CS 188 Spring 2024 — Introduction to Artificial Intelligence — Exam Prep 4 Solutions

Q1. First Order Logic

Consider a vocabulary with the following symbols:

- $Occuption(p, o)$: Predicate. Person p has occuption o.
- $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(i) Emily is either a surgeon or a lawyer.

$O(E, S) \vee O(E, L)$

(ii) Joe is an actor, but he also holds another job.

$O(J, A) \wedge \exists o\ o \neq A \wedge O(J, o)$

(iii) All surgeons are doctors.

$\forall p\ O(p, S) \Rightarrow O(p, D)$

(iv) Joe does not have a lawyer (i.e., is not a customer of any lawyer).

$\neg \exists p\ C(J, p) \wedge O(p, L)$

(v) Emily has a boss who is a lawyer.

$\exists p\ B(p, E) \wedge O(p, L)$

(vi) There exists a lawyer all of whose customers are doctors.

$\exists\ pO(p, L) \wedge \forall\ qC(q, p) \Rightarrow O(q, D)$

(vii) Every surgeon has a lawyer.

$\forall p\ O(p, S) \Rightarrow \exists\ qO(q, L) \wedge C(p, q)$

Q2. Logic

(a) Prove, or find a counterexample to, each of the following assertions:

(i) If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$

True. This follows from monotonicity.

(ii) If $(\alpha \wedge \beta) \models \gamma$ then $\alpha \models \gamma$ or $\beta \models \gamma$ (or both).

False. Consider Consider $\alpha \equiv A$, $\beta \equiv B$, $\gamma \equiv (A \wedge B)$.

(iii) If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

False. Consider $\beta \equiv A$, $\gamma \equiv \neg A$.

(b) Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(i) $Smoke \implies Smoke$

Valid

(ii) $Smoke \implies Fire$

Neither

(iii) $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

Neither

(iv) $Smoke \vee Fire \vee \neg Fire$

Valid

(v) $((Smoke \wedge Heat) \implies Fire) \Longleftrightarrow ((Smoke \implies Fire) \vee (Heat \implies Fire))$

Valid

(vi) $(Smoke \implies Fire) \implies ((Smoke \wedge Heat) \implies Fire)$

Valid

(vii) $Big \vee Dumb \vee (Big \implies Dumb)$

Valid

(c) Suppose an agent inhabits a world with two states, S and $\neg S$, and can do exactly one of two actions, a and b. Action a does nothing and action b flips from one state to the other. Let S^t be the proposition that the agent is in state S at time t, and let a^t be the proposition that the agent does action a at time t (similarly for b^t).

(i) Write a successor-state axiom for S^{t+1}.

$S^{t+1} \Longleftrightarrow [(S^t \wedge a^t) \vee (\neg S^t \wedge b^t)]$.

(ii) Convert the sentence in the previous part into CNF.

Because the agent can do exactly one action, we know that $b^t \equiv \neg a^t$ so we replace b^t throughout. We obtain four clauses:
1: $(\neg S^{t+1} \vee S^t \vee \neg a^t)$
2: $(\neg S^{t+1} \vee \neg S^t \vee a^t)$
3: $(S^{t+1} \vee \neg S^t \vee \neg a^t)$
4: $(S^{t+1} \vee S^t \vee a^t)$