The New York Times

I'm a Congressman Who Codes. A.I. Freaks Me Out.

By Ted Lieu

Jan. 23, 2023

GRAPH THEORY

Finally, a Fast Algorithm for Shortest Paths on Negative Graphs

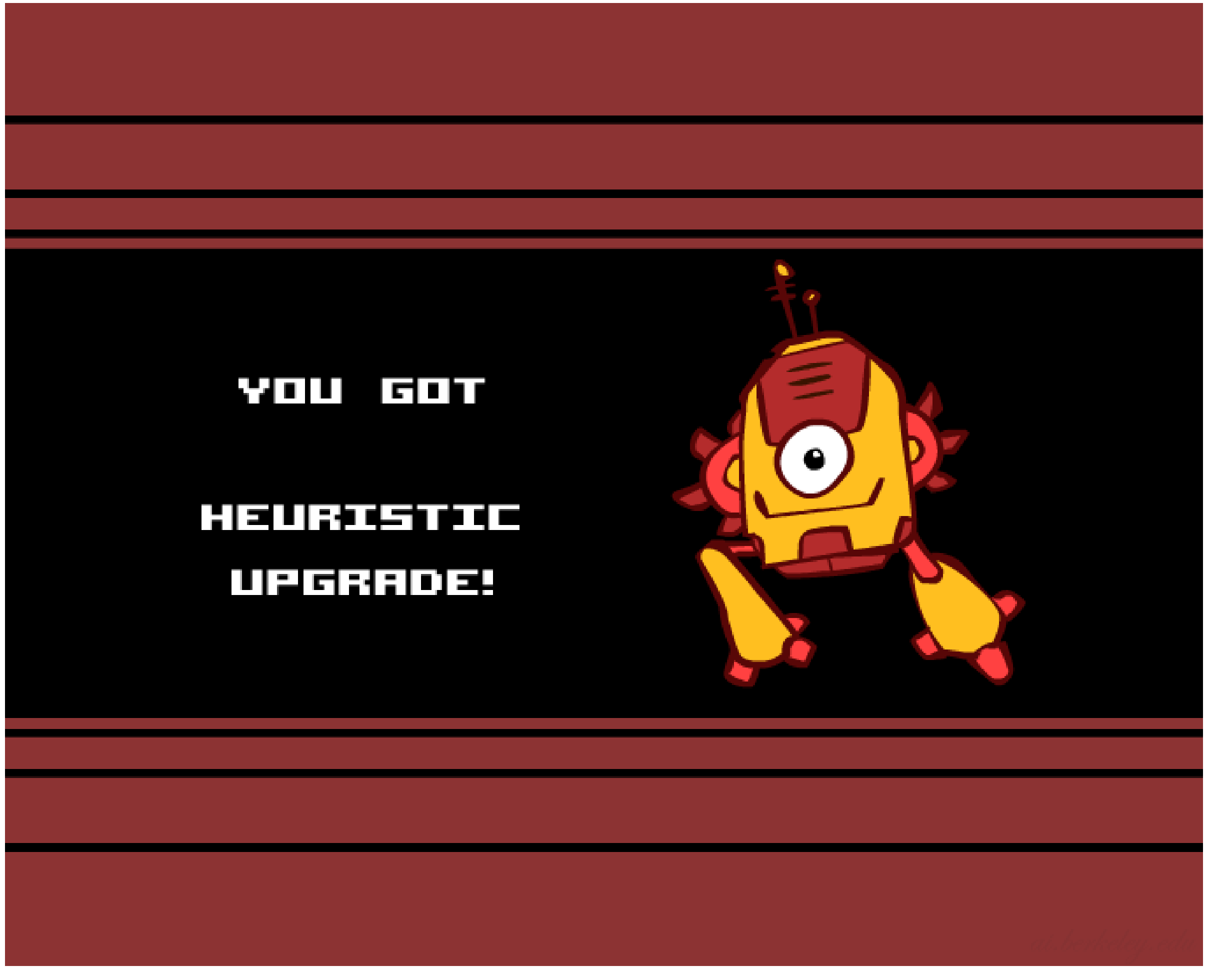
ChatGPT – calm down everyone

- Q: which is bigger? an elephant or a cat?
- A: An elephant is bigger than a cat.
- Q: which is not bigger than the other? an elephant or a cat?
- A: Neither an elephant nor a cat is bigger than the other.
- Q: what is the largest even number?
- A: The largest even number is 9,999,999,998.
- Q: I started with 20 dollars and gave 10 dollars to my friend. How much do we have in total?
- A: You and your friend have a total of 30 dollars.

Recap

- A* expands the fringe node with lowest f value where
  - $f(n) = g(n) + h(n)$
  - $g(n)$ is the cost to reach n
  - $h(n)$ is an admissible estimate of the least cost from n to a goal node: $0 \leq h(n) \leq h^*(n)$
- A* tree search is optimal
- Its performance depends heavily on the heuristic h

Creating Heuristics



Creating Admissible Heuristics

- Often, admissible heuristics are solutions to ***relaxed problems***, where new actions are available

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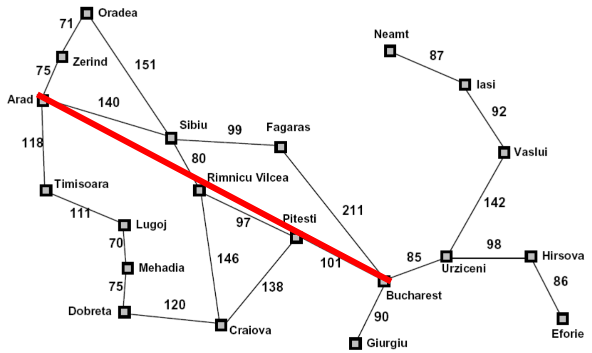
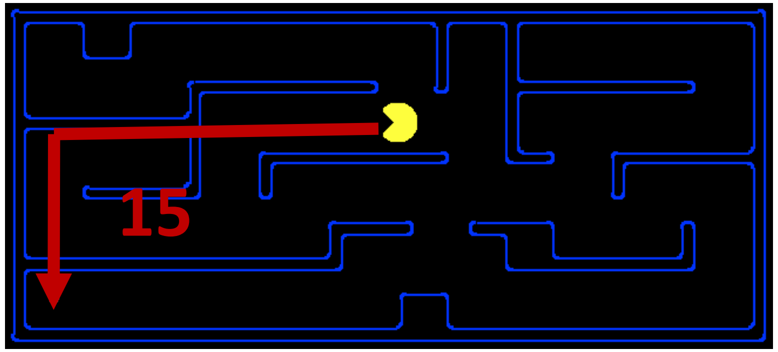
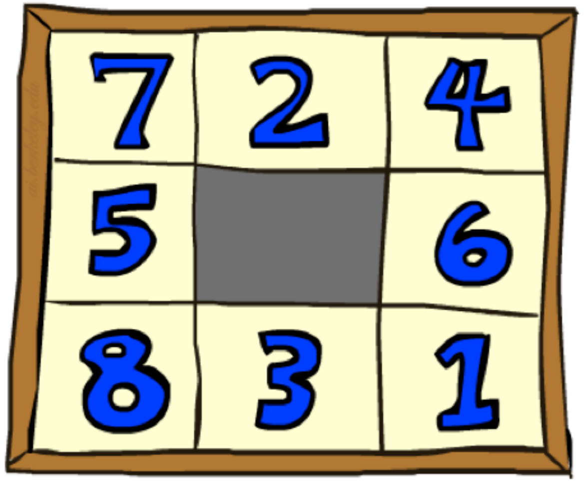


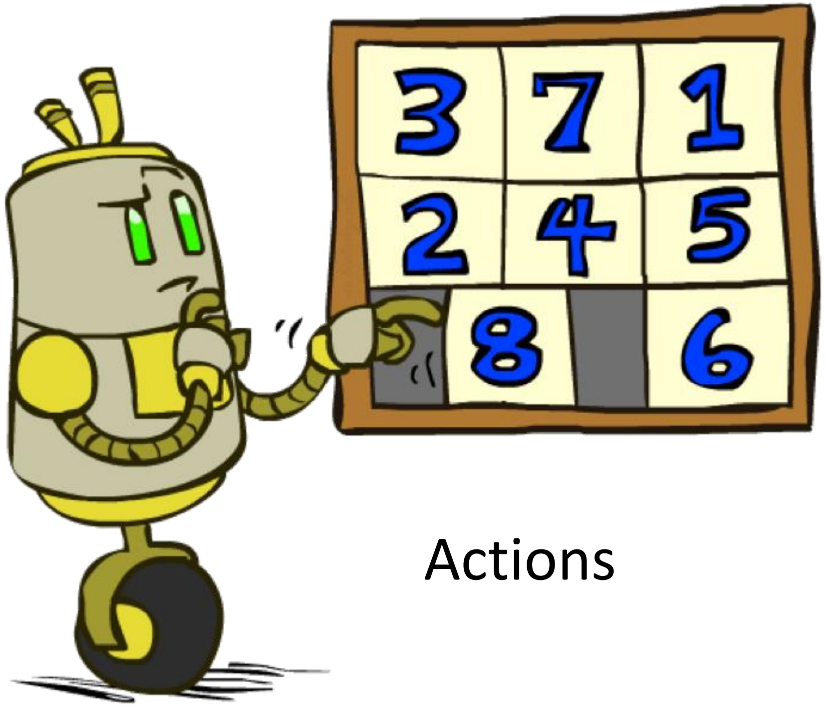


- Problem P_2 is a relaxed version of P_1 if $\mathcal{A}_2(s) \supseteq \mathcal{A}_1(s)$ for every s
- Theorem: $h_2^*(s) \leq h_1^*(s)$ for every s, so $h_2^*(s)$ is admissible for P_1

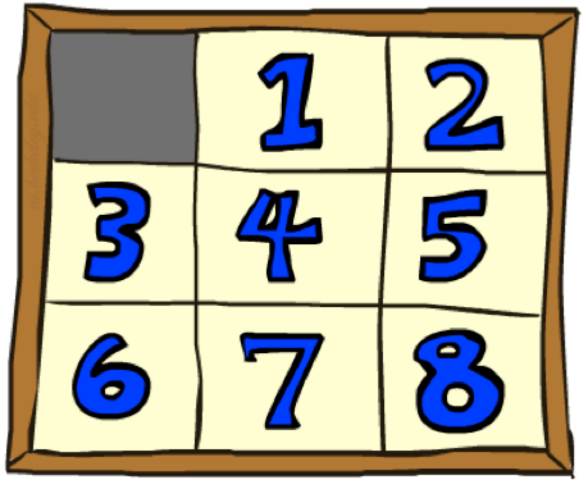
Example: 8 Puzzle



Start State



Actions

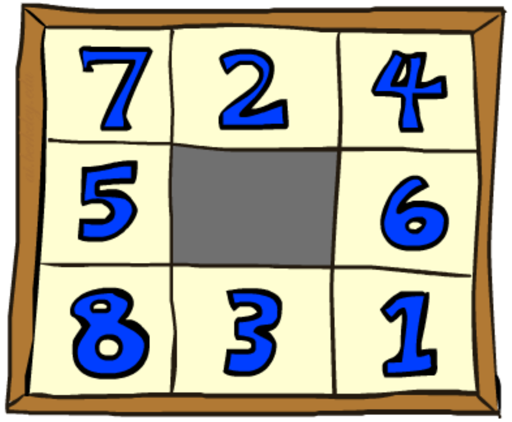


Goal State

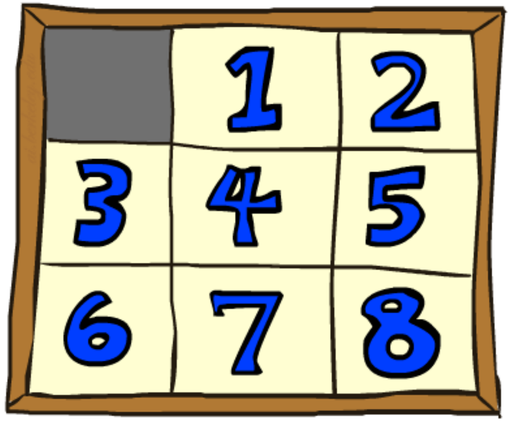
- What are the states?
- How many states?
- What are the actions?
- What are the step costs?

8 Puzzle I

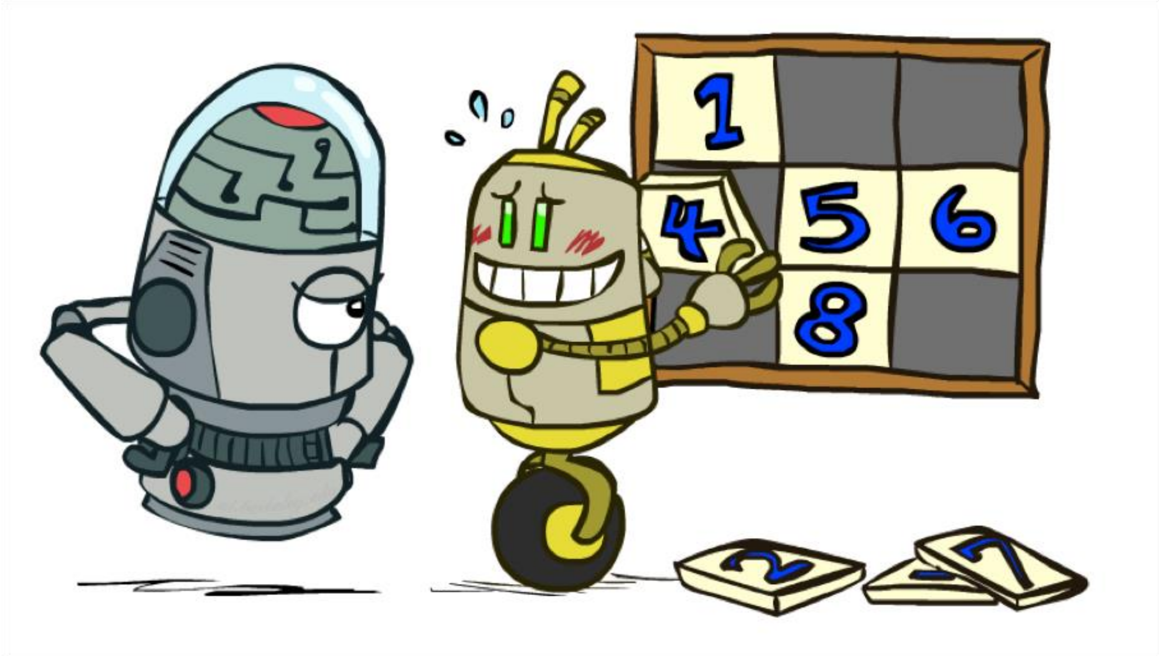
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) = 8



Start State



Goal State

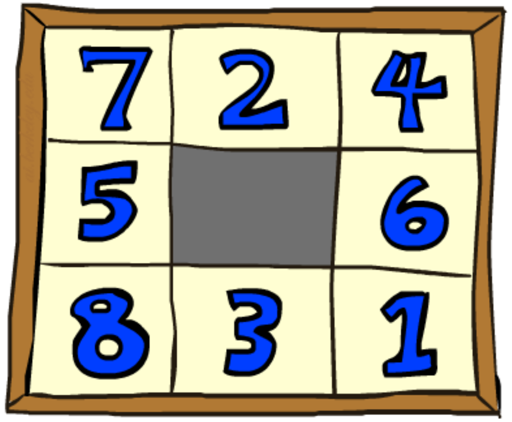


	Average nodes expanded when the optimal path has...		
	...4 steps	...8 steps	...12 steps
UCS	112	6,300	3.6×10^6
A*TILES	13	39	227

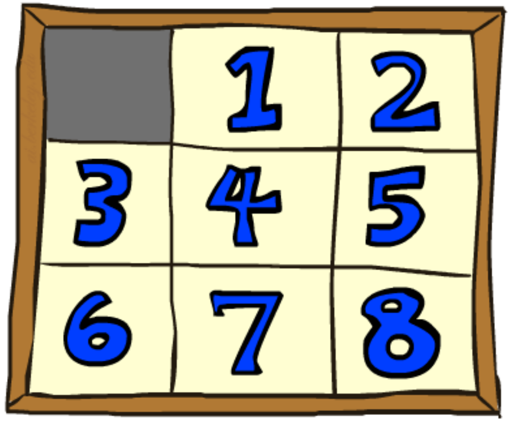
Statistics from Andrew Moore

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State



Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
A*TILES	13	39	227
A*MANHATTAN	12	25	73

Combining heuristics

- Dominance: $h_1 \geq h_2$ if

  $$\forall n \;\; h_1(n) \geq h_2(n)$$

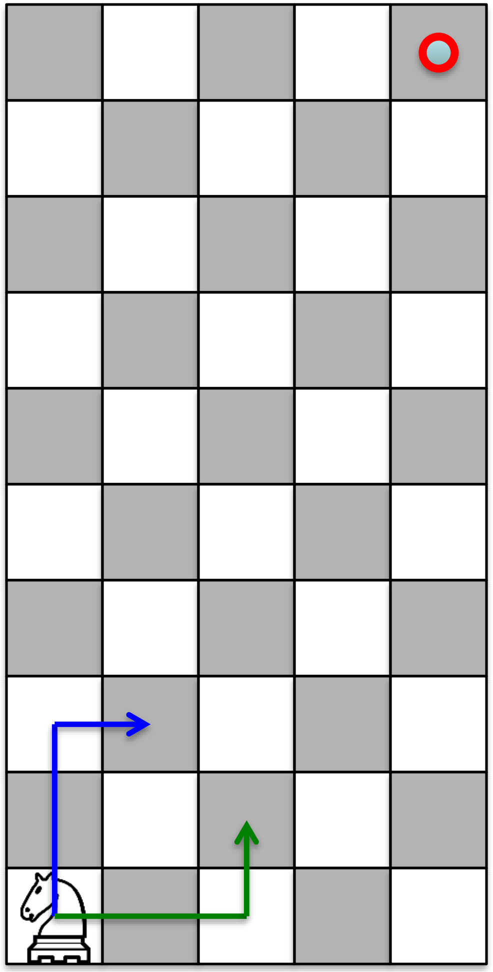
  - Roughly speaking, larger is better as long as both are admissible
  - The zero heuristic is pretty bad (what does A* do with h=0?)
  - The exact heuristic is pretty good, but usually too expensive!
- What if we have two heuristics, neither dominates the other?
  - Form a new heuristic by taking the max of both:

    $$h(n) = \max(\, h_1(n), h_2(n))$$

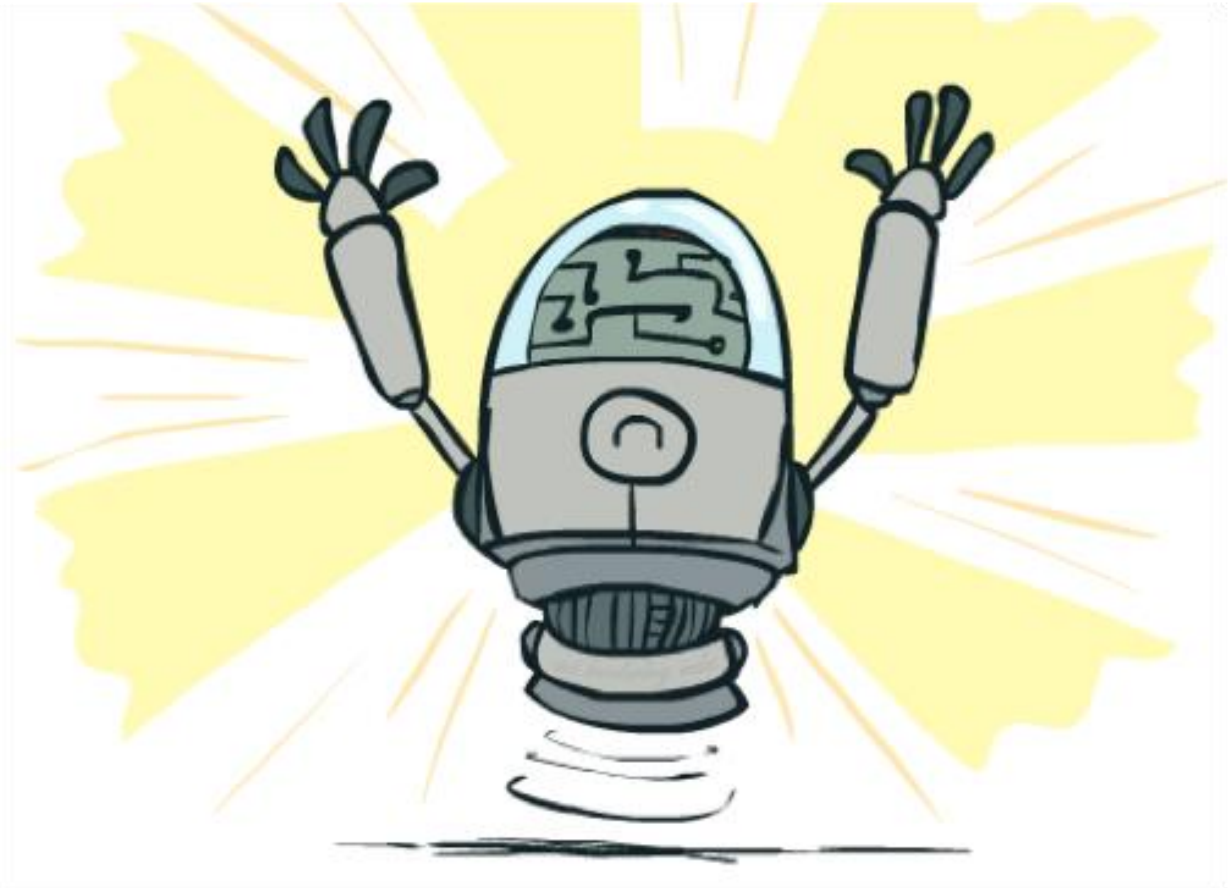
  - Max of admissible heuristics is admissible and dominates both!

Example: Knight's moves

- Minimum number of knight's moves to get from A to B?
  - h_1 = (Manhattan distance)/3
    - h_1' = h_1 rounded up to correct parity (even if A, B same color, odd otherwise)
  - h_2 = (Euclidean distance)/√5 (rounded up to correct parity)
  - h_3 = (max x or y shift)/2 (rounded up to correct parity)
- $h(n) = \max(h_1'(n), h_2(n), h_3(n))$ is admissible!



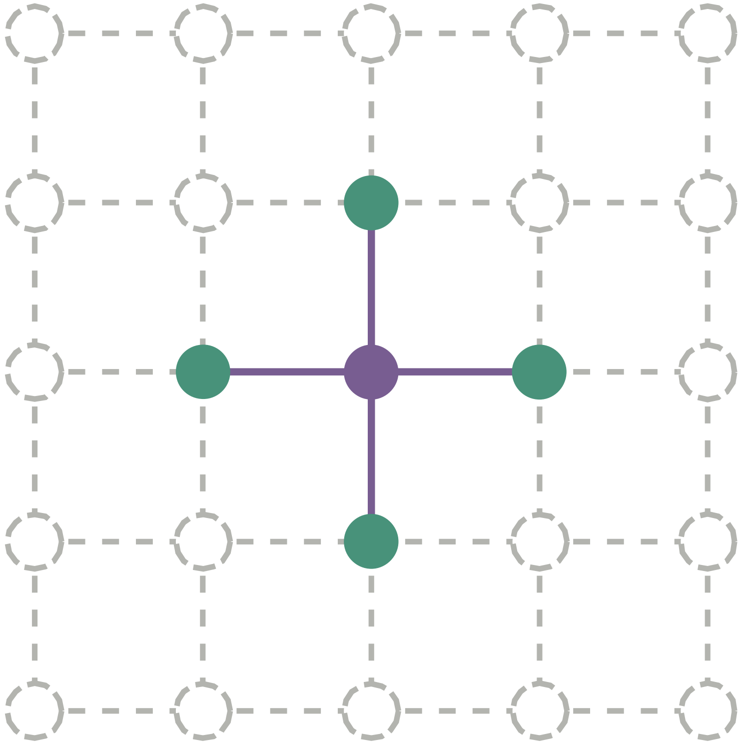
Optimality of A* Graph Search



This part is a bit fiddly, sorry about that

Quiz: State Space Graphs vs. Search Trees

Consider a rectangular grid:



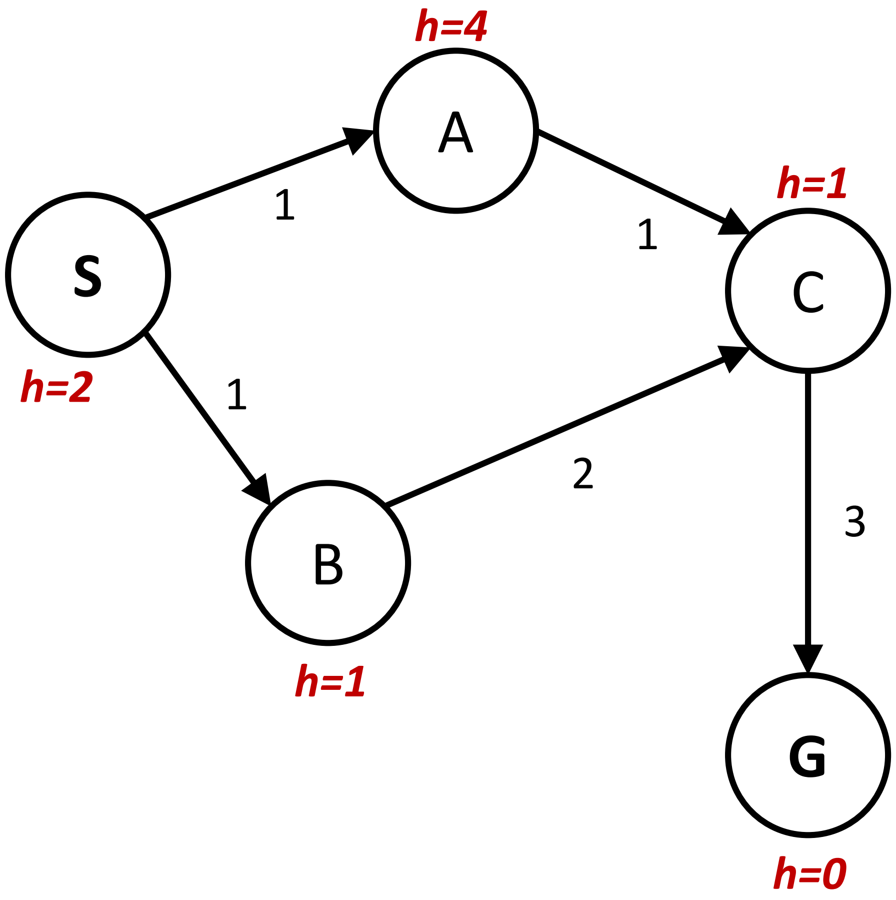
How many states within *d* steps of start?

How many states in search tree of depth *d*?

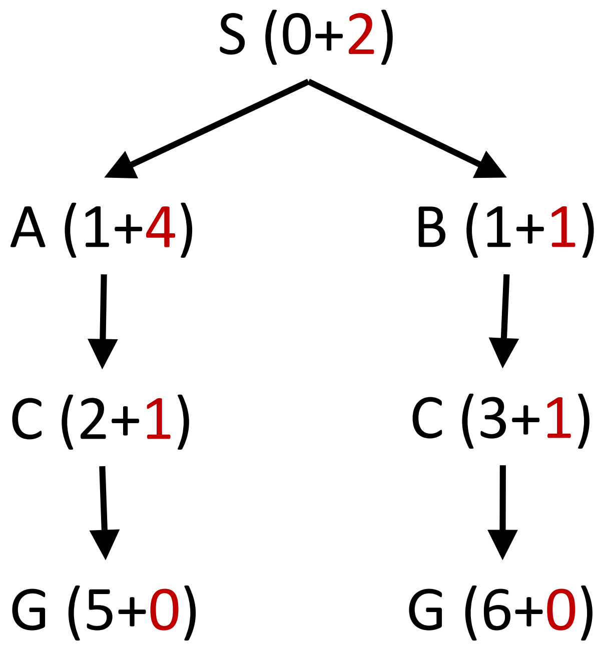
Basic idea of graph search: don't re-expand a state that has been expanded previously

A* Graph Search Gone Wrong?

State space graph

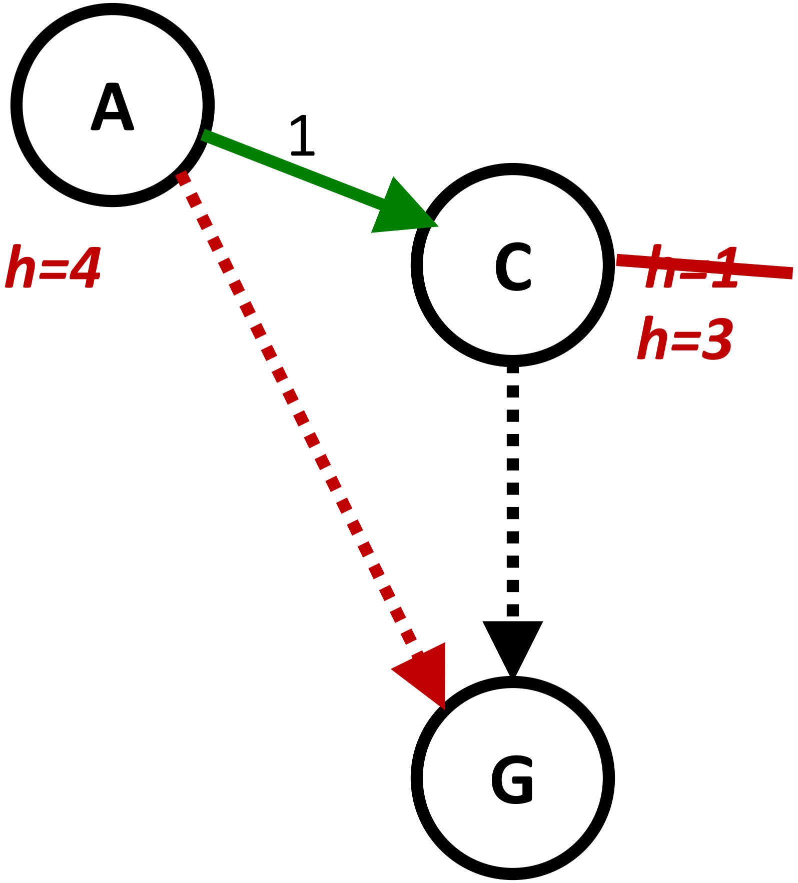


Search tree



Simple check against expanded set blocks C
Fancy check allows new C if cheaper than old
but requires recalculating C's descendants

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    $h(A) \leq h^*(A)$
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

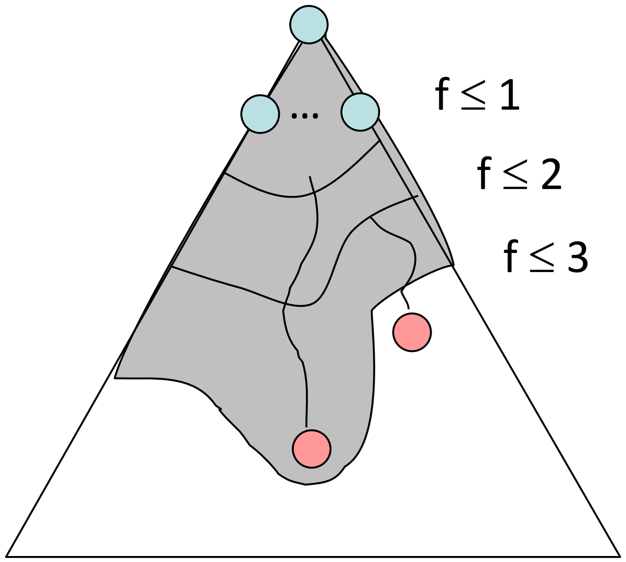
    $h(A) - h(C) \leq c(A,C)$

    or $h(A) \leq c(A,C) + h(C)$ (triangle inequality)
    - Note: h* *necessarily* satisfies triangle inequality
- Consequences of consistency:
  - The *f* value along a path never decreases:

    $h(A) \leq c(A,C) + h(C) \quad \Rightarrow g(A) + h(A) \leq g(A) + c(A,C) + h(C)$
  - A* graph search is optimal

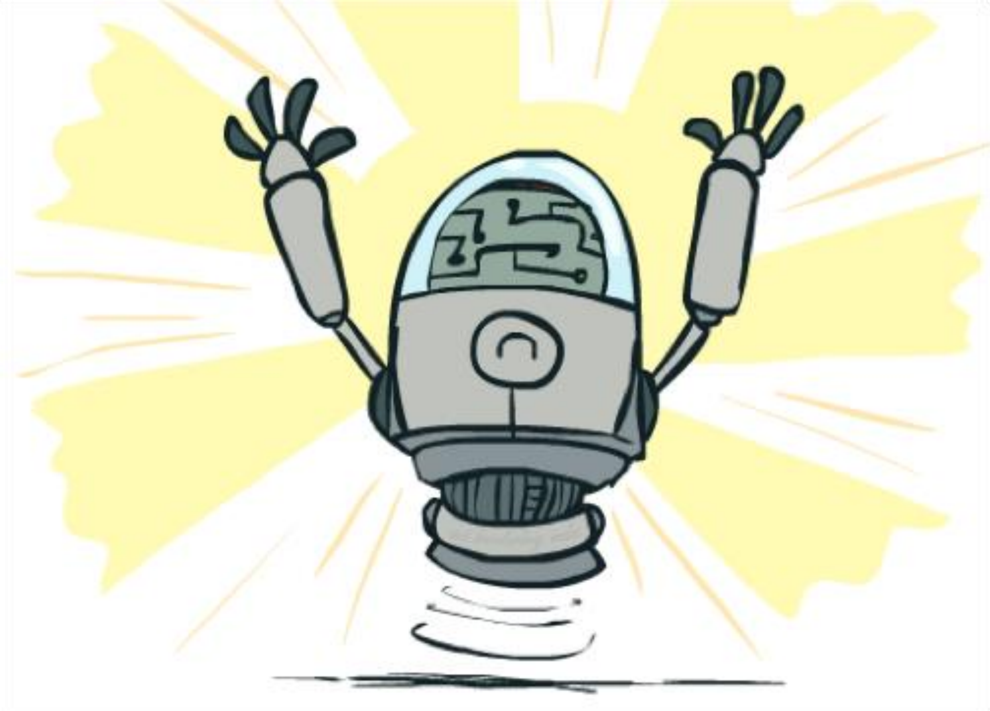
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
- Graph search:
  - A* optimal if heuristic is consistent
- Consistency implies admissibility
- Most natural admissible heuristics tend to be consistent, especially if from relaxed problems



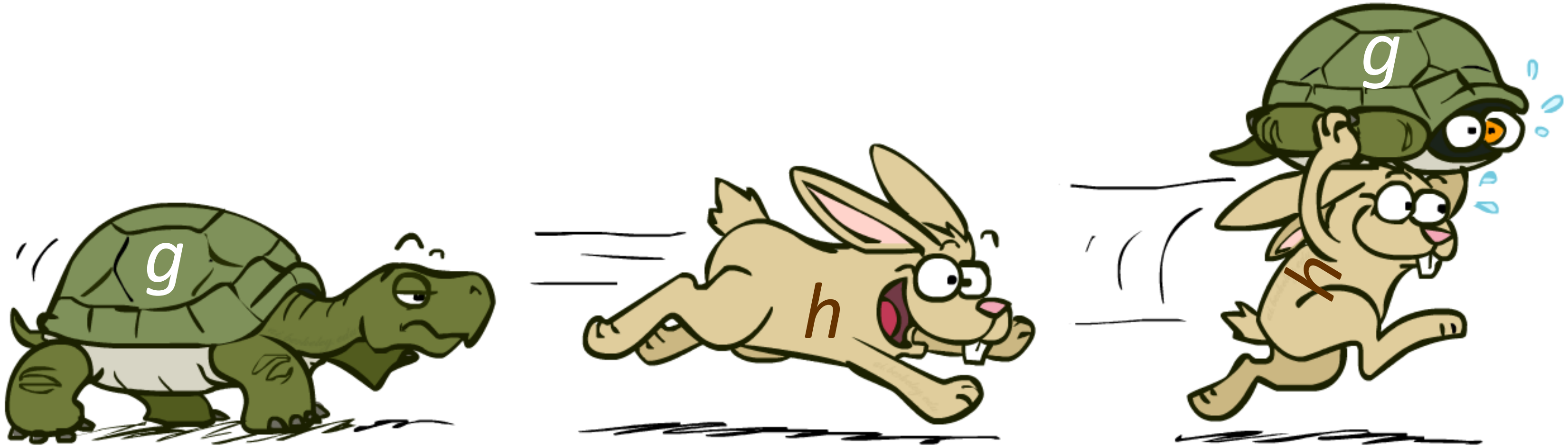
But…

- A* keeps the entire explored region in memory
- => will run out of space before you get bored waiting for the answer
- There are variants that use less memory (Section 3.5.5):
  - IDA* works like iterative deepening, except it uses an *f*-limit instead of a depth limit
    - On each iteration, remember the smallest *f*-value that exceeds the current limit, use as new limit
    - Very inefficient when *f* is real-valued and each node has a unique value
  - RBFS is a recursive depth-first search that uses an *f*-limit = the *f*-value of the best alternative path available from any ancestor of the current node
    - When the limit is exceeded, the recursion unwinds but remembers the best reachable *f*-value on that branch
  - SMA* uses *all available memory* for the queue, minimizing thrashing
    - When full, drop worst node on the queue but remember its value in the parent

A*: Summary

- A* orders nodes in the queue by $f(n) = g(n) + h(n)$
- A* is optimal for trees/graphs with admissible/consistent heuristics

- Heuristic design is key: often use relaxed problems



CS 188: Artificial Intelligence

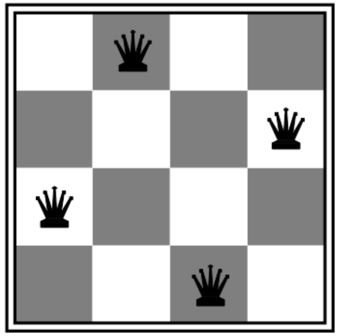
Local search

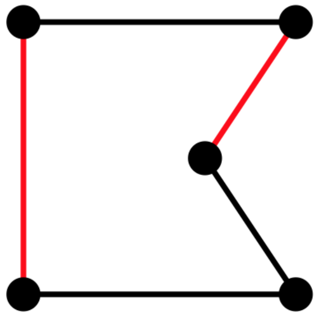
Instructors: Stuart Russell and Peyrin Kao

University of California, Berkeley

Local search algorithms

- In many optimization problems, ***path*** is irrelevant; the goal state ***is*** the solution
- Then state space = set of "complete" configurations;
  find ***configuration satisfying constraints***, e.g., n-queens problem; or, find ***optimal configuration***, e.g., travelling salesperson problem

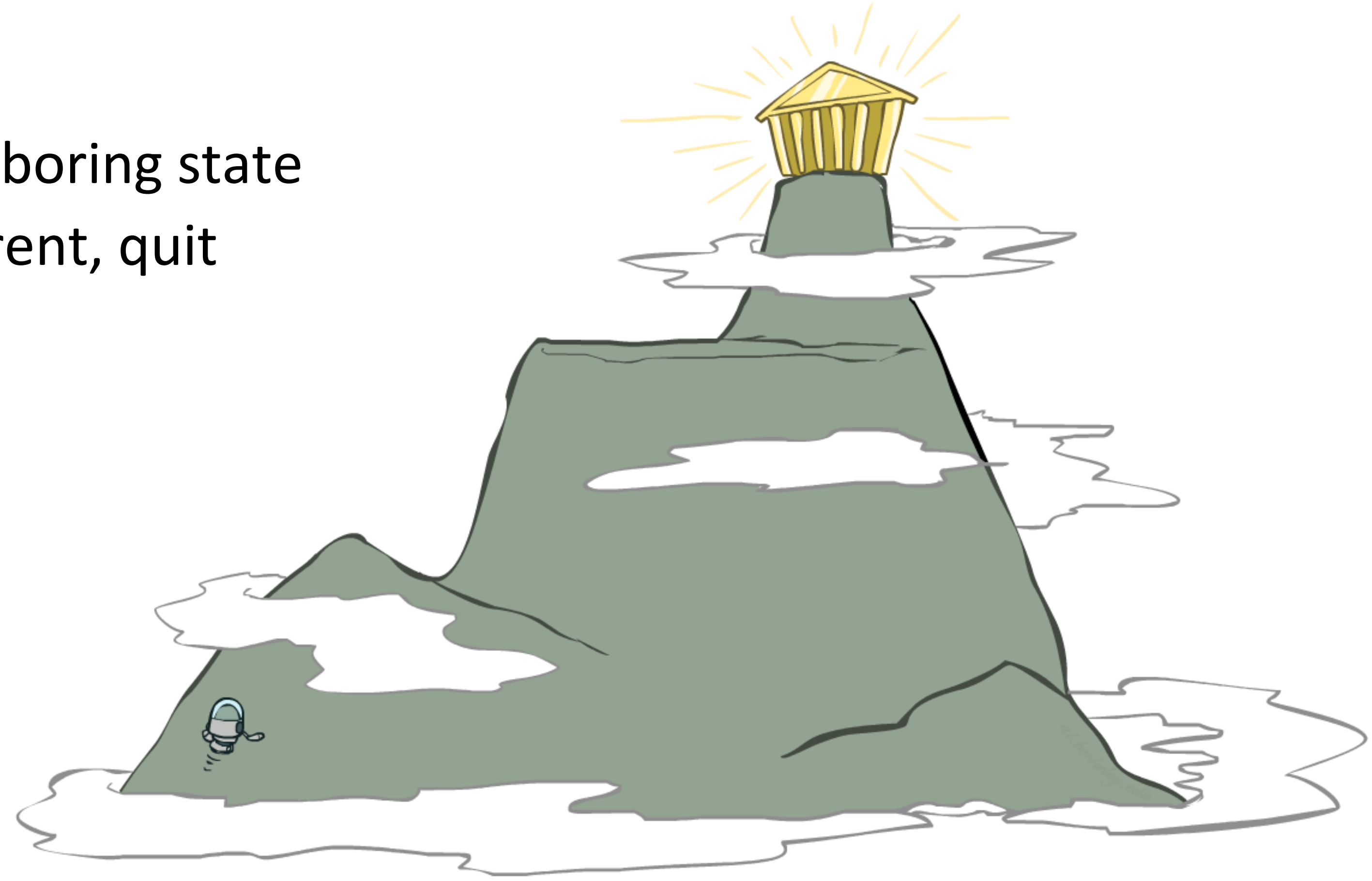




- In such cases, can use ***iterative improvement*** algorithms: keep a single "current" state, try to improve it
- Constant space, suitable for online as well as offline search
- More or less unavoidable if the "state" is yourself (i.e., learning)

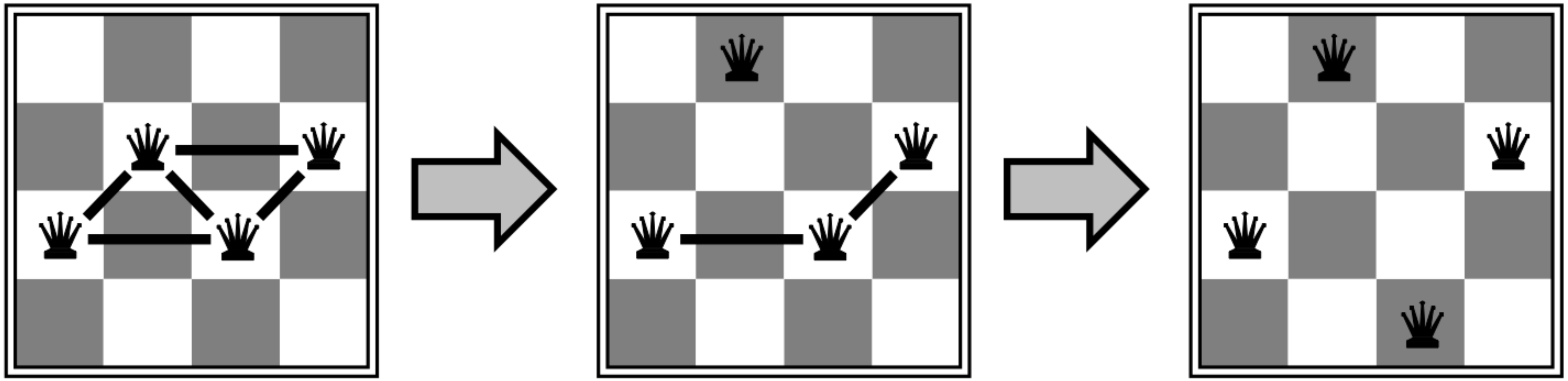
Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit



Heuristic for *n*-queens problem

- Goal: n queens on board with no ***conflicts***, i.e., no queen attacking another
- States: n queens on board, one per column
- Actions: move a queen in its column
- Heuristic value function: number of conflicts



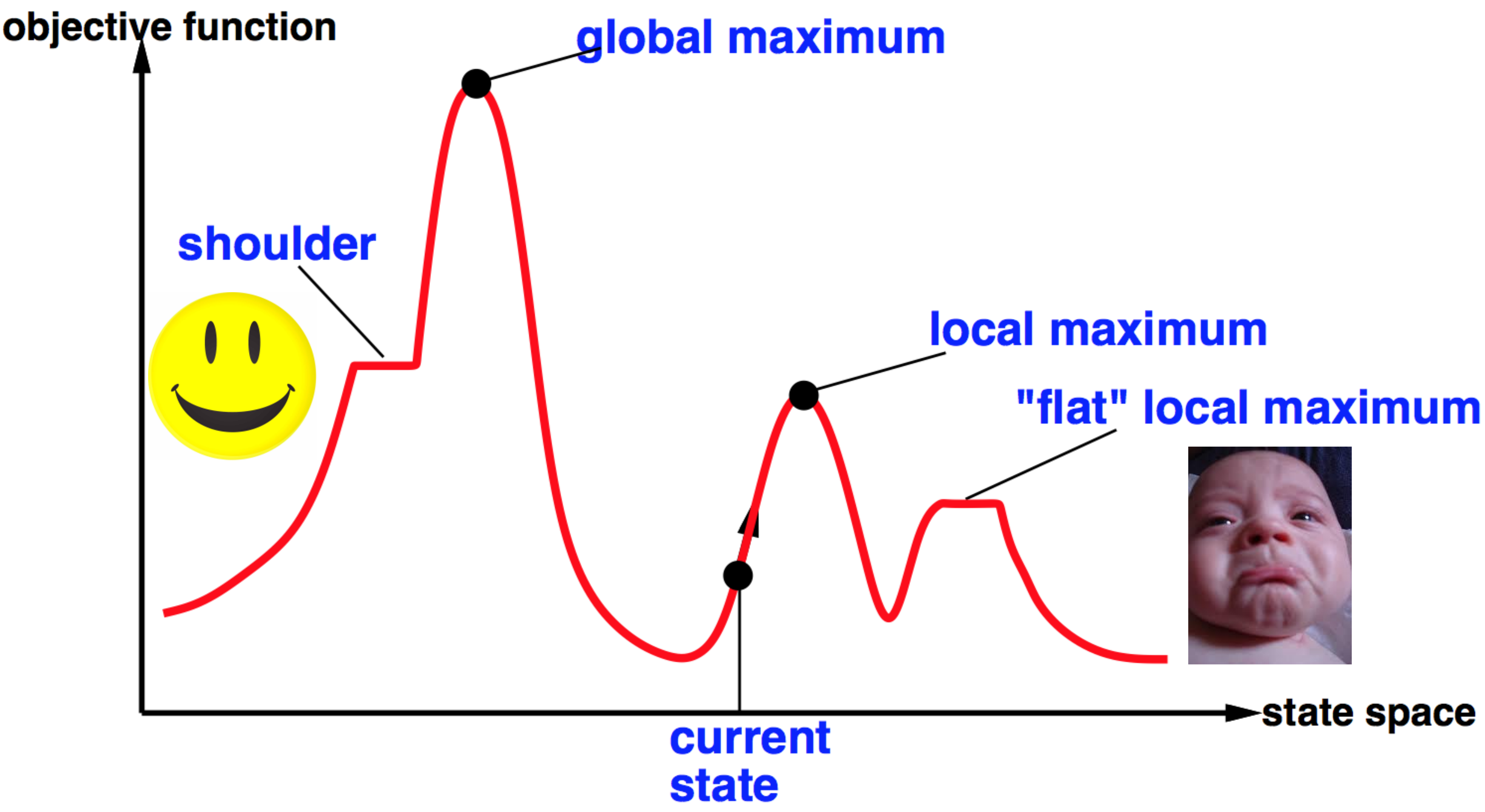
h = 5 h = 2 h = 0

Hill-climbing algorithm

```
function HILL-CLIMBING(problem) returns a state
    current ← make-node(problem.initial-state)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbor.value ≤ current.value then
            return current.state
        current ← neighbor
```

"Like climbing Everest in thick fog with amnesia"

Global and local maxima



Random restarts

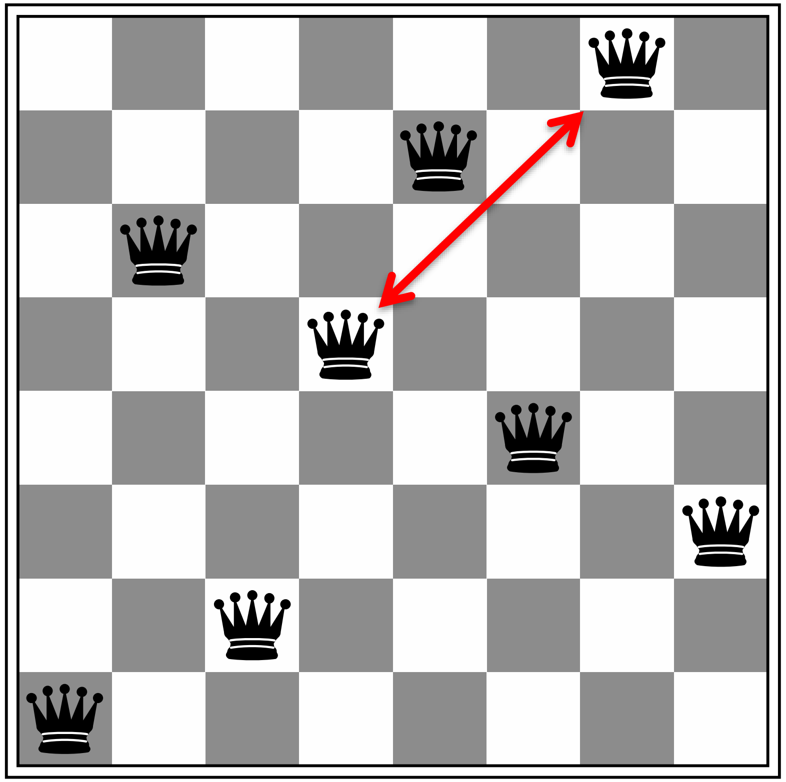
- find global optimum
- duh

Random sideways moves

- Escape from shoulders
- Loop forever on flat local maxima

Hill-climbing on the 8-queens problem

- No sideways moves:
  - Succeeds w/ prob. 0.14
  - Average number of moves per trial:
    - 4 when succeeding, 3 when getting stuck
  - Expected total number of moves needed:
    - 3(1-p)/p + 4 =~ 22 moves
- Allowing 100 sideways moves:
  - Succeeds w/ prob. 0.94
  - Average number of moves per trial:
    - 21 when succeeding, 65 when getting stuck
  - Expected total number of moves needed:
    - 65(1-p)/p + 21 =~ 25 moves



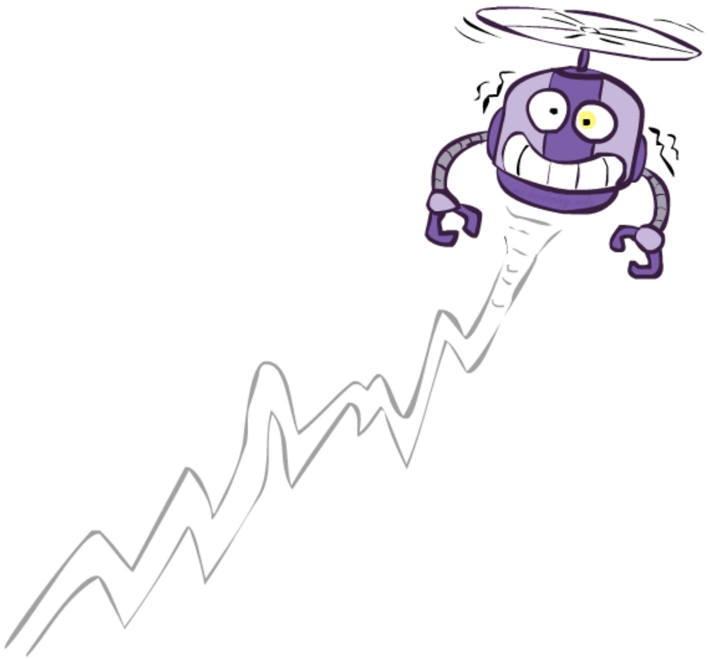
Moral: algorithms with knobs to twiddle are irritating

Simulated annealing

- Resembles the annealing process used to cool metals slowly to reach an ordered (low-energy) state
- Basic idea:
  - Allow “bad” moves occasionally, depending on “temperature”
  - High temperature => more bad moves allowed, shake the system out of its local minimum
  - Gradually reduce temperature according to some schedule
  - Sounds pretty flaky, doesn’t it?

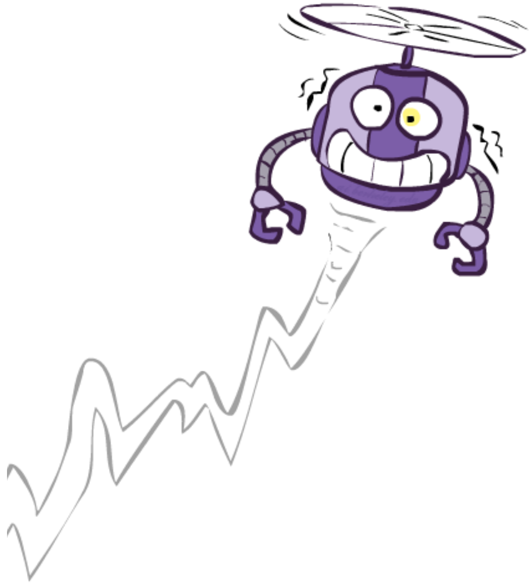
Simulated annealing algorithm

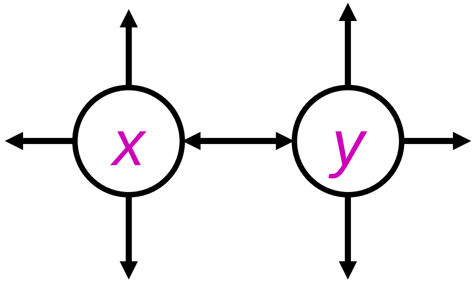
```
function SIMULATED-ANNEALING(problem,schedule) returns a  state
current ← problem.initial-state
for t = 1 to ∞ do
        T ←schedule(t)
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← next.value – current.value
        if ΔE > 0 then current ← next
                else current ← next only with probability e^(ΔE/T)
```



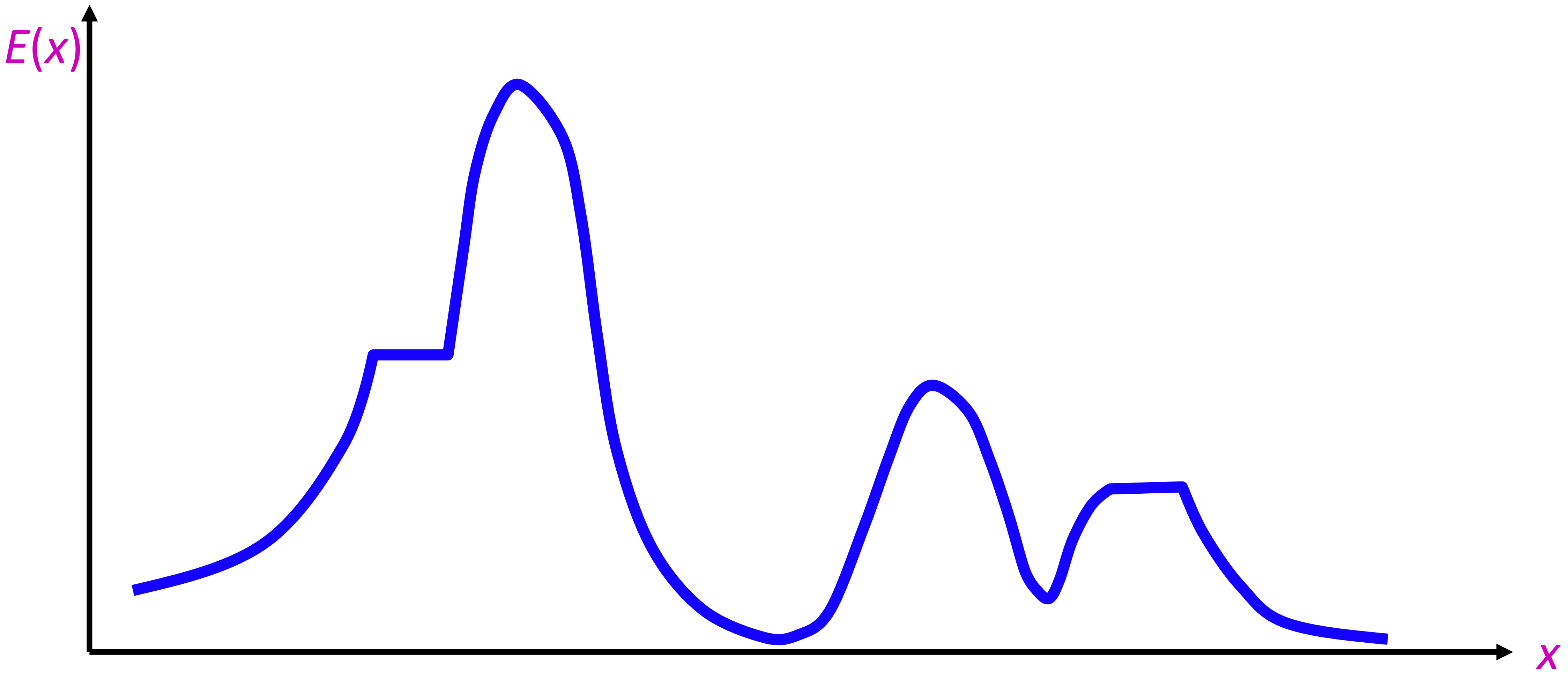
Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution (Boltzmann): $P(x) \propto e^{E(x)/T}$
  - If T decreased slowly enough, will converge to optimal state!
- Proof sketch
  - Consider two adjacent states x, y with $E(y) > E(x)$ [high is good]
  - Assume $x \rightarrow y$ and $y \rightarrow x$ and outdegrees $D(x) = D(y) = D$
  - Let $P(x)$, $P(y)$ be the equilibrium occupancy probabilities at T
  - Let $P(x \rightarrow y)$ be the probability that state x transitions to state y



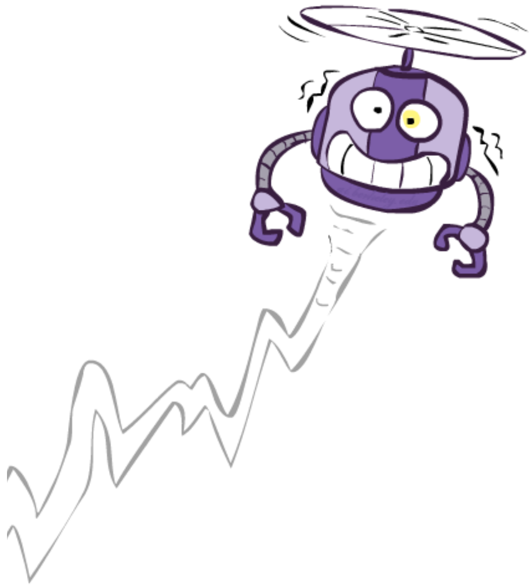


Occupation probability as a function of T



Simulated Annealing

- Is this convergence an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - “Slowly enough” may mean exponentially slowly
  - Random restart hillclimbing also converges to optimal state…
- Simulated annealing and its relatives are a key workhorse in VLSI layout and other optimal configuration problems

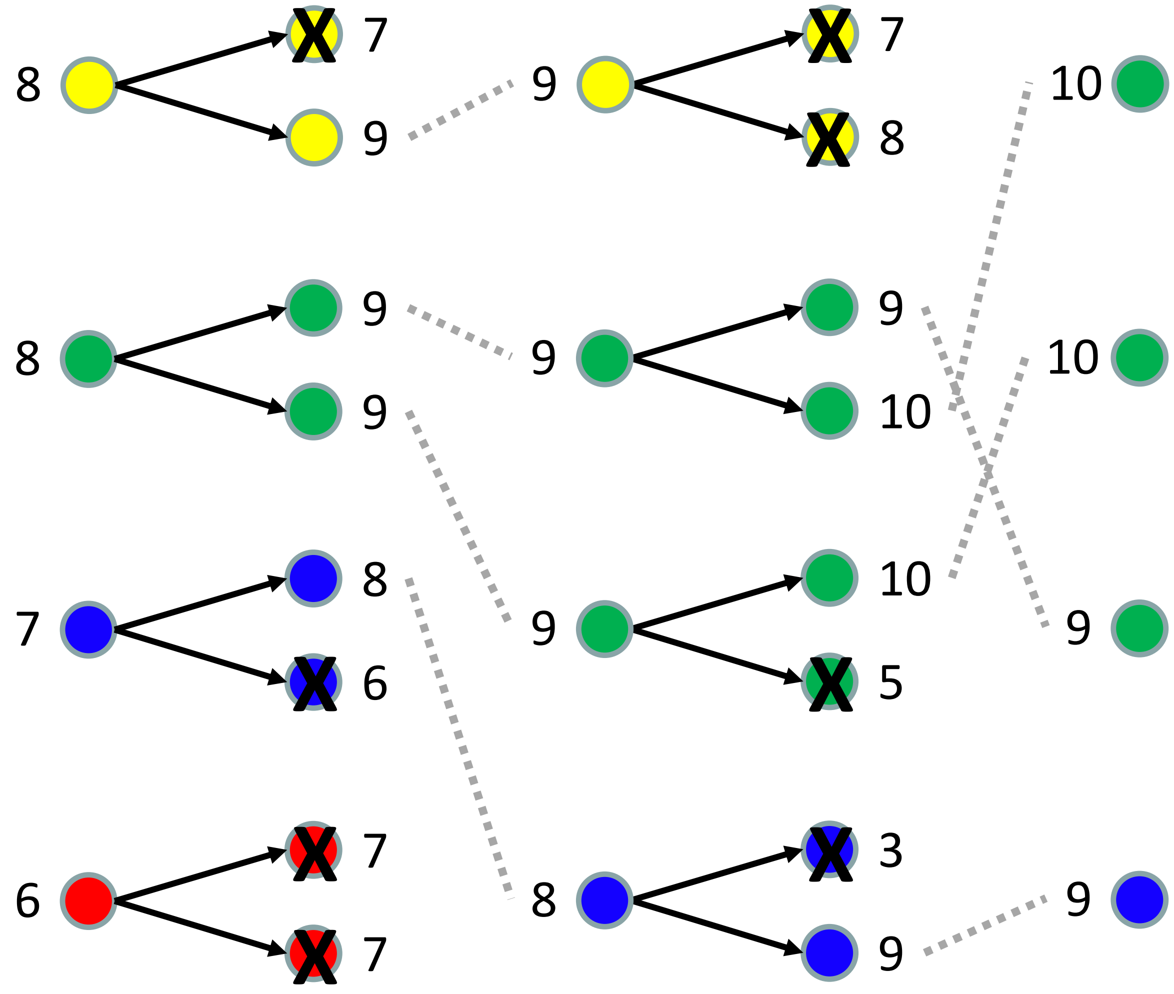


Local beam search

- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose $\boxed{\text{best } K}$ of these to be the new current states

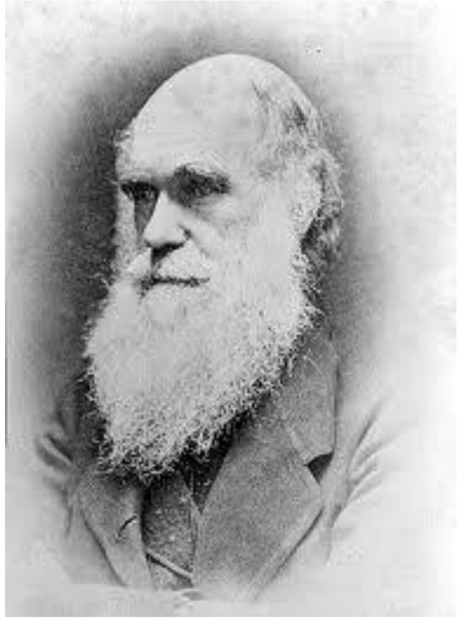
Or, K chosen randomly with a bias towards good ones

Beam search example (K=4)

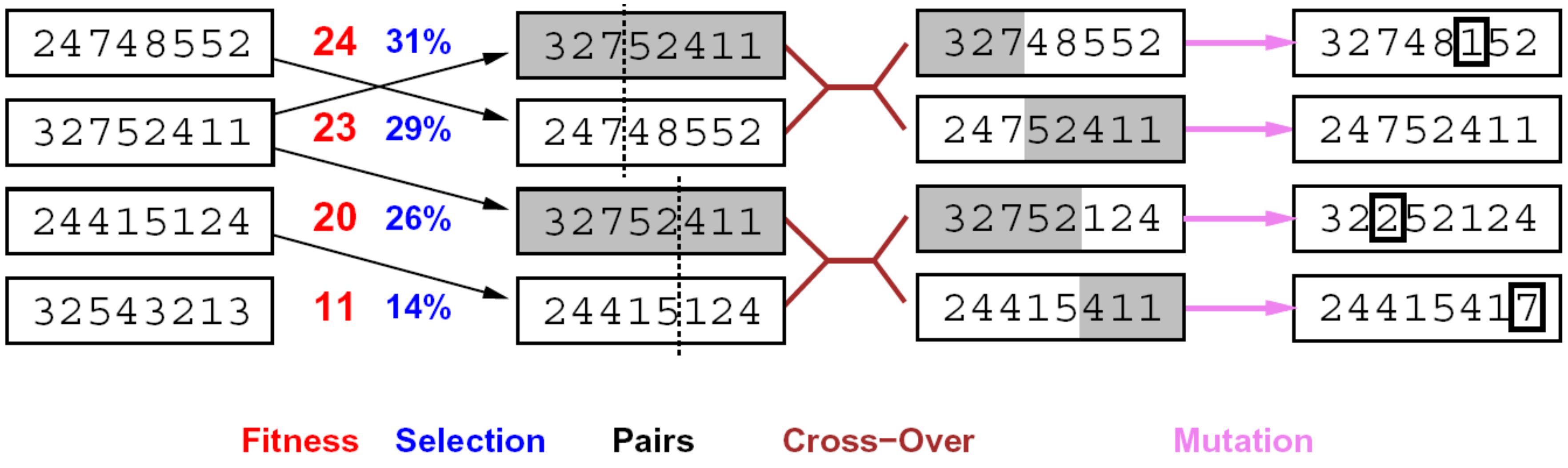


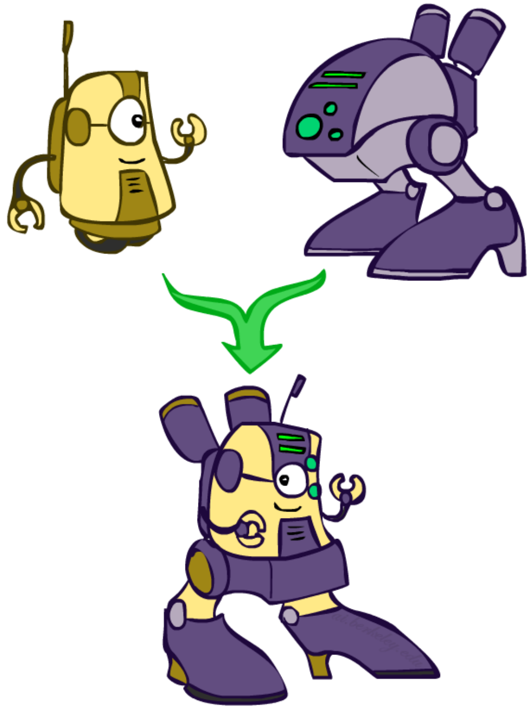
Local beam search

- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! "Come over here, the grass is greener!"
- What other well-known algorithm does this remind you of?
  - Evolution!



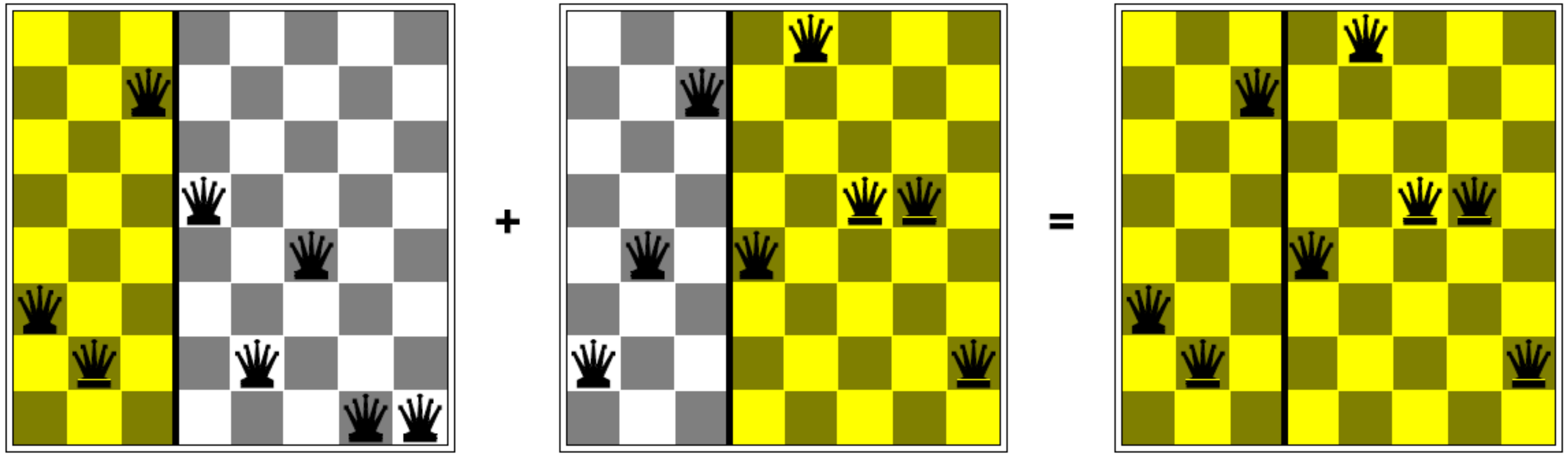
Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety

Example: N-Queens

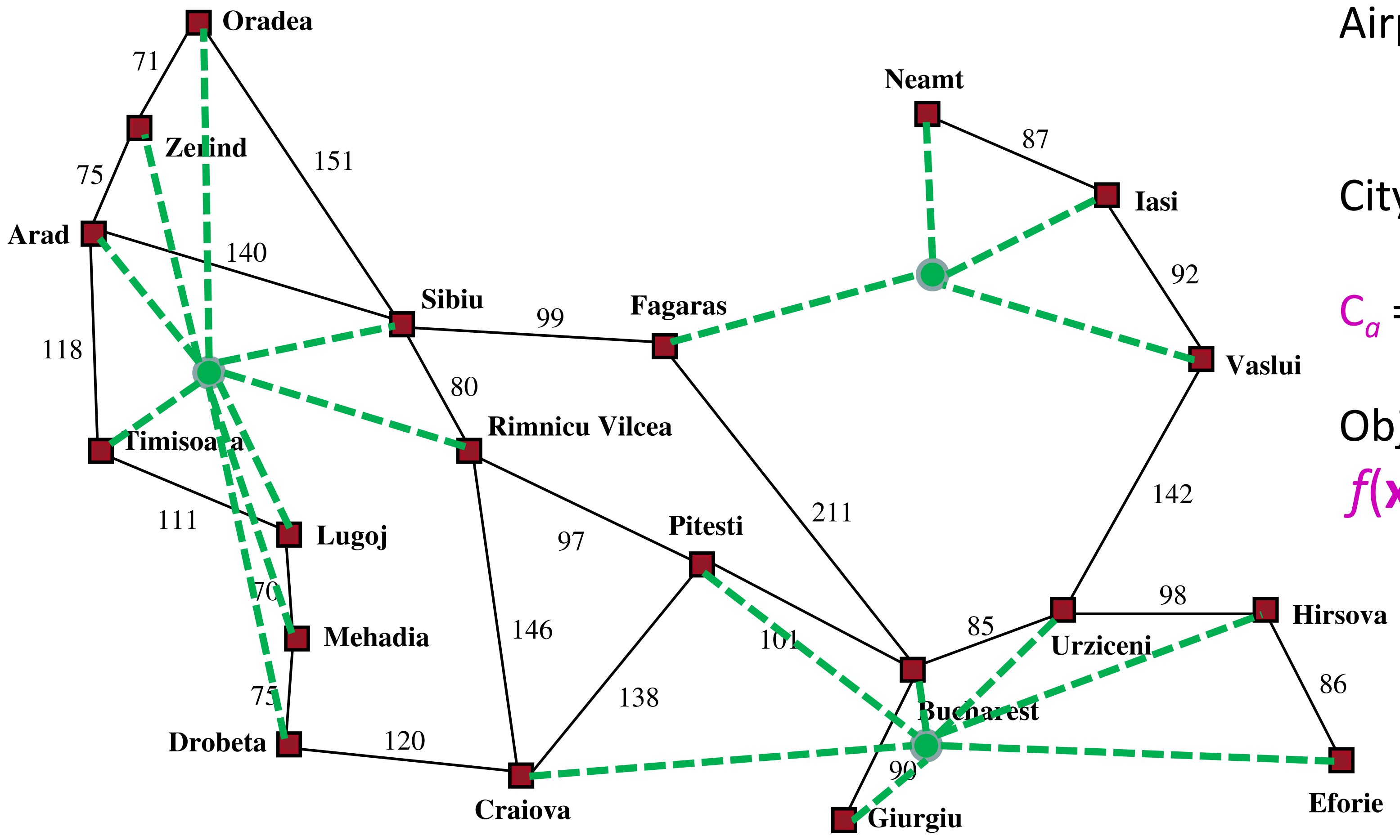


- Does crossover make sense here?
- What would mutation be?
- What would a good fitness function be?

Local search in continuous spaces

Example: Siting airports in Romania

Place 3 airports to minimize the sum of squared distances from each city to its nearest airport



Airport locations

$\mathbf{x} = (x_1,y_1), (x_2,y_2), (x_3,y_3)$

City locations (x_c,y_c)

C_a = cities closest to airport a

Objective: minimize

$f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$

Handling a continuous state/action space

1. Discretize it!
   - Define a grid with increment δ, use any of the discrete algorithms
2. Choose random perturbations to the state
   a. First-choice hill-climbing: keep trying until something improves the state
   b. Simulated annealing
3. Compute gradient of $f(\mathbf{x})$ analytically

Finding extrema in continuous space

- Gradient vector $\nabla f(\mathbf{x}) = (\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial x_2, \ldots)^T$
- For the airports, $f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$
- $\partial f/\partial x_1 = \Sigma_{c \in C_1} 2(x_1 - x_c)$
- At an extremum, $\nabla f(\mathbf{x}) = 0$
- Can sometimes solve in closed form: $x_1 = (\Sigma_{c \in C_1} x_c)/|C_1|$
- Is this a local or global minimum of f?
- Gradient descent: $\mathbf{x} \leftarrow \mathbf{x} - \alpha \nabla f(\mathbf{x})$
  - Huge range of algorithms for finding extrema using gradients

Summary

- Many configuration and optimization problems can be formulated as local search
- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches