are equivalent to $P(E_T = e_T|X_{T-1})$? More than one answer might be correct.

A. $\sum_{x_t} P(X_T = x_t|X_{T-1})P(E_T = e_T|X_T = x_t)$

B. $\sum_{x_t} P(X_T = x_t|X_{T-1})P(E_T = e_T|X_T = x_t, X_{T-1})$

C. $P(X_T = x_t|X_{T-1})P(E_T = e_T|X_T = x_t)$

D. $P(X_T = x_t|X_{T-1})P(E_T = e_T|X_T = x_t, X_{T-1})$

A,B.

$$\begin{aligned} P(E_T = e_T|X_{T-1}) &= \sum_{x_t} P(E_T = e_T, X_T = x_t|X_{T-1}) \\ &= \sum_{x_t} P(X_T = x_t|X_{T-1})P(E_T = e_T|X_T = x_t, X_{T-1}) \\ &= \sum_{x_t} P(X_T = x_t|X_{T-1})P(E_T = e_T|X_T = x_t) \end{aligned}$$

Where in the first line we use the marginalization rule, in the second the chain rule and in the third the fact that E_T is independent of X_{T-1} given X_T.

Q5) (9 pts) This is continuing from Q4. A similar approach to forward recursion for filtering can be used to compute $\alpha(X_1), \ldots, \alpha(X_T)$. We want to derive a *backward* recursion to compute the βs. For each blank (i),...,(iv) choose the correct expression to generate the formula below:

$$P(E_{t+1} = e_{t+1}, \ldots, E_T = e_T|X_t) = \boxed{\text{(i)}}\boxed{\text{(ii)}}\boxed{\text{(iii)}}\boxed{\text{(iv)}}$$

If for a particular location no term is needed, select *None*. Each part (i)-(iv) is worth 1 point.

Q5.1) (i)

A. $\sum_{x_{t-1}}$

B. $\sum_{x_t}$

C. $\sum_{x_{t+1}}$

D. *None*

C

Q5.2) (ii)

A. $\alpha(X_{t-1} = x_{t-1})$

B. $\alpha(X_t = x_t)$

C. $\alpha(X_{t+1} = x_{t+1})$

D. $\beta(X_{t-1} = x_{t-1})$

E. $\beta(X_{t+1} = x_{t+1})$

F. *None*

E

Q5.3) (iii)

A. $P(X_t = x_t | X_{t-1})$

B. $P(X_{t+1} = x_{t+1} | X_t)$

C. $P(X_t | X_{t-1} = x_{t-1})$

D. $P(X_{t+1} | X_t = x_t)$

E. *None*

B

Q5.4) (iv)

A. $P(E_{t-1} = e_{t-1} | X_{t-1})$

B. $P(E_t = e_t | X_t)$

C. $P(E_{t+1} = e_{t+1} | X_{t+1})$

D. $P(E_{t-1} = e_{t-1} | X_{t-1} = x_{t-1})$

E. $P(E_t = e_t | X_t = x_t)$

F. $P(E_{t+1} = e_{t+1} | X_{t+1} = x_{t+1})$

G. *None*

F

Q5.5) Please show your work for questions Q5.1-Q5.4 below by uploading a screenshot of your work (using pen paper, a tablet, LaTex pdf, etc.)

Justification for all choices:

$$P(E_{t+1:T} = e_{t+1:T} | X_t) = \sum_{x_{t+1}} P(E_{t+1:T} = e_{t+1:T}, X_{t+1} = x_{t+1} | X_t)$$

$$= \sum_{x_{t+1}} P(E_{t+1:T} = e_{t+1:T} | X_{t+1} = x_{t+1}, X_t) P(X_{t+1} = x_{t+1} | X_t)$$

$$= \sum_{x_{t+1}} P(E_{t+1:T} = e_{t+1:T} | X_{t+1} = x_{t+1}) P(X_{t+1} = x_{t+1} | X_t)$$

$$= \sum_{x_{t+1}} P(E_{t+2:T} = e_{t+2:T} | X_{t+1} = x_{t+1}) P(E_{t+1} = e_{t+1} | X_{t+1} = x_{t+1}, E_{t+2:T} = e_{t+2:T}) P(X_{t+1} = x_{t+1} | X_t)$$

$$= \sum_{x_{t+1}} P(E_{t+2:T} = e_{t+2:T} | X_{t+1} = x_{t+1}) P(E_{t+1} = e_{t+1} | X_{t+1} = x_{t+1}) P(X_{t+1} = x_{t+1} | X_t)$$

$$= \sum_{x_{t+1}} \beta(X_{t+1} = x_{t+1}) P(E_{t+1} = e_{t+1} | X_{t+1} = x_{t+1}) P(X_{t+1} = x_{t+1} | X_t)$$

Where in the first line we use the marginalization rule, in the second the chain rule, in the third the fact that E_T is independent of X_t given X_{t+1}. In the fourth line we use the chain rule, in the fifth the fact that E_{t+1} is independent of $E_{t+2:T}$ given X_{t+1} and in the last one we use the definition of β.

Q5.6) Which of the following expressions are equivalent to $P(X_t = x_t | E_{1:T} = e_{1:T})$? More than one choice might be correct.

A. $\sum_{x'_t} \alpha(X_t = x'_t)\beta(X_t = x'_t)$

B. $\alpha(X_t = x_t)\beta(X_t = x_t)$

C. $\frac{\alpha(X_t=x_t)\beta(X_t=x_t)}{\sum_{x'_t} \alpha(X_t=x'_t)\beta(X_t=x'_t)}$

D. $\frac{\alpha(X_t=x_t)\beta(X_t=x_t)}{\sum_{x'_T} \alpha(X_T=x'_T)}$

C, D.

The answer follows from the fact that $P(E_{1:T} = e_{1:T}) = \sum_{x'_t} \alpha(X_t = x'_t)\beta(X_t = x'_t) = \sum_{x'_T} \alpha(X_T = x'_T)$. Furthermore, $P(E_{1:T} = e_{1:T}, X_t = x_t) = \alpha(X_t = x_t)\beta(X_t = x_t)$. Using Bayes rule we obtain:

$$P(X_t = x_t | E_{1:T} = e_{1:T}) = \frac{P(X_t = x_t, E_{1:T} = e_{1:T})}{P(E_{1:T} = e_{1:T})}$$