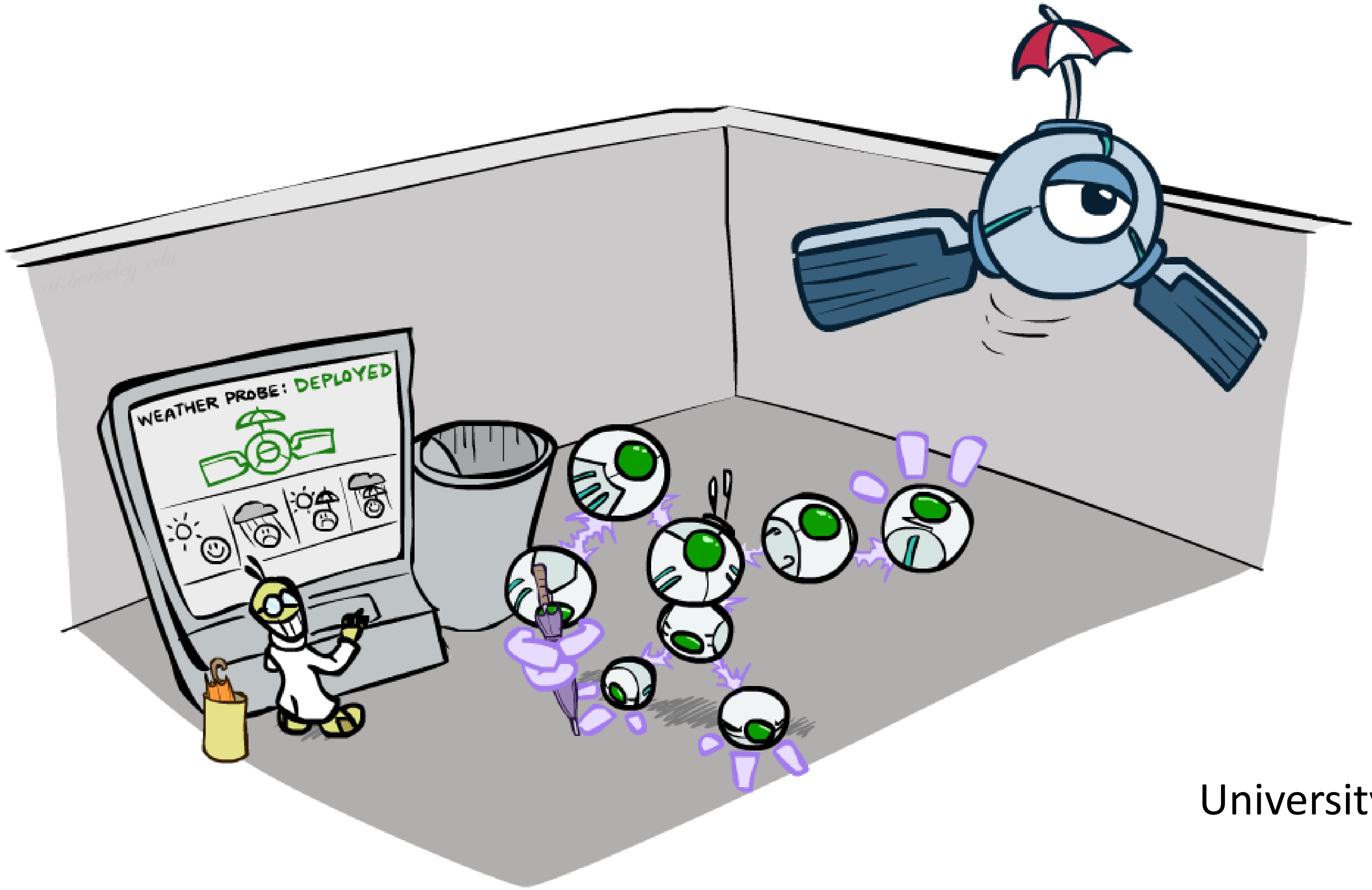
CS 188: Artificial Intelligence

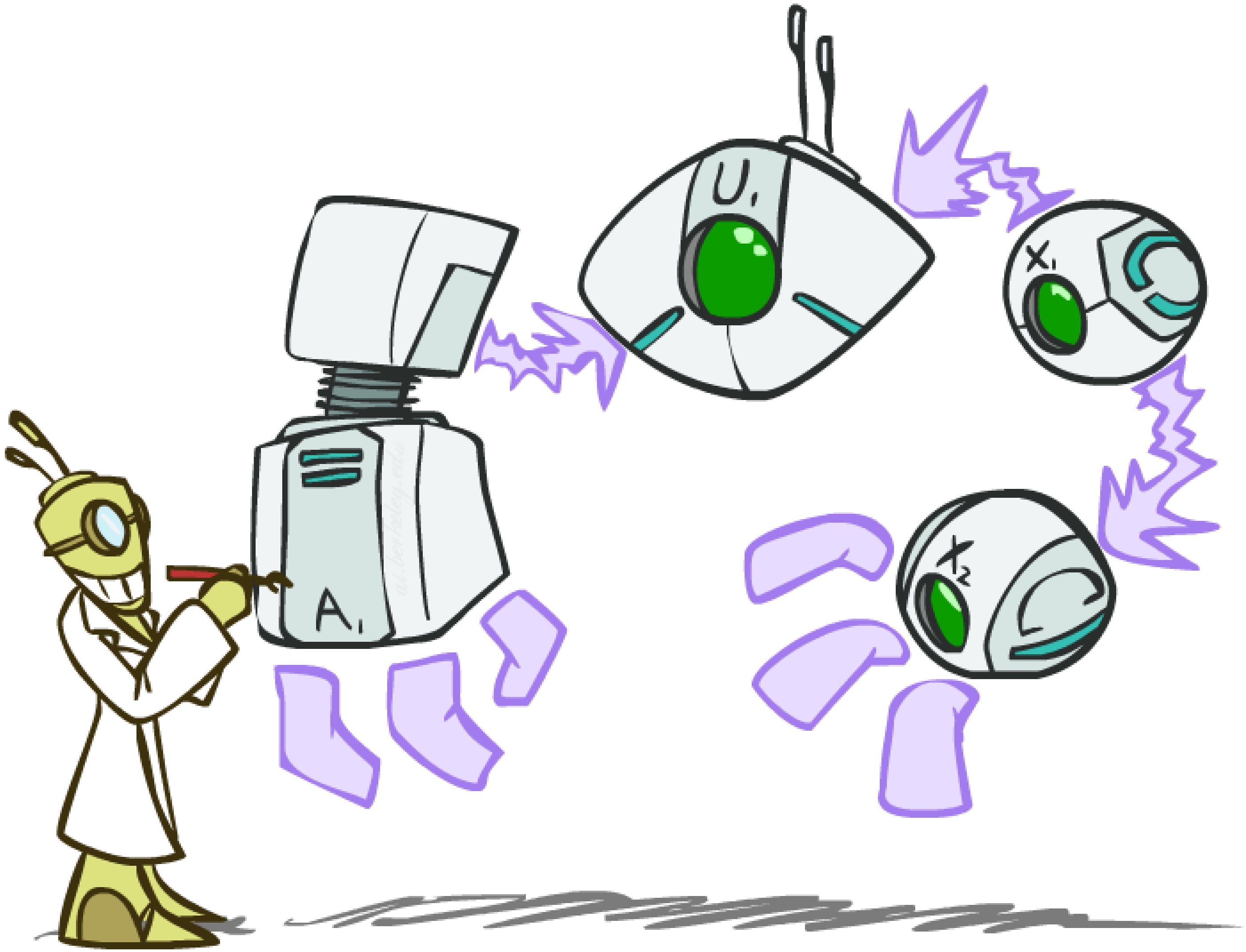
Decision Networks and Value of Information



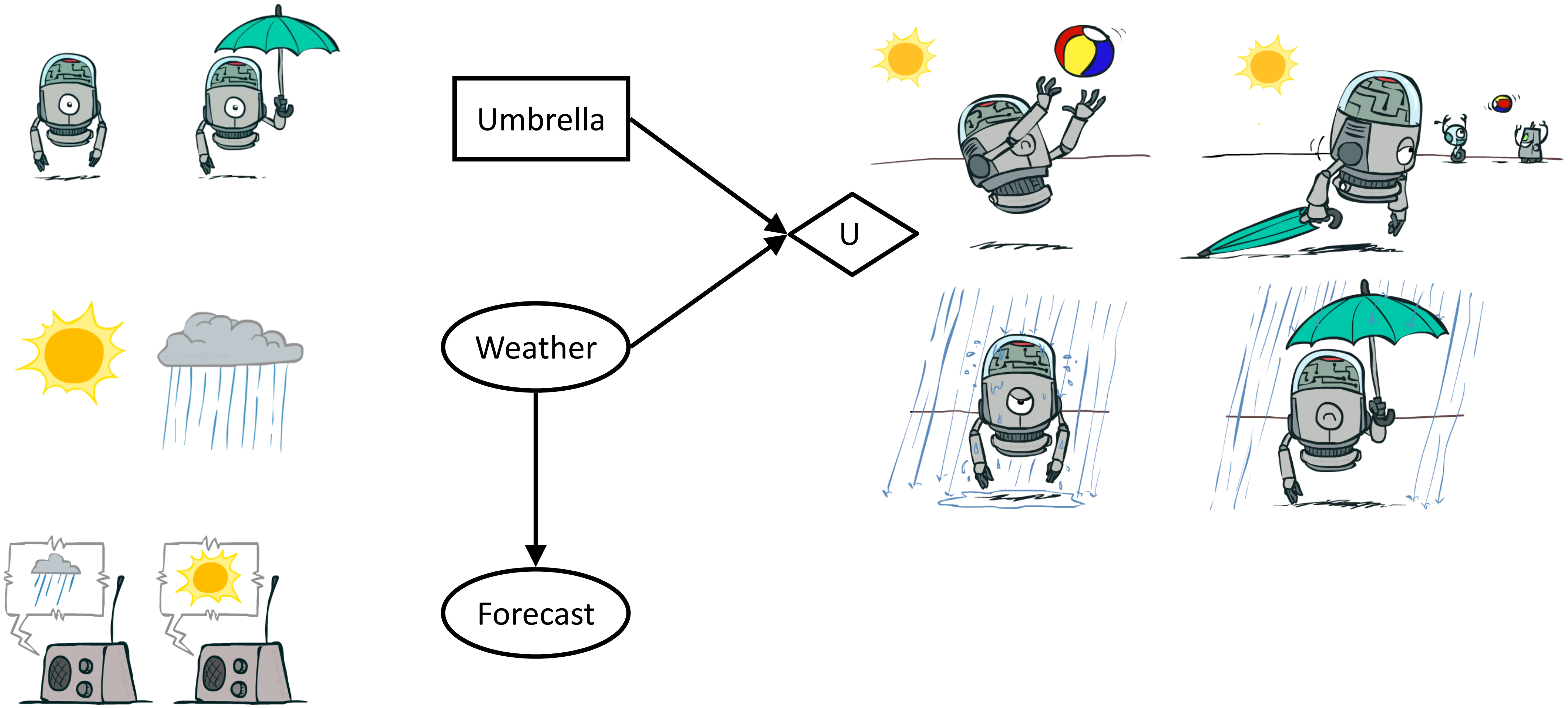
Spring 2023

University of California, Berkeley

Decision Networks



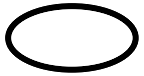
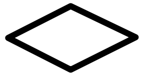
Decision Networks

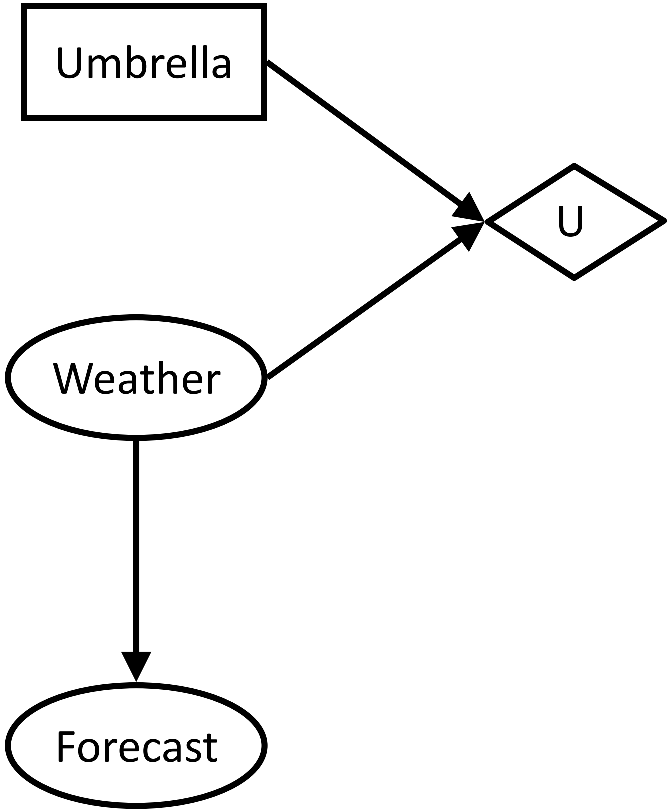


Decision Networks

- **MEU: choose the action which maximizes the expected utility given the evidence**

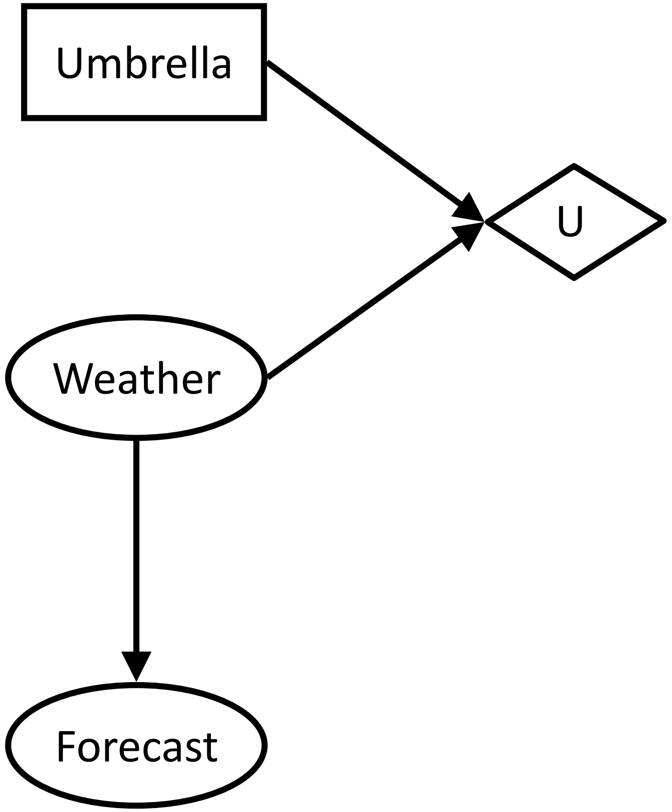
- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action

- New node types:
  - Chance nodes (just like BNs) 
  - Actions (rectangles, cannot have parents, act as observed evidence) 
  - Utility node (diamond, depends on action and chance nodes) 



Decision Networks

- Action selection
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_w P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

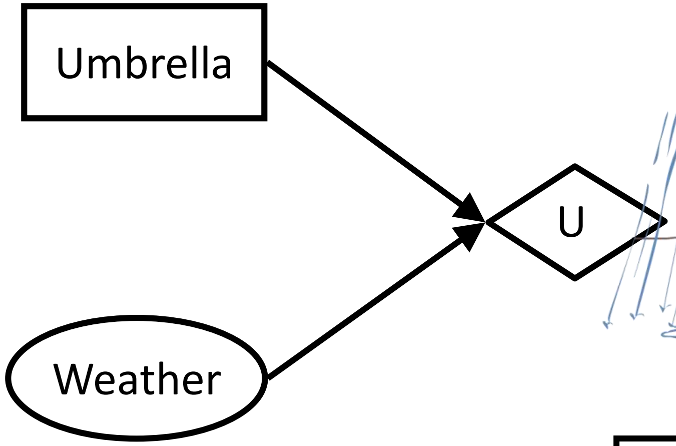
Umbrella = take

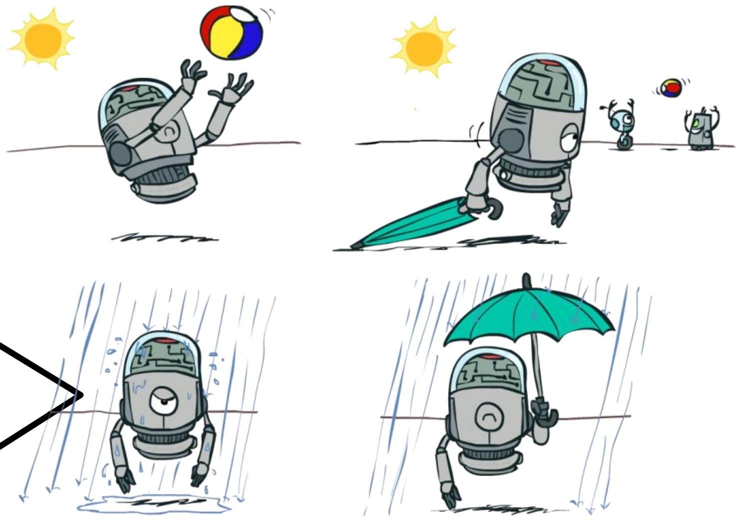
$$\mathrm{EU}(\mathrm{take}) = \sum_w P(w)U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\emptyset) = \max_a \mathrm{EU}(a) = 70$$





W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decision Networks: Notation

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_w P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

Umbrella = take

$$\mathrm{EU}(\mathrm{take}) = \sum_w P(w)U(\mathrm{take}, w)$$

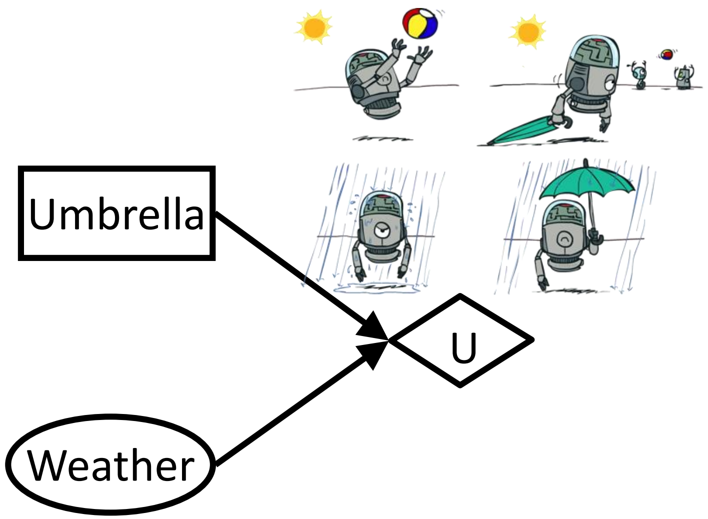
$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

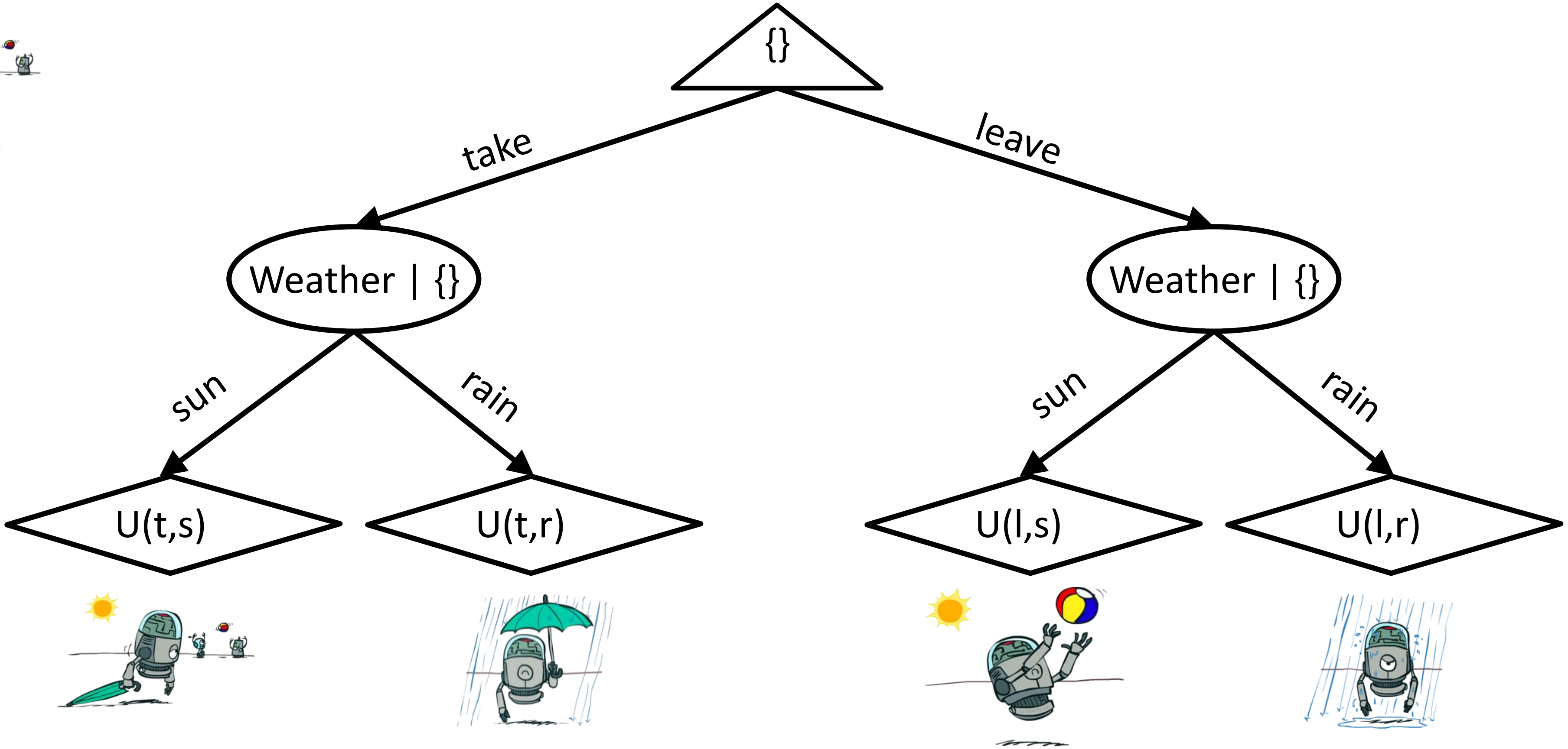
Optimal decision = leave

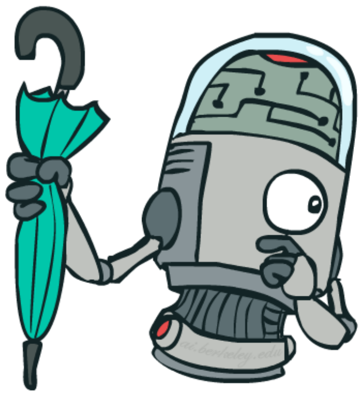
$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

- EU(leave) = Expected Utility of taking action leave
  - In the parentheses, we write an action
  - Calculating EU requires taking an expectation over chance node outcomes
- MEU(ø) = Maximum Expected Utility, given no information
  - In the parentheses, we write the evidence (which nodes we know)
  - Calculating MEU requires taking a maximum over several expectations (one EU per action)

Decisions as Outcome Trees







- Almost exactly like expectimax / MDPs
- What’s changed?

Example: Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

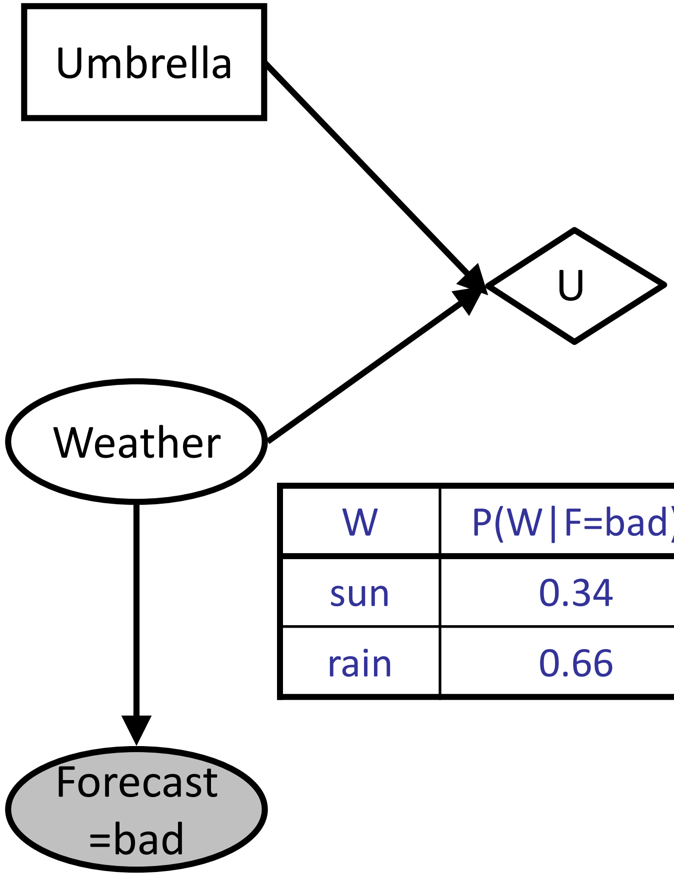
Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

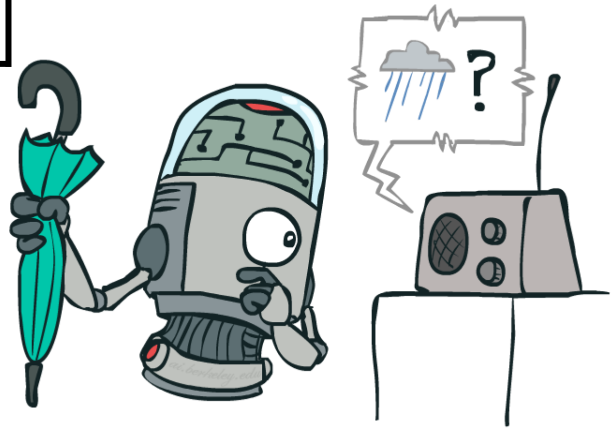
Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$



A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

W	P(W|F=bad)
sun	0.34
rain	0.66



Decision Networks: Notation

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

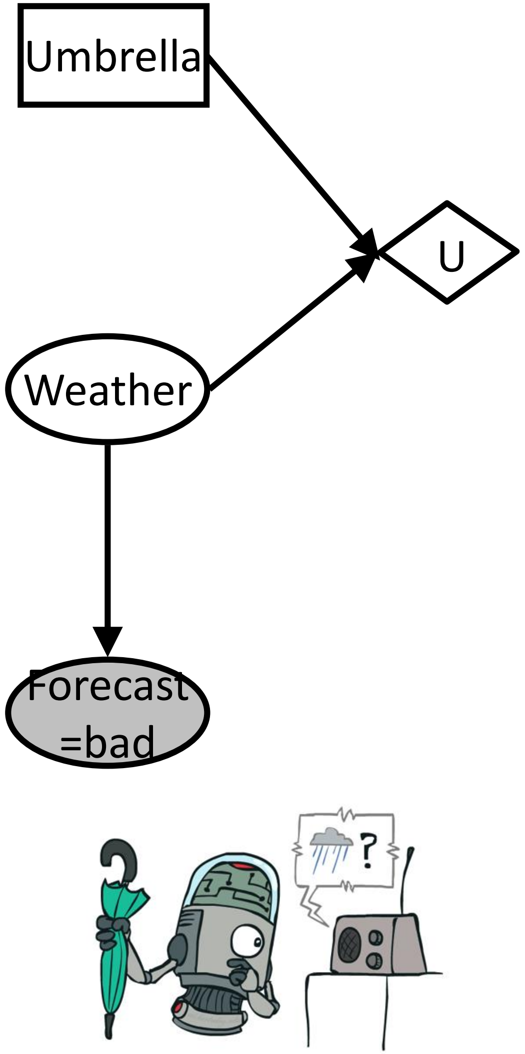
$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

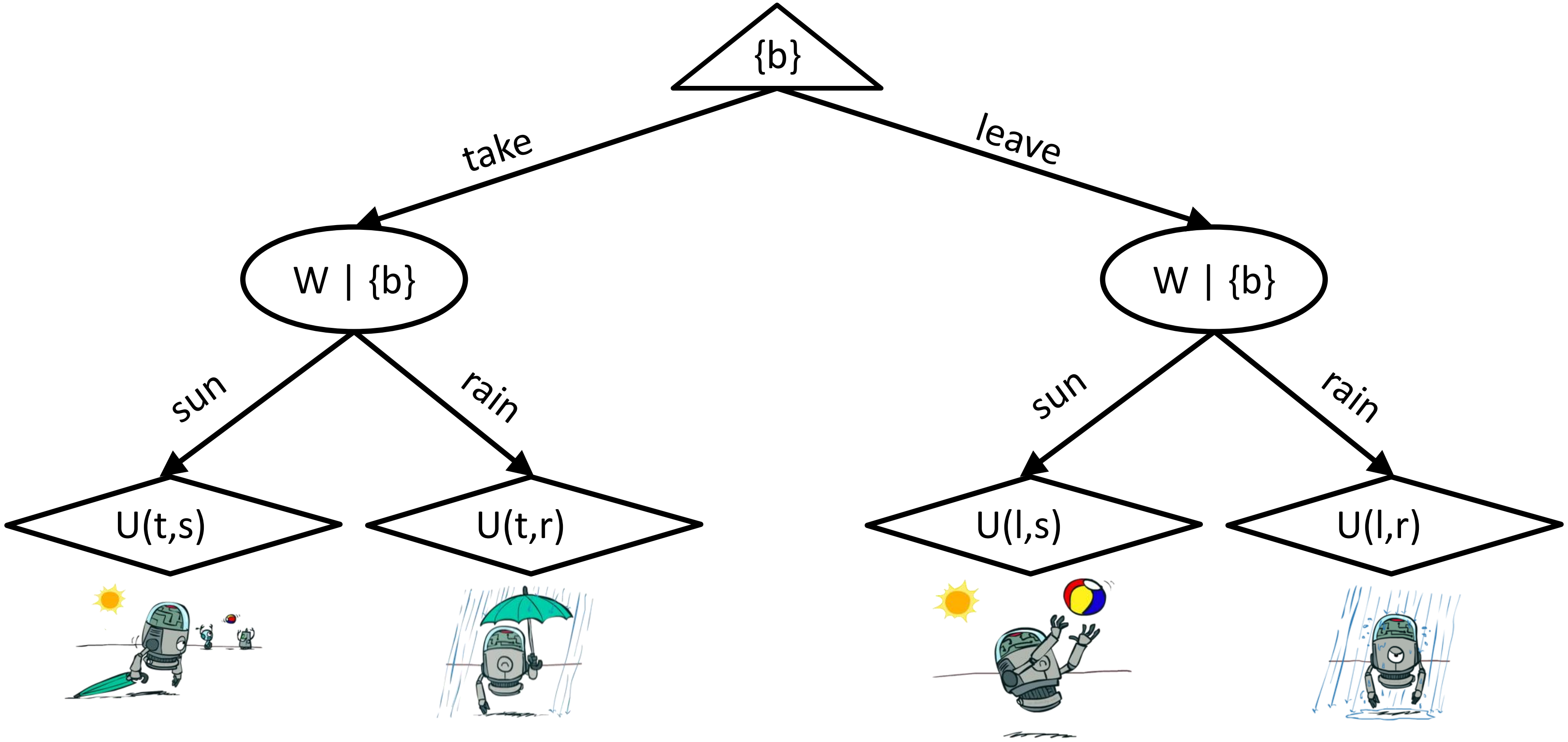
Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

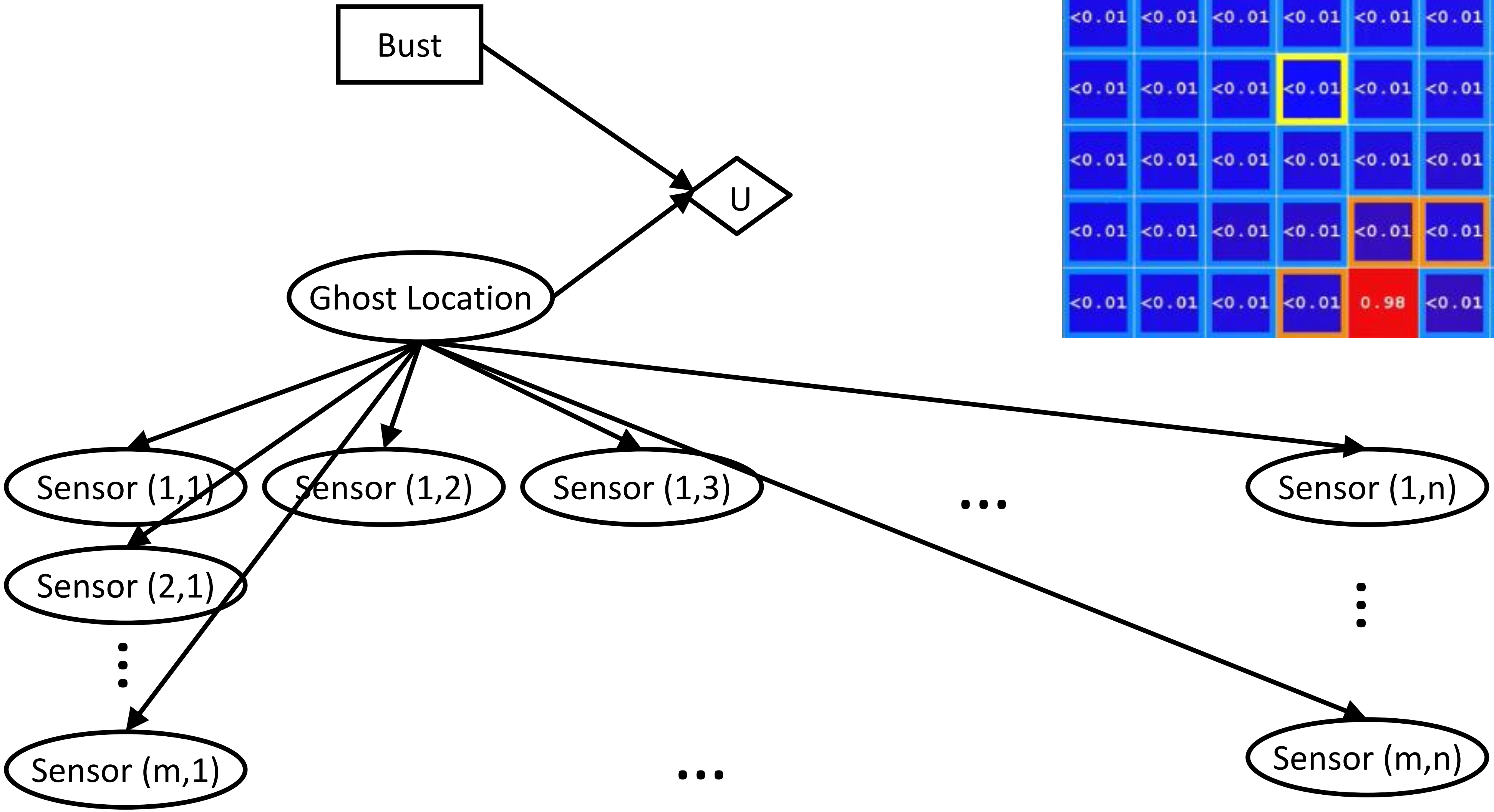
- EU(leave|bad) = Expected Utility of taking action leave, given you know the forecast is bad
  - Left side of conditioning bar: Action being taken
  - Right side of conditioning bar: The random variable(s) we know the value of (evidence)
- MEU(F=bad) = Maximum Expected Utility, given you know the forecast is bad
  - In the parentheses, we write the evidence (which nodes we know)

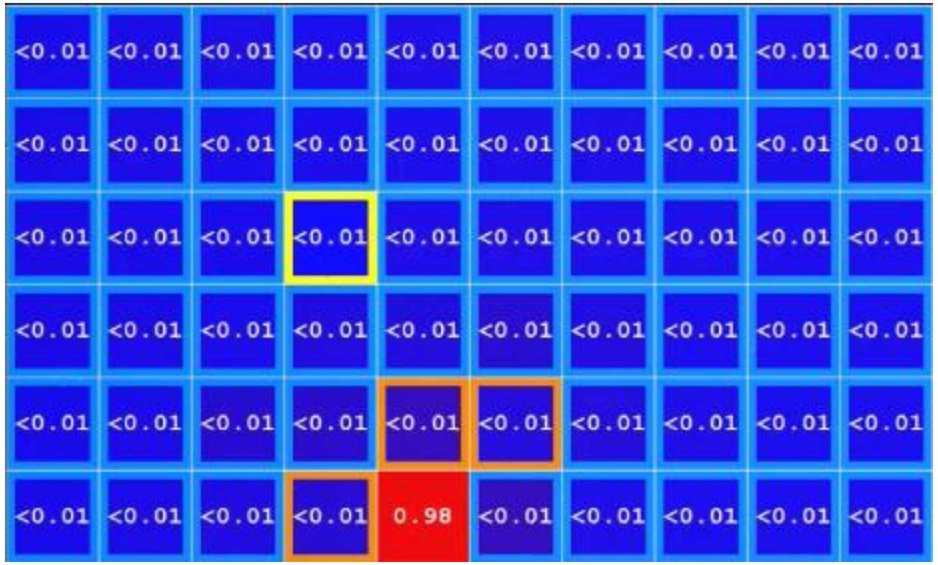
Decisions as Outcome Trees



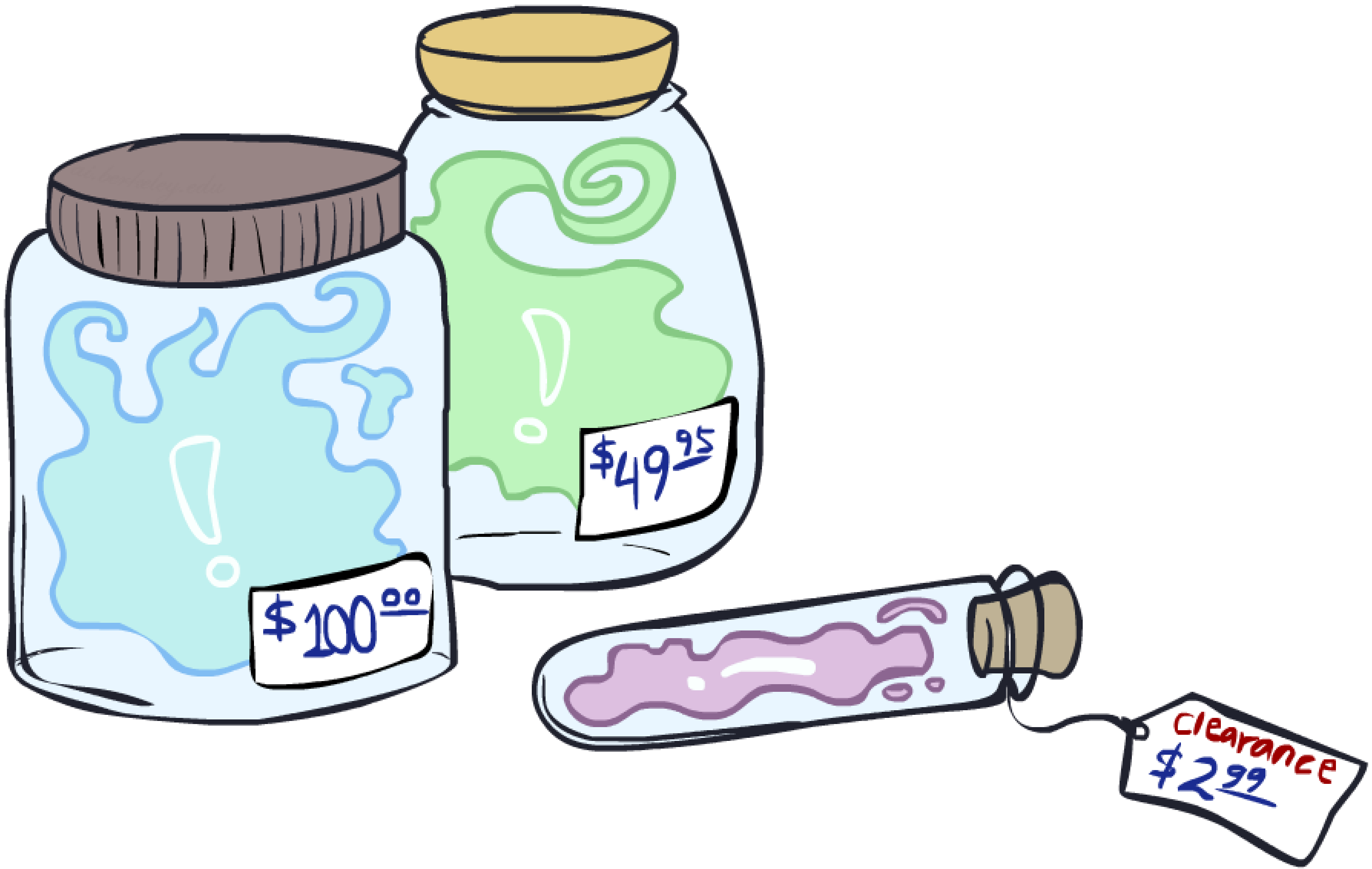


Ghostbusters Decision Network





Value of Information



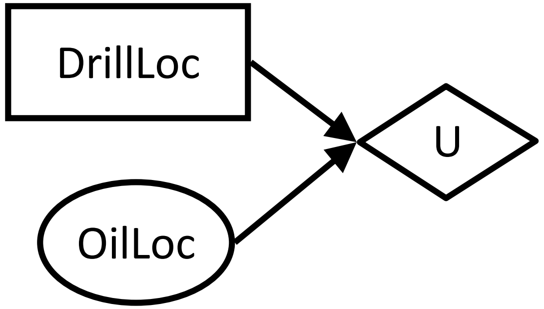
Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

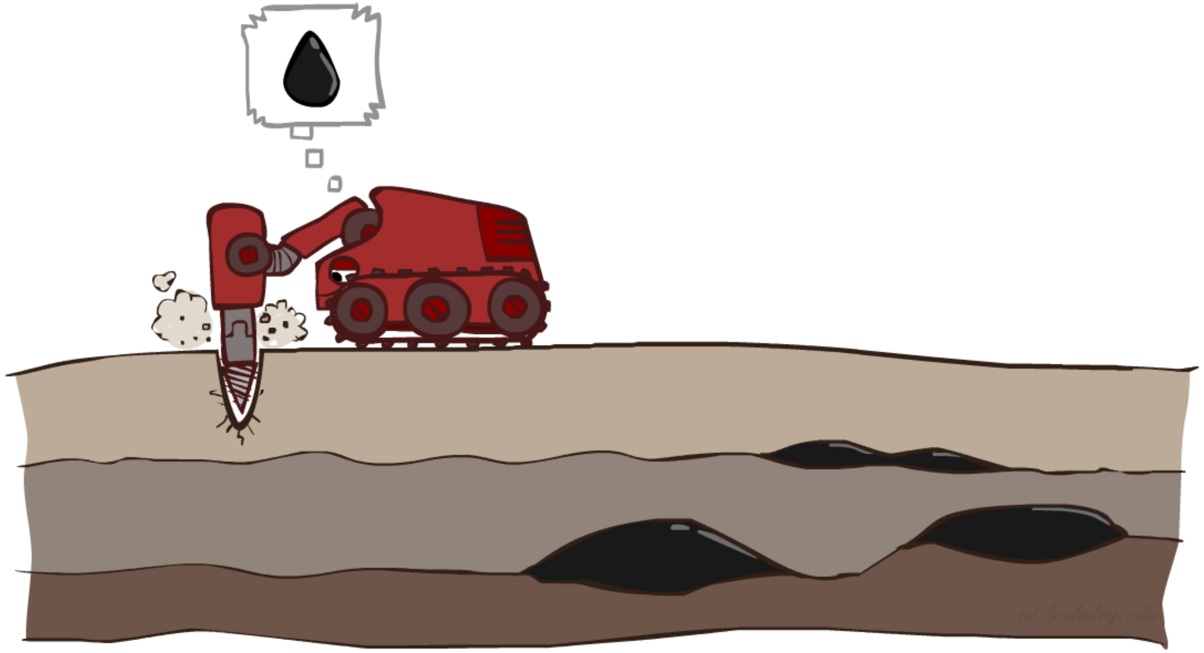
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b", prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2

O	P
a	1/2
b	1/2



D	O	U
a	a	k
a	b	0
b	a	0
b	b	k



VPI Example: Weather

MEU with no evidence

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

MEU if forecast is good

$$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

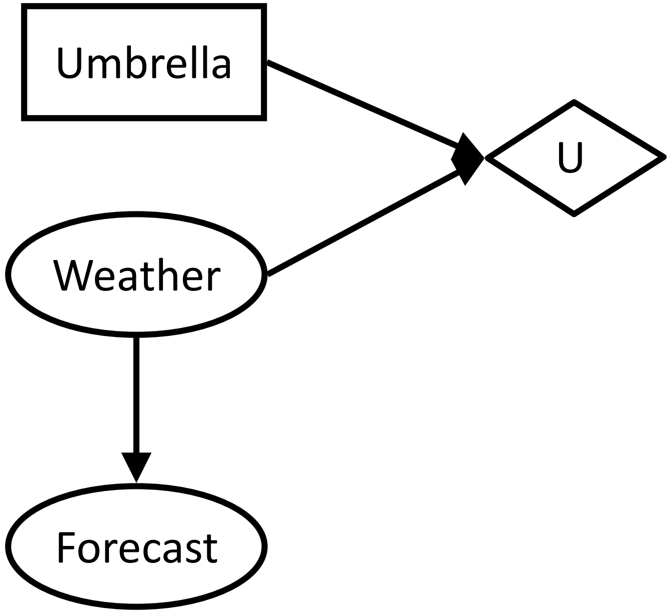
Forecast distribution

F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

$$\mathrm{VPI}(E'|e) = \left(\sum_{e'} P(e'|e) \mathrm{MEU}(e, e') \right) - \mathrm{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
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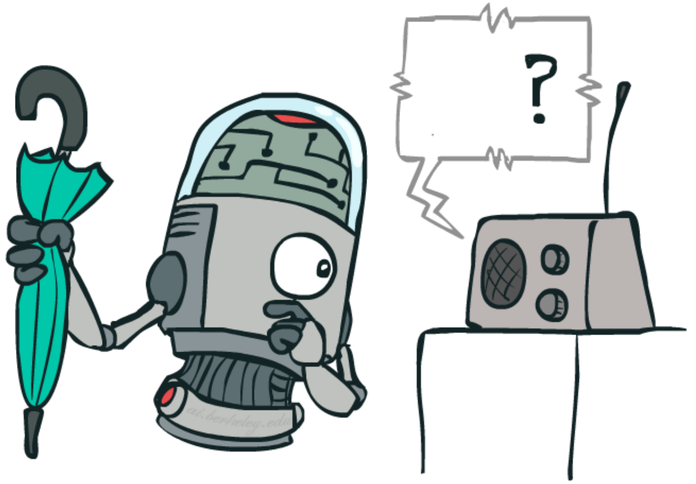


Value of Information

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- Assume we see that E' = e'. Value if we act then:

$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

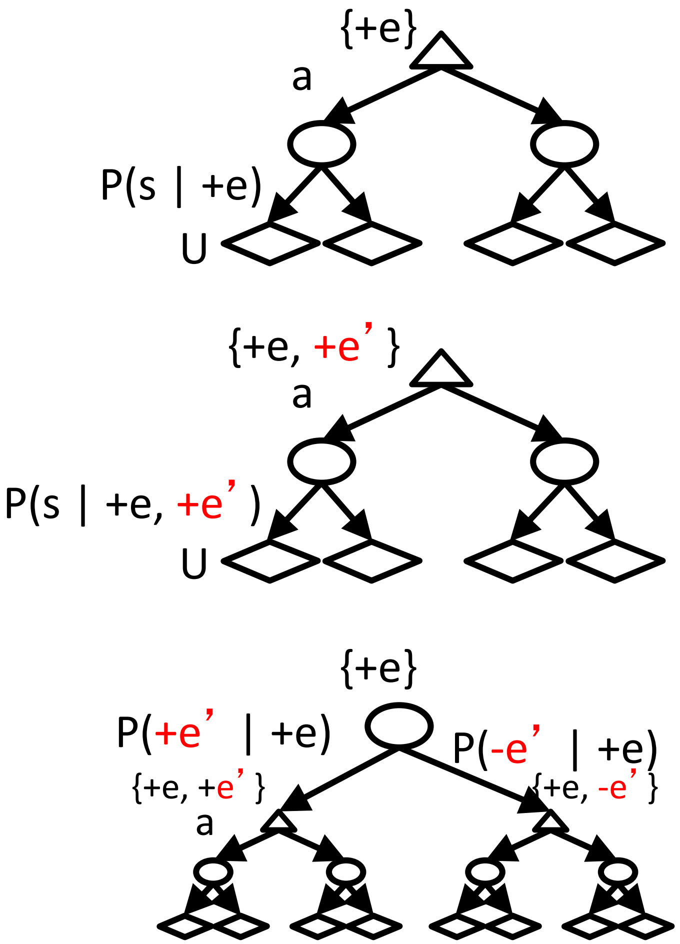
- BUT E' is a random variable whose value is unknown, so we don't know what e' will be

- Expected value if E' is revealed and then we act:

$$\text{MEU}(e,E') = \sum_{e'} P(e'|e)\text{MEU}(e,e')$$

- Value of information: how much MEU goes up by revealing E' first then acting, over acting now:

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$



VPI: Notation

- MEU(e) = Maximum Expected Utility, given evidence E=e
  - In the parentheses, we write the evidence (which nodes we know)
  - Calculating MEU requires taking a maximum over several expectations (one EU per action)
- VPI(E'|e) = Expected gain in utility for knowing the value of E', given that I know the value of e so far
  - Left side of conditioning bar: The random variable(s) we want to know the value of revealing
  - Right side of conditioning bar: The random variable(s) we already know the value of
  - Calculating VPI requires taking an expectation over several MEUs (one MEU per possible outcome of E', because we don't know the value of E')

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

$$\text{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\text{MEU}(e,e') \right) - \text{MEU}(e)$$

VPI: Computation Workflow

$$\text{MEU}(e, E') \quad - \text{MEU}(e) \quad = \text{VPI}(E'|e)$$

$$\text{MEU}(e, E') = \sum_{e'} P(e'|e)\text{MEU}(e, e')$$

Expectation over E' (we don’t know what the value of the new information will be)

$$\text{MEU}(e, e') = \max_a \text{EU}(a|e, e')$$

Maximum over actions (we control action, and take the best one)

$$\text{EU}(a|e, e') = \sum_x P(x|e, e')U(a, x)$$

Expectation over x (outcome of the chance nodes that affect utility)

$$\text{MEU}(e) = \max_a \text{EU}(a|e)$$

Maximum over actions (we control action, and take the best one)

$$\text{EU}(a|e) = \sum_x P(x|e)U(a, x)$$

Expectation over x (outcome of the chance nodes that affect utility)

VPI Properties

- Nonnegative

$$\forall E', e : \mathrm{VPI}(E'|e) \geq 0$$

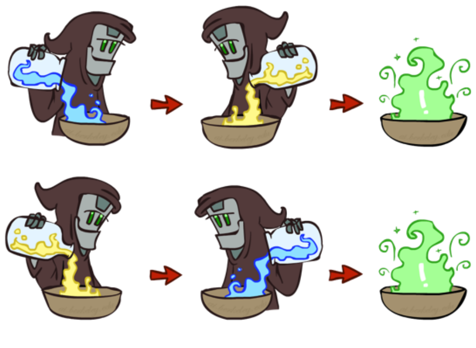
- Nonadditive

  (think of observing E_j twice)

$$\mathrm{VPI}(E_j, E_k|e) \neq \mathrm{VPI}(E_j|e) + \mathrm{VPI}(E_k|e)$$

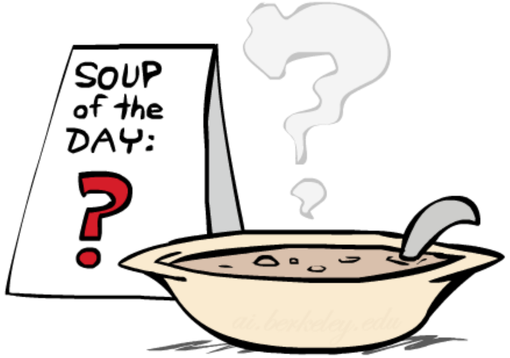
- Order-independent

$$\begin{aligned}\mathrm{VPI}(E_j, E_k|e) &= \mathrm{VPI}(E_j|e) + \mathrm{VPI}(E_k|e, E_j) \\ &= \mathrm{VPI}(E_k|e) + \mathrm{VPI}(E_j|e, E_k)\end{aligned}$$

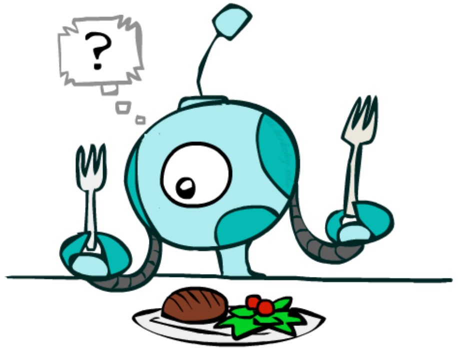


Quick VPI Questions

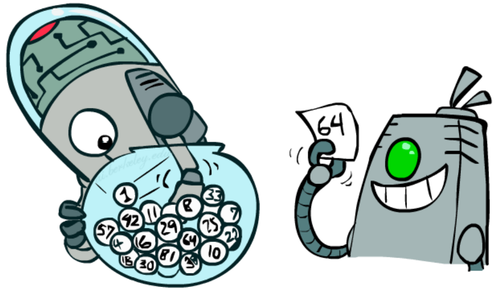
- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?



- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?



- You're playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?

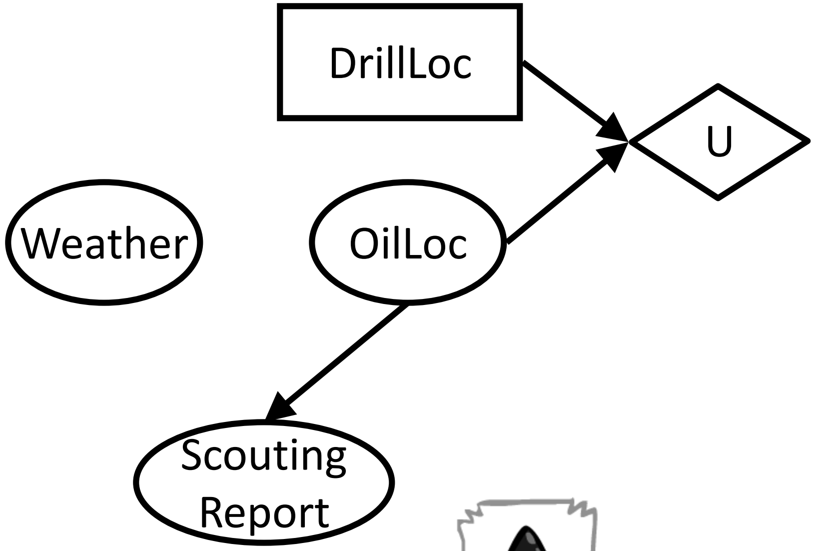


Value of Imperfect Information?

- No such thing (as we formulate it)
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

VPI Question

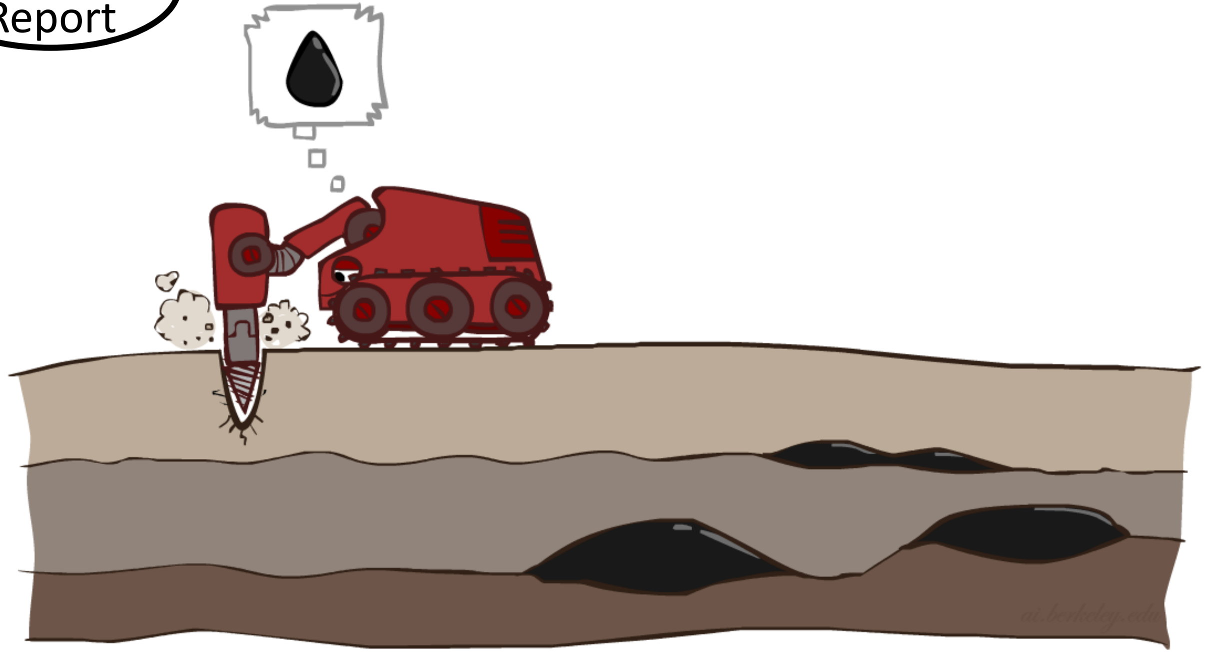
- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Weather) ?
- VPI(Weather | ScoutingReport) ?



- Generally:

  If $\quad$ Parents(U) $\perp\!\!\!\perp$ Z | CurrentEvidence

  Then $\quad$ VPI(Z | CurrentEvidence) = 0



Next Time: Machine Learning