CS 188 Spring 2024 | Introduction to Artificial Intelligence | HW6B Solutions

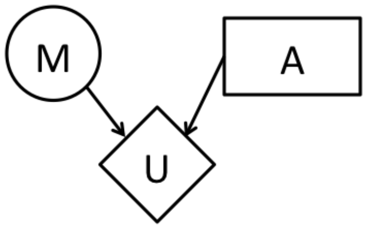
- **Due:** Friday 11/4 at 11:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

| | |
|---|---|
| First name | |
| Last name | |
| SID | |
| Collaborators | |

**For staff use only:**

# Q6. [29 pts] Decision Networks and HMMs

Assume that you want to watch a film $M$ that can either be great $+m$ or pretty bad $-m$. You can either watch the film in a theater or at home by renting it. This is controlled by your choice $A$. Consider the following decision network and tables:



| M | P(M) |
|---|---|
| +m | 0.5 |
| -m | 0.5 |

| M | A | U(M,A) |
|---|---|---|
| +m | *theater* | 100 |
| -m | *theater* | 10 |
| +m | *rent* | 80 |
| -m | *rent* | 40 |

Figure 1: Decision network and tables.

**6.1)** (5 pts) Compute the following quantities. $EU$ and $MEU$ stand for expected and maximum expected utility respectively.

$EU(theater) = P(+m)U(+m, theater) + P(-m)U(-m, theater) = 0.5 \cdot 100 + 0.5 \cdot 10 = 55$

$EU(rent) = P(+m)U(+m, rent) + P(-m)U(-m, rent) = 0.5 \cdot 80 + 0.5 \cdot 40 = 60$

$MEU(\emptyset) = 60$

$\text{argmax}_A EU(A) = rent$

**6.2)** (5 pts) You would like obtain more information about whether the film is good or not. For that, we introduce another variable $F$ which designates the "fullness" (how sold-out the tickets are) in the theaters. This variable is affected by another variable $S$ which designates possible Covid-19 restrictions. The prior of $M$ and the utilities are the same as before. Assuming that both $F$ and $S$ are binary, consider the following network and tables:

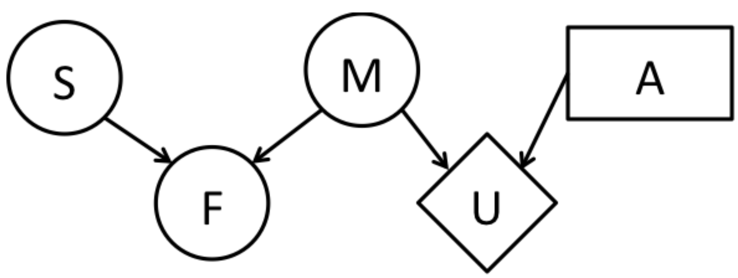


Figure 2: Decision network.

| S | M | F | $P(F\|S,M)$ |
|---|---|---|---|
| +s | +m | +f | 0.6 |
| +s | +m | -f | 0.4 |
| +s | -m | +f | 0.0 |
| +s | -m | -f | 1.0 |

| S | M | F | $P(F\|S,M)$ |
|---|---|---|---|
| -s | +m | +f | 1.0 |
| -s | +m | -f | 0.0 |
| -s | -m | +f | 0.3 |
| -s | -m | -f | 0.7 |

| S | $P(S)$ |
|---|---|
| +s | 0.2 |
| -s | 0.8 |

Figure 3: Tables.

We want to figure out the value of revealing the Covid-19 restrictions $S$. Compute the values of the following quantities.

$EU(theater| + s) = 55$

The shortage variable is independent of the parents of the utility node when no additional evidence is present; thus, the same values hold: $EU(theater|+s) = EU(theater) = 55$

$EU(rent|+s) = EU(rent) = 60$

$MEU(\{+s\}) = 60$

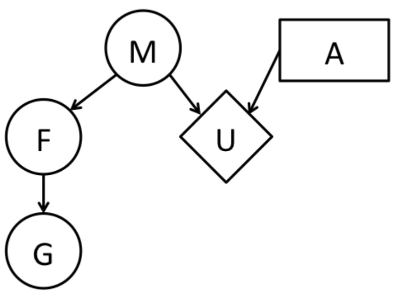
Optimal action for $+s = rent$

$MEU(\{-s\}) = 60$

Optimal action for $-s = rent$

$VPI(S) = 0$, since the Value of Perfect Information is the expected difference in MEU given the evidence vs. without the evidence and here the evidence is uninformative.

**6.3)** (5 pts) Now let's assume that we want to determine the "fullness" of the theaters $F$ without using information about the Covid-19 restrictions but using information about the garbage disposal $G$ outside the theaters. This new variable is also binary and the new decision network and tables are as follows:




| G | F | $P(G\|F)$ |
|---|---|---|
| +g | +f | 0.8 |
| -g | +f | 0.2 |
| +g | -f | 0.3 |
| -g | -f | 0.7 |

| M | P(M) |
|---|---|
| +m | 0.5 |
| -m | 0.5 |

| F | M | $P(F\|M)$ |
|---|---|---|
| +f | +m | 0.92 |
| -f | +m | 0.08 |
| +f | -m | 0.24 |
| -f | -m | 0.76 |

| M | A | U(M,A) |
|---|---|---|
| +m | *theater* | 100 |
| -m | *theater* | 10 |
| +m | *rent* | 80 |
| -m | *rent* | 40 |

Figure 4: Decision network and tables.

We also provide you with the following extra tables that might help you to answer the question.

| F | $P(F)$ |
|---|---|
| +f | 0.58 |
| -f | 0.42 |

| G | $P(G)$ |
|---|---|
| +g | 0.59 |
| -g | 0.41 |

| M | G | $P(M\|G)$ |
|---|---|---|
| +m | +g | 0.644 |
| -m | +g | 0.356 |
| +m | -g | 0.293 |
| -m | -g | 0.707 |

| G | M | $P(G\|M)$ |
|---|---|---|
| +g | +m | 0.760 |
| -g | +m | 0.240 |
| +g | -m | 0.420 |
| -g | -m | 0.580 |

| M | F | $P(M\|F)$ |
|---|---|---|
| +m | +f | 0.793 |
| -m | +f | 0.207 |
| +m | -f | 0.095 |
| -m | -f | 0.905 |

Figure 5: Extra tables.

Fill in the following values:

$MEU(+g) = max(EU(theater|+g), EU(rent|+g)) = 67.96$

$EU(theater|+g) = P(+m|+g) \cdot U(+m, theater) + P(-m|+g) \cdot U(-m, theater) = 0.644 \cdot 100 + 0.356 \cdot 10 = 67.96$

$EU(rent|+g) = P(+m|+g) \cdot U(+m, rent) + P(-m|+g) \cdot U(-m, rent) = 0.644 \cdot 80 + 0.356 \cdot 40 = 65.76$

$MEU(-g) = max(EU(theater|-g), EU(rent|-g)) = 51.72$

$EU(theater|-g) = P(+m|-g) \cdot U(+m, theater) + P(-m|-g) \cdot U(-m, theater) = 0.293 \cdot 100 + 0.707 \cdot 10 = 36.37$

$EU(rent|-g) = P(+m|-g) \cdot U(+m, rent) + P(-m|-g) \cdot U(-m, rent) = 0.293 \cdot 80 + 0.707 \cdot 40 = 51.72$

$VPI(G) = P(+g) \cdot MEU(+g) + P(-g)MEU(-g) - MEU(\emptyset) = 0.59 \cdot 67.96 + 0.41 \cdot 51.72 - MEU(\emptyset) = 61.3 - 60 = 1.3$