CS 188 Spring 2024 — Introduction to Artificial Intelligence — HW1B

- **Due:** Tuesday 1/30 at 11:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	
Last name	
SID	
Collaborators	

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Q1. [20 pts] Search

It is training day for Pacbabies, also known as Hungry Running Maze Games day. Each of k Pacbabies starts in its own assigned start location s_i in a large maze of size $M \times N$ and must return to its own Pacdad who is waiting patiently but proudly at g_i along the way, the Pacbabies must, between them, eat all the dots in the maze.

At each step, all k Pacbabies move one unit to any open adjacent square. The only legal actions are Up, Down, Left, or Right. It is illegal for a Pacbaby to wait in a square, attempt to move into a wall, or attempt to occupy the same square as another Pacbaby. To set a record, the Pacbabies must find an optimal collective solution.

1.1) (3 pts) Define a minimal state space representation for this problem.

The minimal state space is defined by the current locations of k Pacbabies and, for each square of the grid, a Boolean variable that indicates whether there is food there or not.

Note that the Pacdad locations are constant, so we don't need to keep track of different configurations of Pacdad locations in the state space. (For example, we could hard-code the Pacdad locations into our goal test.)

1.2) (2 pts) How large is the state space?

Given the minimal state representation defined above an upper bound on the size of the state space is $(MN)^k \cdot 2^{MN}$. The first part is $(MN)^k$ as the pacbabies can move to any state in the state-space. The second term 2^{MN} accounts for all the possible food configurations on the grid. You could also point out that given that two pacbabies cannot be on the same place at the same time the first term is $(MN) * (MN-1) * \cdots (MN-(k-1))$. Both approaches are considered correct.

1.3) (2 pts) What is the maximum branching factor for this problem?

A) 4^k

B) 8^k

C) $4^k 2^{MN}$

D) $4^k 2^4$

For each distinct action of a pacbaby we will end up in a possibly different child node. Given that we have k pacbabies then the answer is 4^k as each of the k Pacbabies has a choice of 4 actions.

1.4) (8 pts) Let $MH(p, q)$ be the Manhattan distance between positions p and q and F be the set of all positions of remaining food pellets and p_i be the current position of Pacbaby i. For the following six heuristics, state whether they are admissible or not and briefly justify.

A) $\frac{\sum_{i=1}^{k} MH(p_i, g_i)}{k}$

B) $\max_{1 \le i \le k} MH(p_i, g_i)$

C) $\max_{1 \le i \le k}[\max_{f \in F} MH(p_i, f)]$

D) $\max_{1 \le i \le k}[\min_{f \in F} MH(p_i, f)]$

E) $\min_{1 \le i \le k}[\min_{f \in F} MH(p_i, f)]$

F) $\min_{f \in F}[\max_{1 \le i \le k} MH(p_i, f)]$

- A) is admissible because to solve the problem, the furthest Pacbaby must at least reach its Pacdad. This requires the furthest Pacbaby to at least travel the Manhattan distance to its Pacdad (and possibly further if there are walls or food dots to eat). Therefore, the furthest distance between any Pacbaby and its Pacdad is admissible. The average distance between Pacbaby and Pacdad is less than the furthest distance between Pacbaby and Pacdad, which we've reasoned is admissible.

- B) is admissible because this expression represents the furthest distance between any Pacbaby and its Pacdad. As explained in the previous subpart, this is admissible.
- C) is inadmissible because it looks at the distance from each Pacbaby to its most distant food square and in the optimal solution we might have another Pacbaby, that is closer, going to that square so this heuristic is inadmissible.
- D) same logic as C).
- E) represents the minimum cost it would take for one food pellet to be eaten by any Pacbaby. To solve the problem, at least one Pacbaby needs to travel to a food pellet to eat it.

  Another way to explain admissibility of this heuristic is to consider a relaxed problem where only a single food pellet needs to be eaten (not all of them), and the Pacbabies don't need to reach their Pacdads. This expression represents the cost to solve this relaxed problem.
- F) is inadmissible because it connects each food square to the most distant Pacbaby, which may not be the one who eats it.

1.5) (2 pts) You'd like to choose two heuristic functions f, g from the heuristics above, such that their maximum, $h(n) = \max(f(n), g(n))$, is an admissible heuristic.

What is a sufficient condition on f and/or g for $h(n)$ to be admissible?

f and g are both admissible. As seen in lecture, if two heuristics are both admissible, then the pointwise maximum of those heuristics is also admissible.

1.6) (3 pts) Now, you'd like to choose two heuristic functions f, g from the heuristics above, such that,

$$h(n) = \alpha f(n) + (1 - \alpha)g(n)$$

is an admissible heuristic for any value a between 0 and 1.

Which is a sufficient condition for $h(n)$ to be admissible? Briefly justify.

A) Any f and g is sufficient

B) At least one of f and g is admissible

C) Both f and g are admissible

D) $h(n)$ is admissible for $a = 0.5$

E) $h(n)$ is admissible for $a = 0$

C)

Intuitively, $h(n)$ is a weighted average between $f(n)$ and $g(n)$, which means that $h(n)$ will fall between $f(n)$ and $g(n)$. If $f(n)$ and $g(n)$ are both admissible, then a heuristic that falls between f and g will also be admissible.

Another way to justify this condition is to note that $h(n) \leq max(f(n), g(n))$, so the condition from the previous part must also be sufficient here.