CS 188: Artificial Intelligence

Decision Networks and Value of Information

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[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]
Decision Networks
Decision Networks

- **MEU:** choose the action which maximizes the expected utility given the evidence

- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action

- New node types:
  - Chance nodes (just like BNs)
  - Actions (rectangles, cannot have parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)
Decision Networks

- **Action selection**
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action
Decision Networks

Umbrella = leave

\[ EU(\text{leave}) = \sum_w P(w)U(\text{leave}, w) \]
\[ = 0.7 \cdot 100 + 0.3 \cdot 0 = 70 \]

Umbrella = take

\[ EU(\text{take}) = \sum_w P(w)U(\text{take}, w) \]
\[ = 0.7 \cdot 20 + 0.3 \cdot 70 = 35 \]

Optimal decision = leave

\[ \text{MEU}(\emptyset) = \max_a EU(a) = 70 \]
Decision Networks: Notation

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\[ \text{MEU}(\emptyset) = \max_a EU(a) = 70 \]

- \( EU(\text{leave}) = \) Expected Utility of taking action leave
  - In the parentheses, we write an action
  - Calculating EU requires taking an expectation over chance node outcomes

- \( \text{MEU}(\emptyset) = \) Maximum Expected Utility, given no information
  - In the parentheses, we write the evidence (which nodes we know)
  - Calculating MEU requires taking a maximum over several expectations (one EU per action)
Decisions as Outcome Trees

- Almost exactly like expectimax / MDPs
- What’s changed?
Example: Decision Networks

Umbrella = leave

\[ EU(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{leave}, w) \]
\[ = 0.34 \cdot 100 + 0.66 \cdot 0 = 34 \]

Umbrella = take

\[ EU(\text{take}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{take}, w) \]
\[ = 0.34 \cdot 20 + 0.66 \cdot 70 = 53 \]

Optimal decision = take

\[ \text{MEU}(F = \text{bad}) = \max_a EU(a|\text{bad}) = 53 \]
Decision Networks: Notation

- Umbrella = leave
  \[ EU(\text{leave}|\text{bad}) = \sum_{w} P(w|\text{bad})U(\text{leave}, w) \]
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- Umbrella = take
  \[ EU(\text{take}|\text{bad}) = \sum_{w} P(w|\text{bad})U(\text{take}, w) \]
  \[ = 0.34 \cdot 20 + 0.66 \cdot 70 = 53 \]

**Optimal decision = take**

\[ \text{MEU}(F = \text{bad}) = \max_{a} EU(a|\text{bad}) = 53 \]

- EU(leave|bad) = Expected Utility of taking action leave, given you know the forecast is bad
  - Left side of conditioning bar: Action being taken
  - Right side of conditioning bar: The random variable(s) we know the value of (evidence)

- MEU(F=bad) = Maximum Expected Utility, given you know the forecast is bad
  - In the parentheses, we write the evidence (which nodes we know)
Decisions as Outcome Trees

Weather Forecast = bad

Umbrella

- Take
  - Sun: U(t,s)
  - Rain: U(t,r)

- Leave
  - Sun: U(l,s)
  - Rain: U(l,r)
Ghostbusters Decision Network

Ghost Location

Bust

Sensor (1,1) → Sensor (1,2) → Sensor (1,3) → ... → Sensor (1,n)

Sensor (2,1) → ... → Sensor (m,1) → ... → Sensor (m,n)
Value of Information
**Value of Information**

- **Idea:** compute value of acquiring evidence
  - Can be done directly from decision network

- **Example:** buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth \( k \)
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = \( k/2 \), MEU = \( k/2 \)

- **Question:** what’s the value of information of \( O \)?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say “oil in a” or “oil in b”, prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - \( VPI(\text{OilLoc}) = k/2 \)
  - Fair price of information: \( k/2 \)
MEU with no evidence

\[
\text{MEU}(\varnothing) = \max_a \text{EU}(a) = 70
\]

MEU if forecast is bad

\[
\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53
\]

MEU if forecast is good

\[
\text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95
\]

Forecast distribution

\[
\begin{array}{c|c}
F & P(F) \\
\hline
\text{good} & 0.59 \\
\text{bad} & 0.41 \\
\end{array}
\]

\[
0.59 \cdot (95) + 0.41 \cdot (53) - 70 = 77.8 - 70 = 7.8
\]

\[
\text{VPI}(E'|e) = \left( \sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e)
\]
Value of Information

- Assume we have evidence $E = e$. Value if we act now:
  \[ MEU(e) = \max_a \sum_s P(s|e) U(s, a) \]

- Assume we see that $E' = e'$. Value if we act then:
  \[ MEU(e, e') = \max_a \sum_s P(s|e, e') U(s, a) \]

- BUT $E'$ is a random variable whose value is unknown, so we don’t know what $e'$ will be

- Expected value if $E'$ is revealed and then we act:
  \[ MEU(e, E') = \sum_{e'} P(e'|e) MEU(e, e') \]

- Value of information: how much MEU goes up by revealing $E'$ first then acting, over acting now:
  \[ VPI(E'|e) = MEU(e, E') - MEU(e) \]
VPI: Notation

- **MEU(e)** = Maximum Expected Utility, given evidence E=e
  - In the parentheses, we write the evidence (which nodes we know)
  - Calculating MEU requires taking a maximum over several expectations (one EU per action)
- **VPI(E'|e)** = Expected gain in utility for knowing the value of E', given that I know the value of e so far
  - Left side of conditioning bar: The random variable(s) we want to know the value of revealing
  - Right side of conditioning bar: The random variable(s) we already know the value of
  - Calculating VPI requires taking an expectation over several MEUs (one MEU per possible outcome of E', because we don’t know the value of E')

\[
\begin{align*}
\text{MEU}(e) &= \max_a \sum_s P(s|e) \ U(s, a) \\
\text{MEU}(e, e') &= \max_a \sum_s P(s|e, e') \ U(s, a) \\
\text{VPI}(E'|e) &= \left( \sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e)
\end{align*}
\]
VPI: Computation Workflow

\[
MEU(e, E') = \sum_{e'} P(e'|e) MEU(e, e')
\]

Expectation over \( E' \) (we don’t know what the value of the new information will be)

\[
MEU(e, e') = \max_a EU(a|e, e')
\]

Maximum over actions (we control action, and take the best one)

\[
EU(a|e, e') = \sum_x P(x|e, e') U(a, x)
\]

Expectation over \( x \) (outcome of the chance nodes that affect utility)

\[
-MEU(e) = \max_a EU(a|e)
\]

Maximum over actions (we control action, and take the best one)

\[
EU(a|e) = \sum_x P(x|e) U(a, x)
\]

Expectation over \( x \) (outcome of the chance nodes that affect utility)

\[
= VPI(E'|e)
\]
VPI Properties

- **Nonnegative**
  \[ \forall E', e : \text{VPI}(E'|e) \geq 0 \]

- **Nonadditive**
  (think of observing $E_j$ twice)
  \[ \text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e) \]

- **Order-independent**
  \[ \text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \]
  \[ = \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k) \]
Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn’t order either one. What’s the value of knowing which it is?

- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What’s the value of knowing which?

- You’re playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?
Value of Imperfect Information?

- No such thing (as we formulate it)
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one
VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Weather) ?
- VPI(Weather | ScoutingReport) ?

Generally:
If $\text{Parents}(U) \perp\!\!\!\!\!\perp Z \mid \text{CurrentEvidence}$
Then $\text{VPI}(Z \mid \text{CurrentEvidence}) = 0$
Next Time: Machine Learning