# Announcements

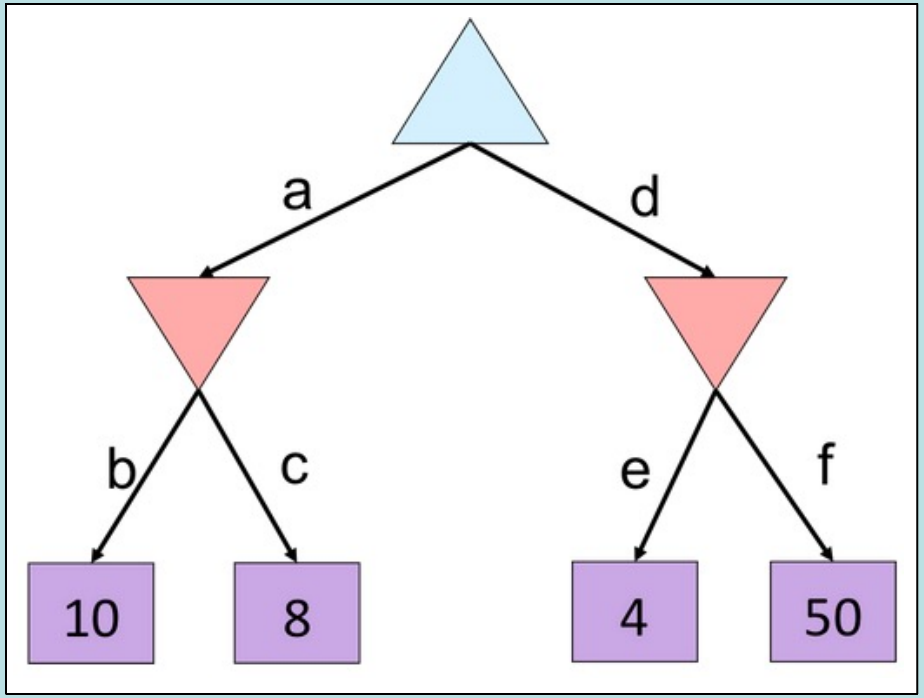
- Project 1 is due **Friday, February 2**, 11:59 PM PT
- HW2 is due **Thursday, February 8**, 11:59 PM PT

Pre-scan attendance QR code now!

(Password appears later)

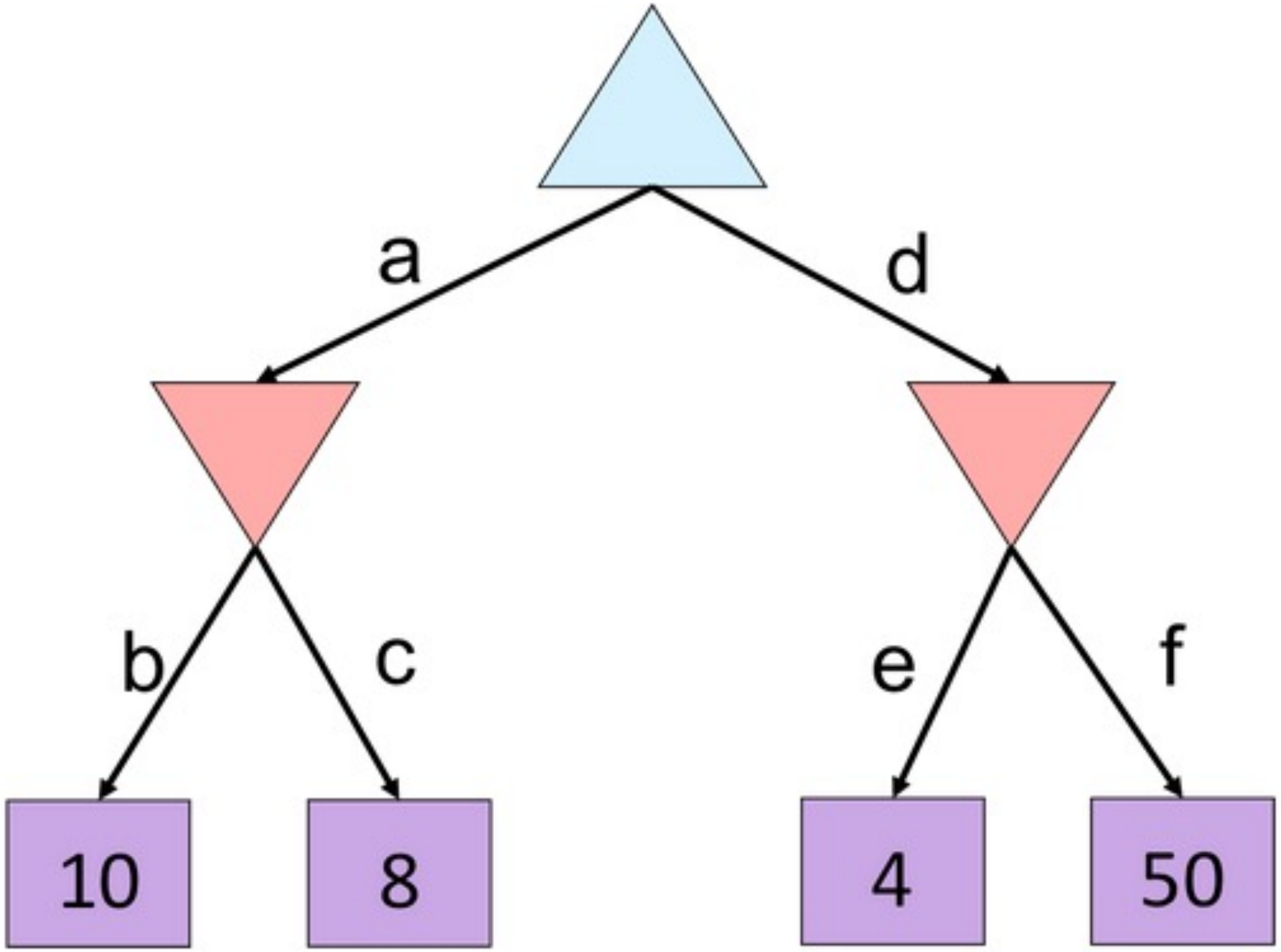
**Alpha-Beta Pruning Quiz**

Which branches can we prune?

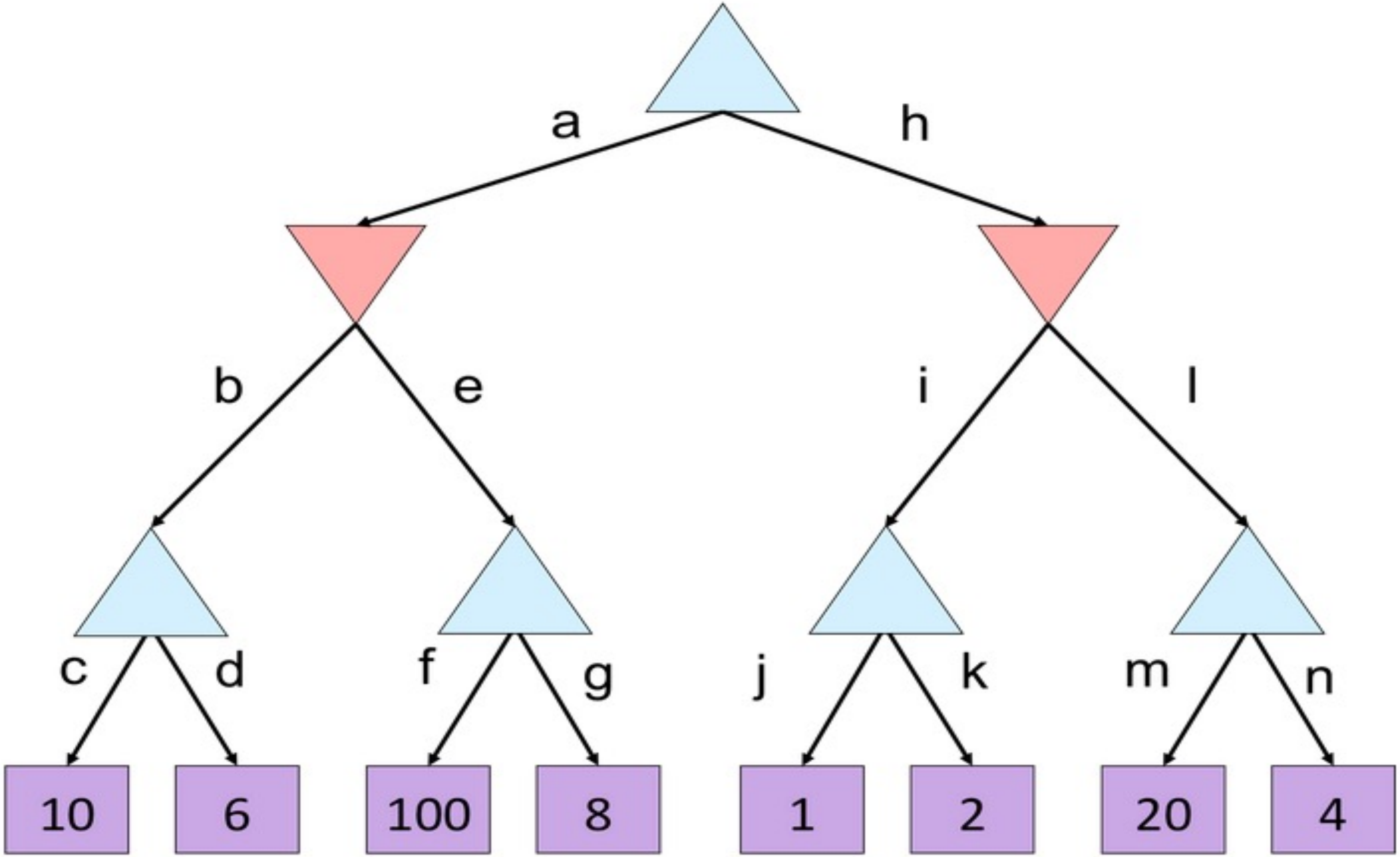



[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

# Alpha-Beta Pruning Quiz

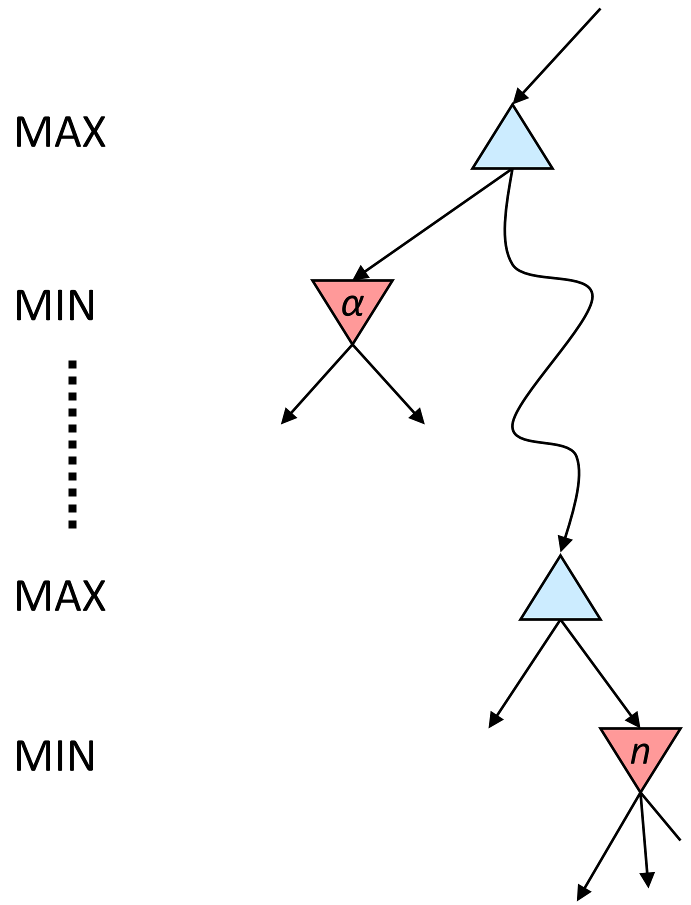


# Alpha-Beta Pruning Quiz 2



# Alpha-Beta Pruning

- General case (pruning children of MIN node)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let ***α*** be the best value that MAX can get so far at any choice point along the current path from the root
  - If *n* becomes worse than ***α***, MAX will avoid it, so we can prune *n*'s other children (it's already bad enough that it won't be played)
- Pruning children of MAX node is symmetric
  - Let **β** be the best value that MIN can get so far at any choice point along the current path from the root



# Alpha-Beta Pruning Implementation

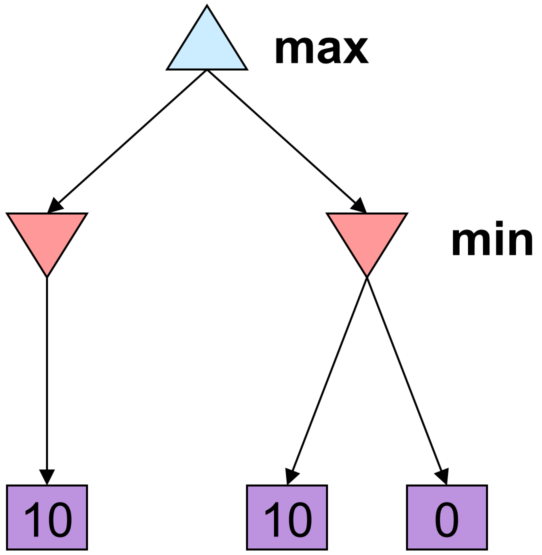
α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

# Alpha-Beta Pruning Properties

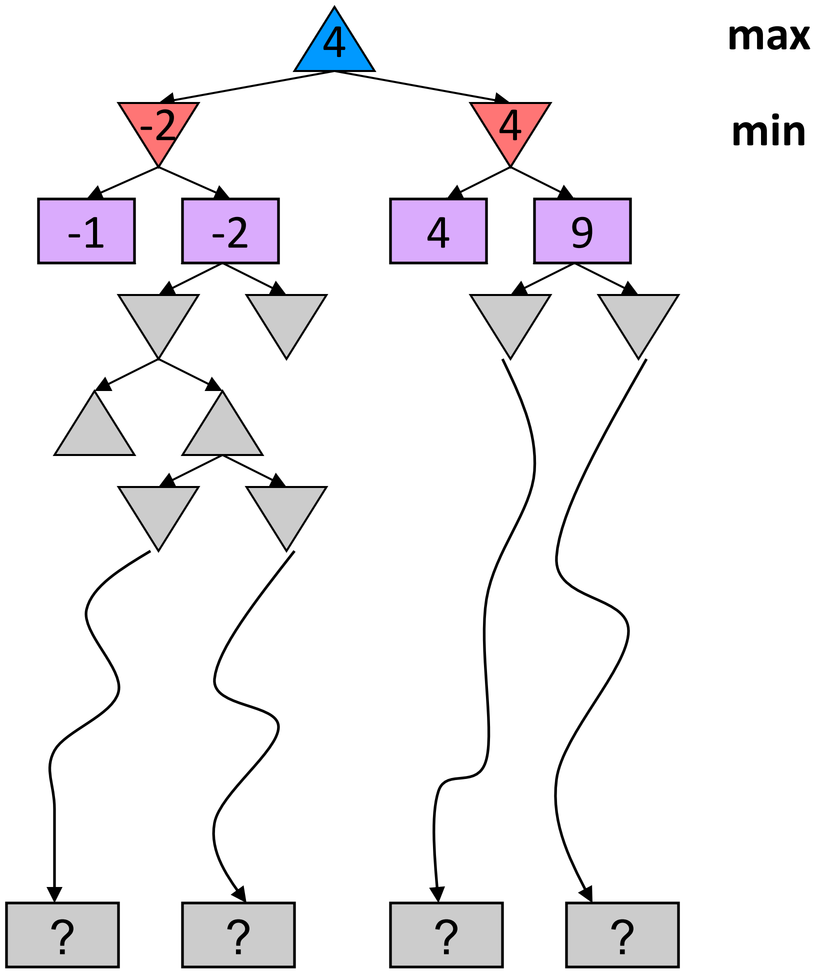
- This pruning has no effect on minimax value computed for the root!
- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value!
- Good child ordering improves effectiveness of pruning
- With “perfect ordering”:
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…
- This is a simple example of metareasoning (computing about what to compute)



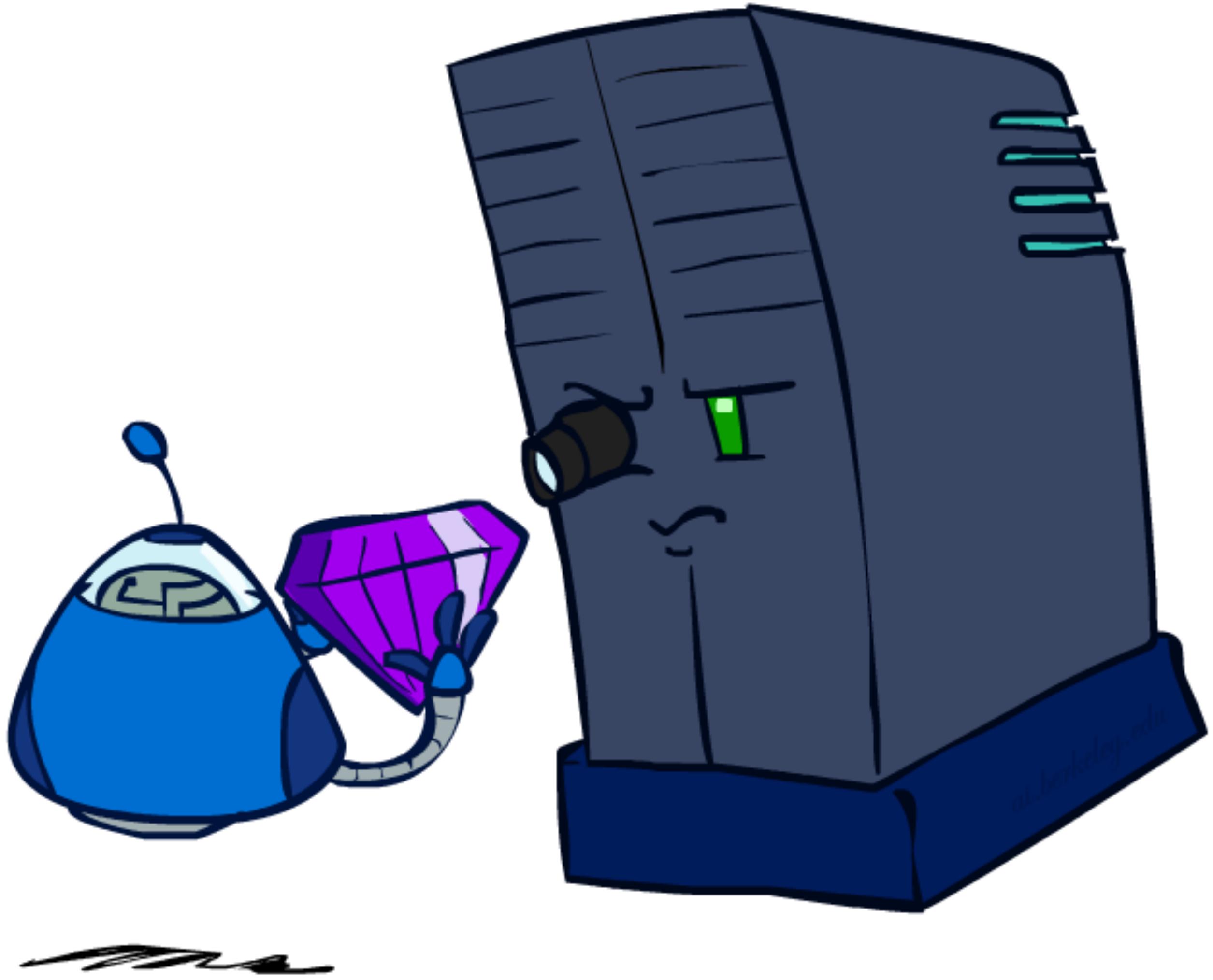
# Resource Limits

# Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - $\alpha$-$\beta$ reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- More plies (layers) makes a BIG difference
- Use iterative deepening for an anytime algorithm

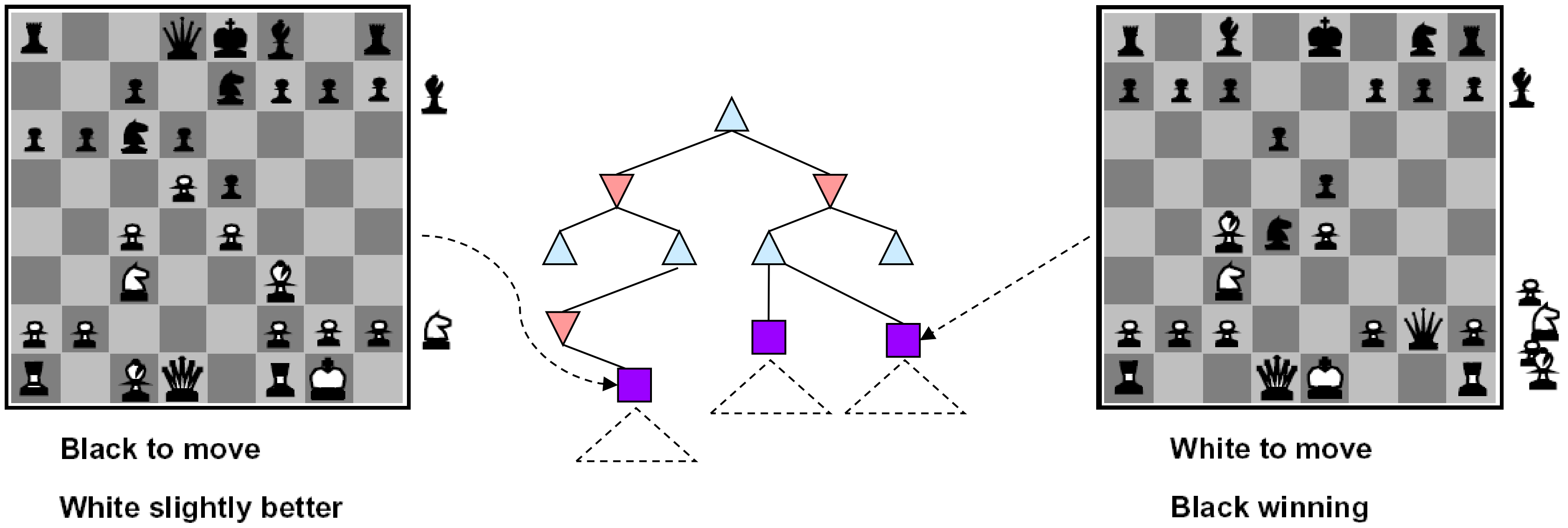


# Evaluation Functions



# Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search

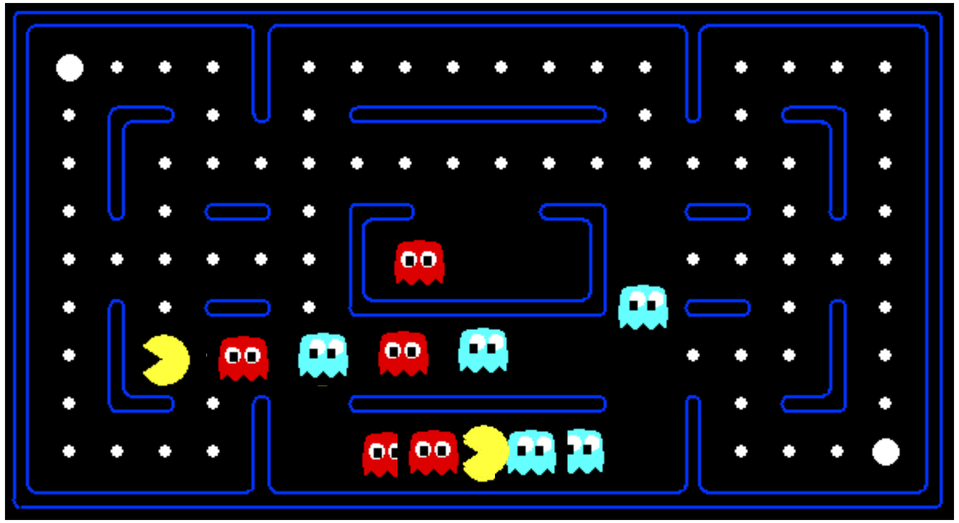



- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

  - E.g. $f_1(s)$ = (num white queens – num black queens), etc.
- Or a more complex nonlinear function (e.g., NN) trained by self-play RL

# Evaluation for Pacman

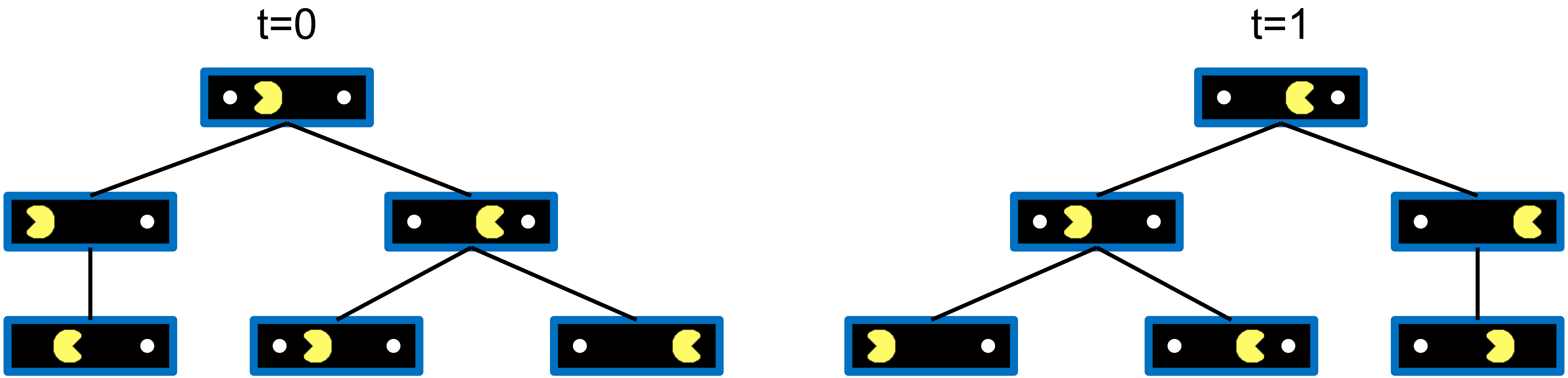


[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate (L6D6,7,8,10)]

# Video of Demo Thrashing (d=2)

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function) (L6D6)]

# Why Pacman Starves



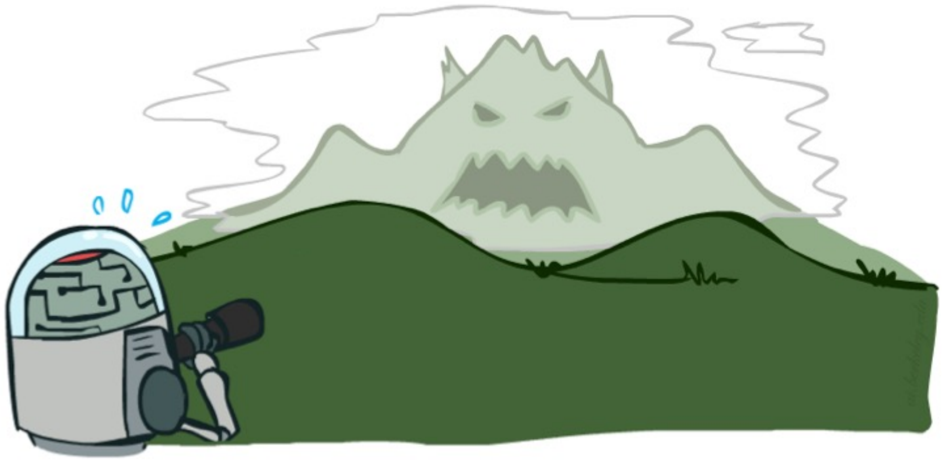

- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, d=2)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

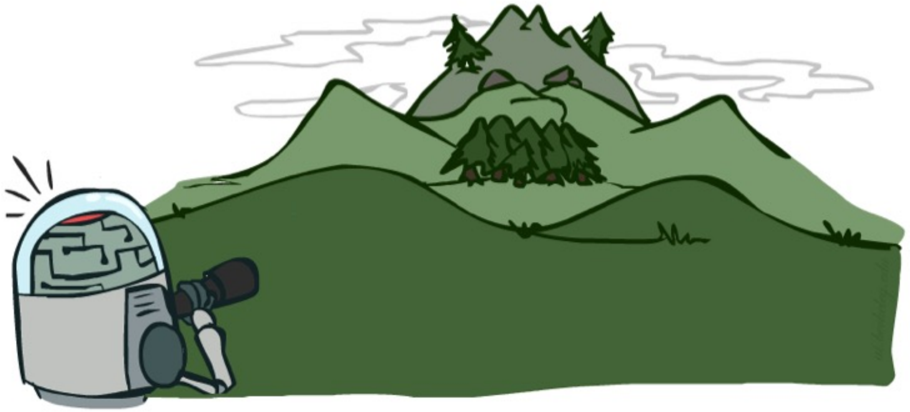
# Video of Demo Thrashing -- Fixed (d=2)

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function) (L6D7)]

# Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation





[Demo: depth limited (L6D4, L6D5)]

# Video of Demo Limited Depth (2)

# Video of Demo Limited Depth (10)

# Synergies between Evaluation Function and Alpha-Beta?

- Alpha-Beta: amount of pruning depends on expansion ordering
  - Evaluation function can provide guidance to expand most promising nodes first (which later makes it more likely there is already a good alternative on the path to the root)
    - (somewhat similar to role of A* heuristic)

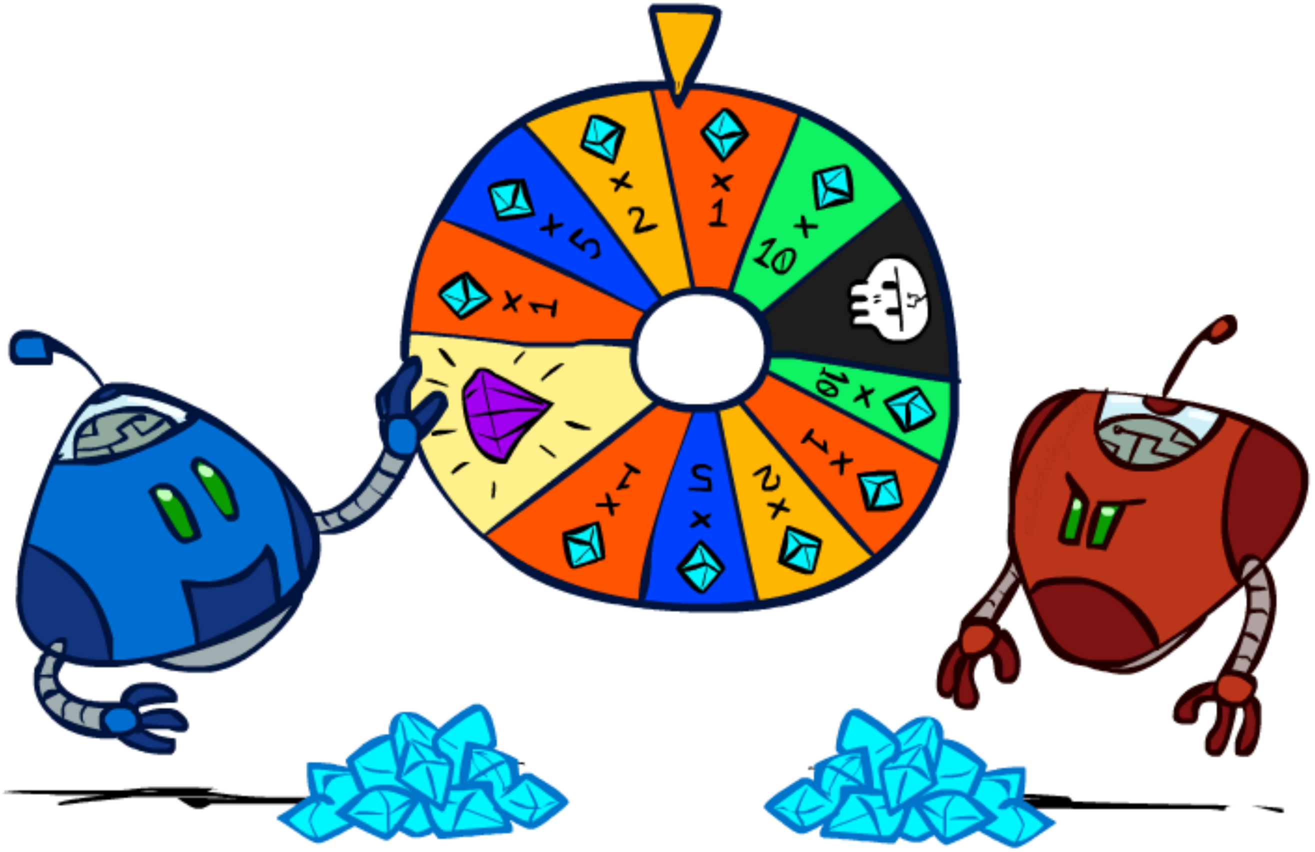
- Alpha-Beta: (similar for roles of min-max swapped)
  - Value at a min-node will only keep going down
  - Once value of min-node lower than better option for max along path to root, can prune
  - Hence: IF evaluation function provides upper-bound on value at min-node, and upper-bound already lower than better option for max along path to root THEN can prune

# Summary

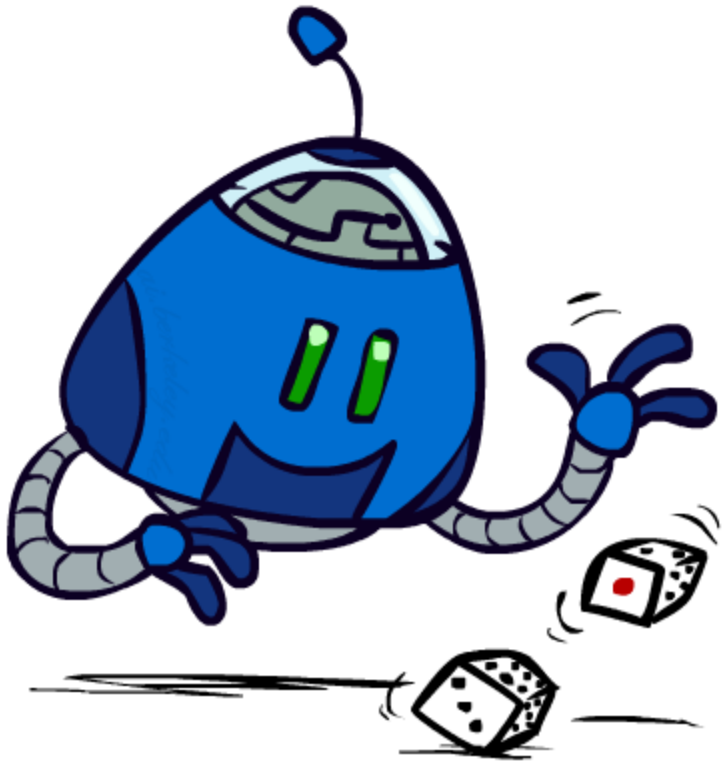
- Games are decision problems with $\geq$ 2 agents
  - Huge variety of issues and phenomena depending on details of interactions and payoffs
- For zero-sum games, optimal decisions defined by minimax
  - Simple extension to n-player "rotating" max with vectors of utilities
  - Implementable as a depth-first traversal of the game tree
  - Time complexity $O(b^m)$, space complexity $O(bm)$
- Alpha-beta pruning
  - Preserves optimal choice at the root
  - Alpha/beta values keep track of best obtainable values from any max/min nodes on path from root to current node
  - Time complexity drops to $O(b^{m/2})$ with ideal node ordering
- Exact solution is impossible even for "small" games like chess

# Next: Uncertainty!

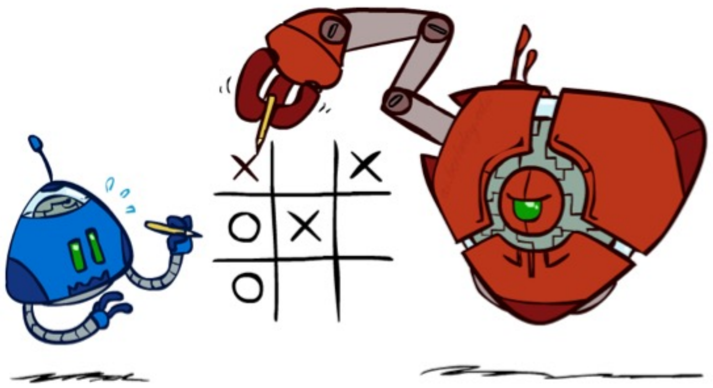
Expectimax, Monte Carlo Tree Search

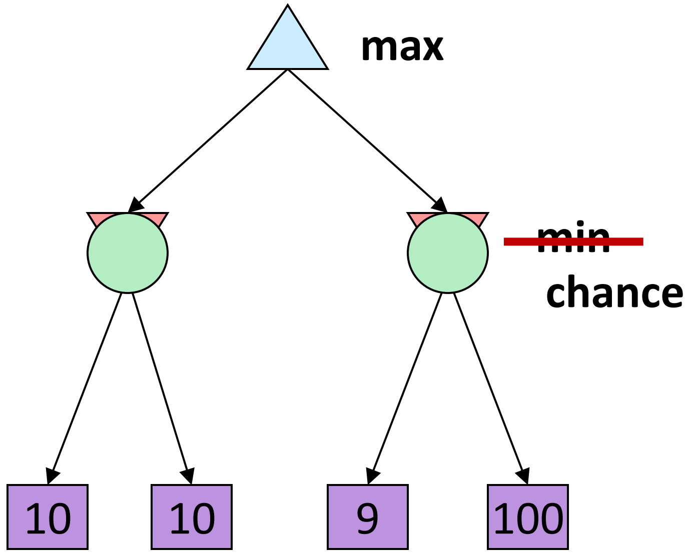


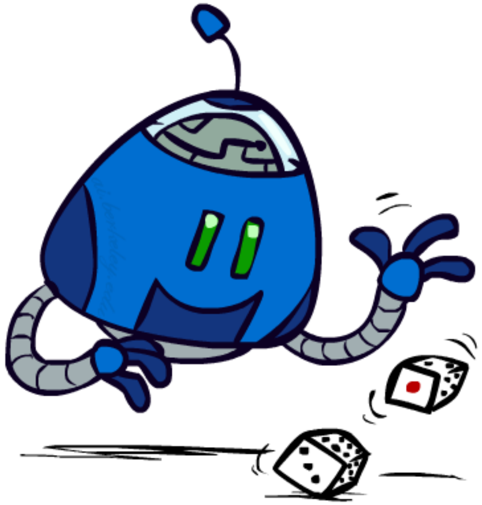
# Uncertain Outcomes



# Worst-Case vs. Average Case





Idea: Uncertain outcomes controlled by chance, not an adversary!

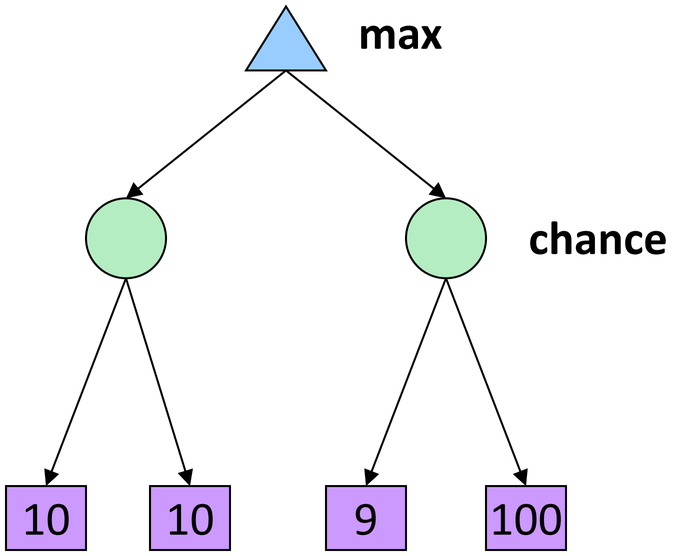
# Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip

- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes

- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children

- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes




[Demo: min vs exp (L7D1,2)]

# Expectimax Pseudocode

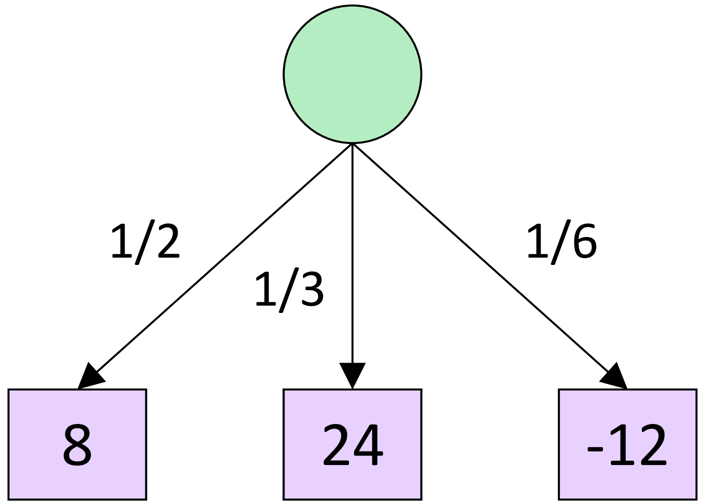
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

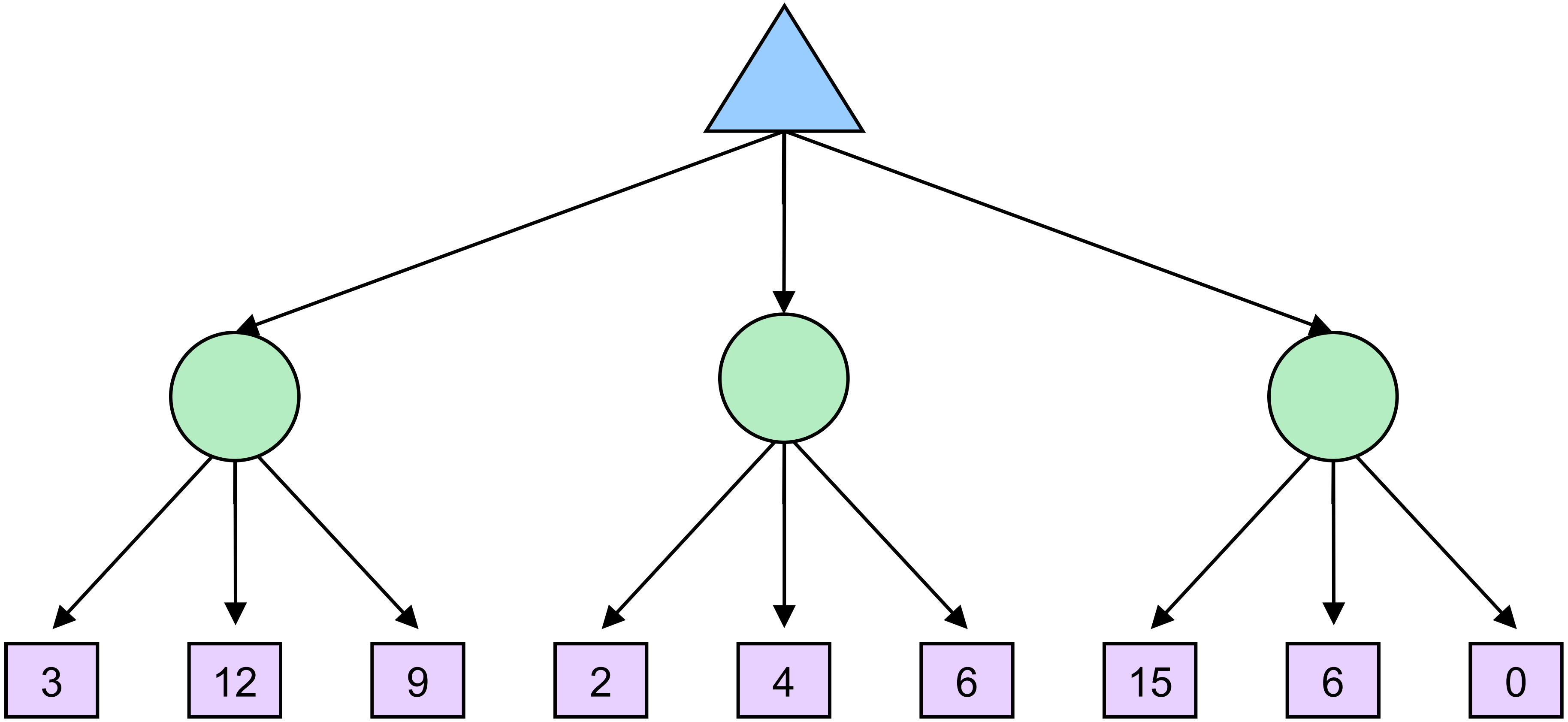
# Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

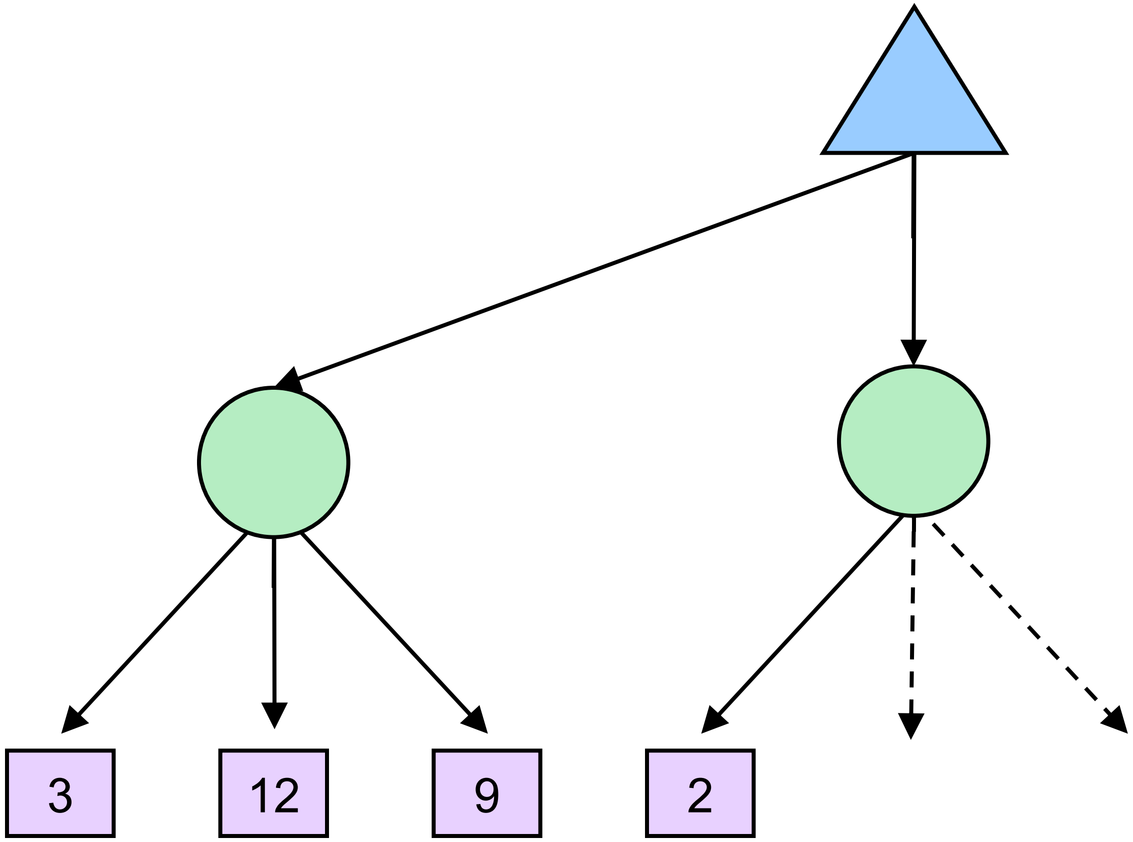


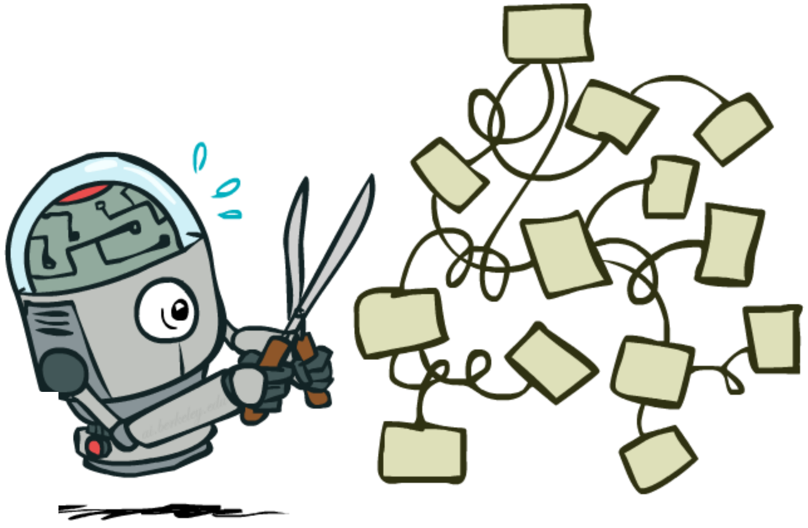

$$v = (1/2)\ (8) + (1/3)\ (24) + (1/6)\ (-12) = 10$$

# Expectimax Example

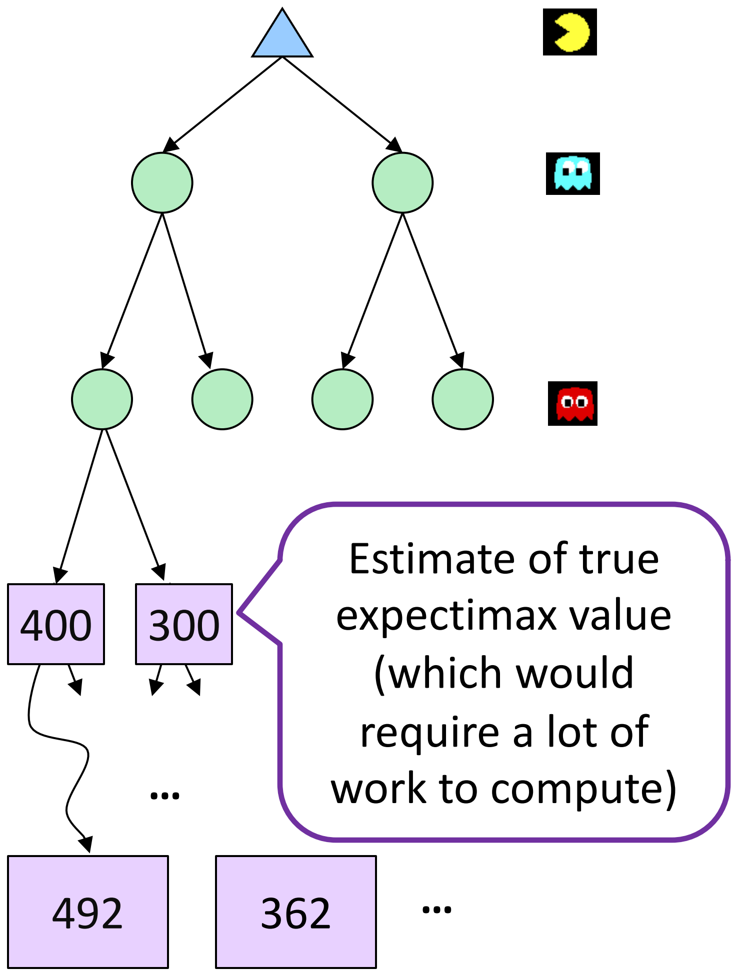


# Expectimax Pruning?

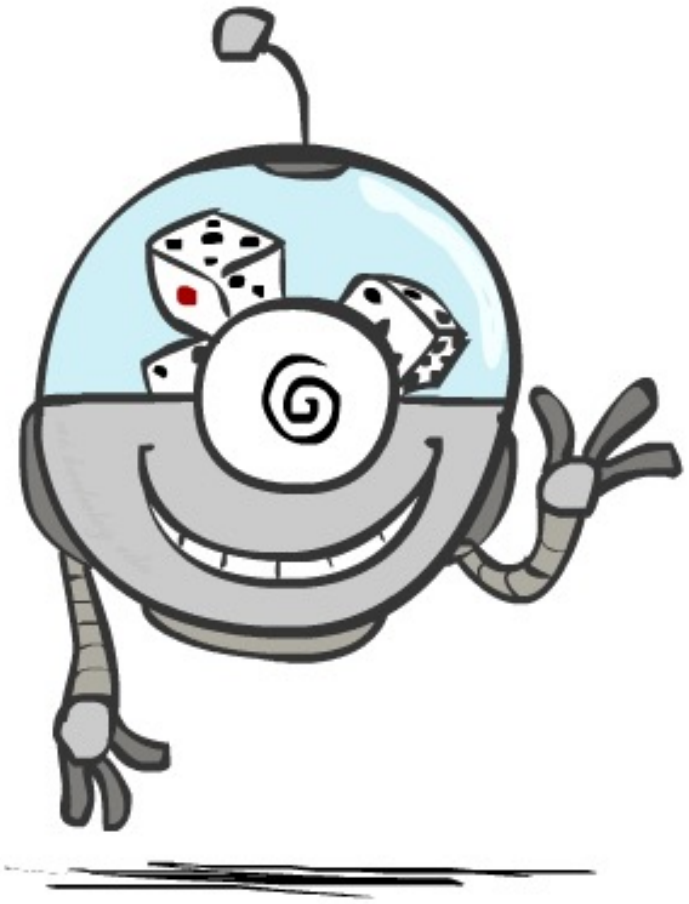


# Depth-Limited Expectimax

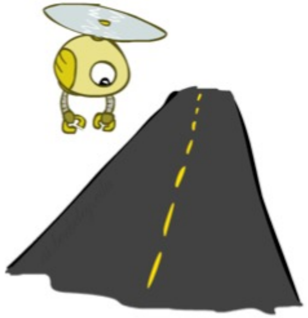


# Probabilities

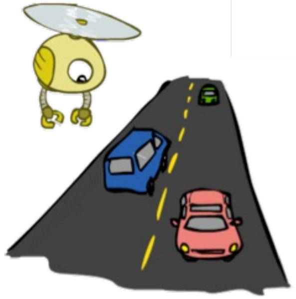


# Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes
- Example: Traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later



0.25

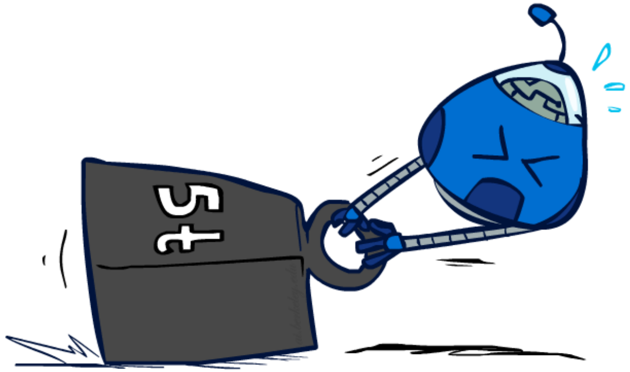


0.50

0.25

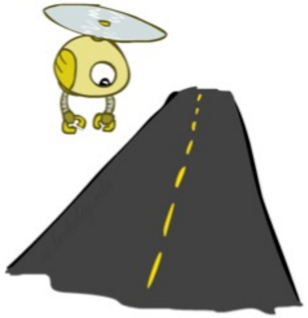
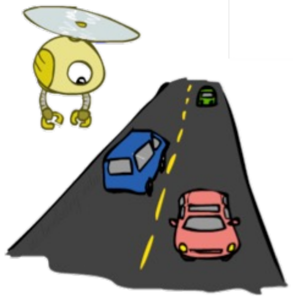
# Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes



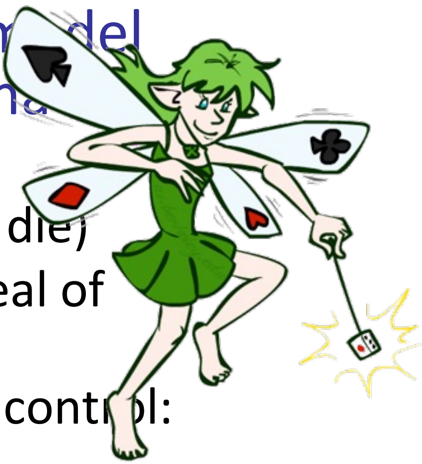

- Example: How long to get to the airport?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Time: | 20 min | | 30 min | | 60 min | | |
| | x | + | x | + | x | ⇒ | 35 min |
| Probability: | 0.25 | | 0.50 | | 0.25 | | |

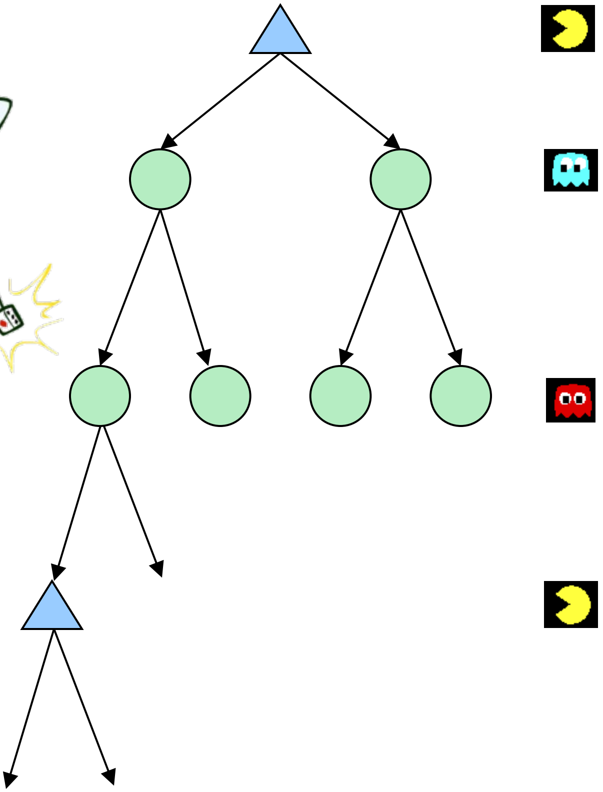




# What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!



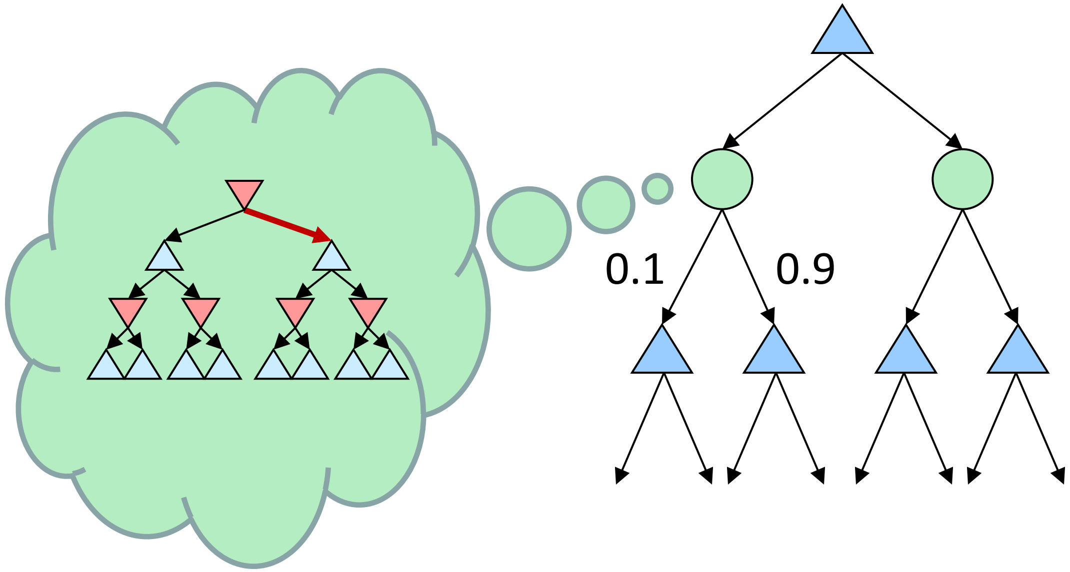
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



*Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!*

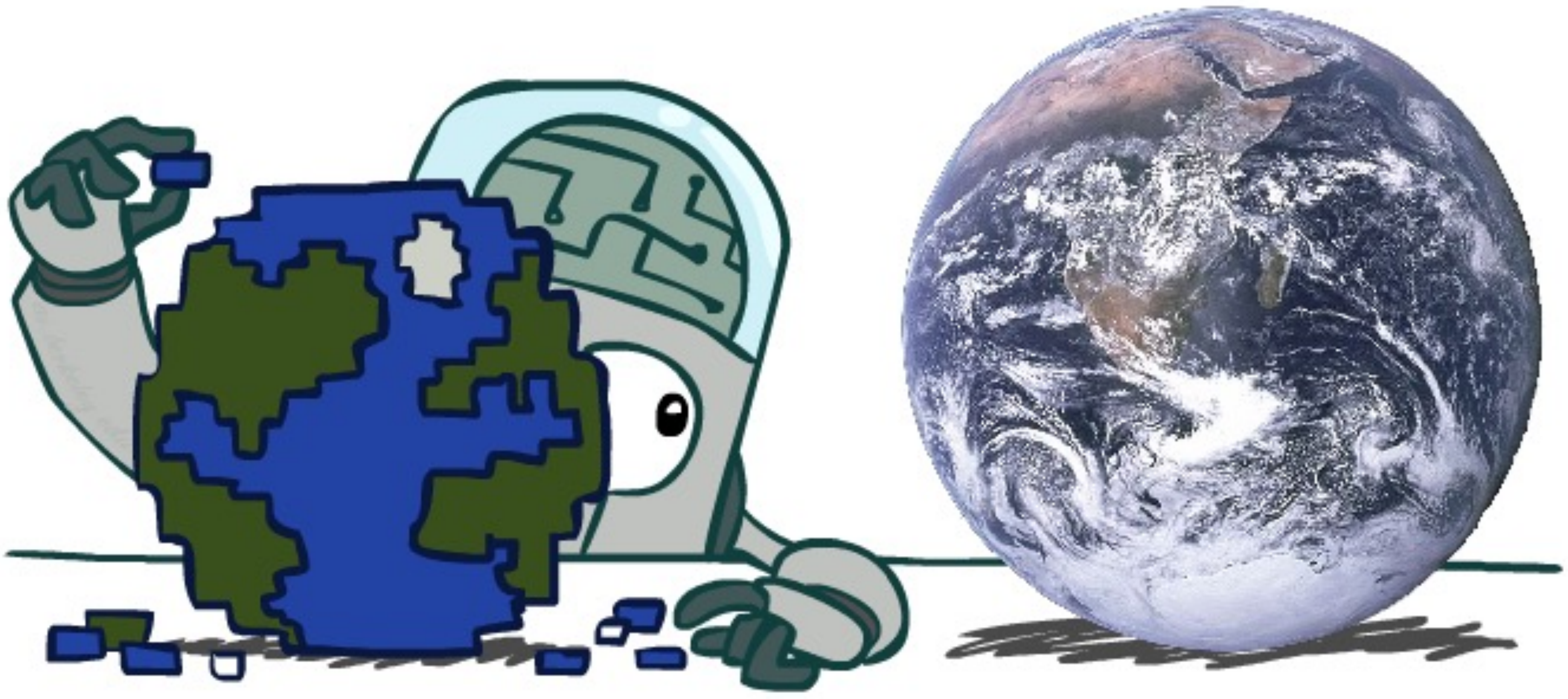
# Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?




- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

# Modeling Assumptions



# The Dangers of Optimism and Pessimism

## Dangerous Optimism

Assuming chance when the world is adversarial

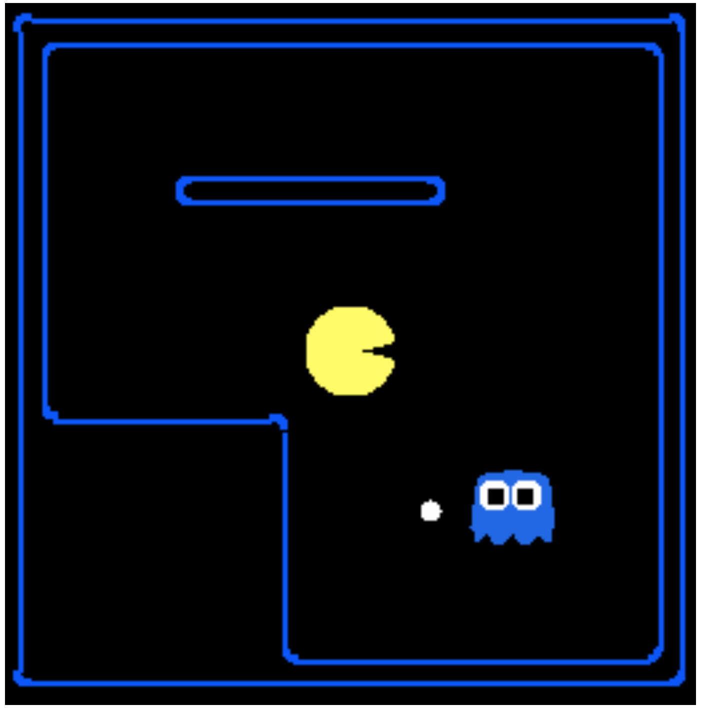

## Dangerous Pessimism

Assuming the worst case when it's not likely

# Video of Demo Minimax vs Expectimax (Min)

# Video of Demo Minimax vs Expectimax (Exp)

# Assumptions vs. Reality



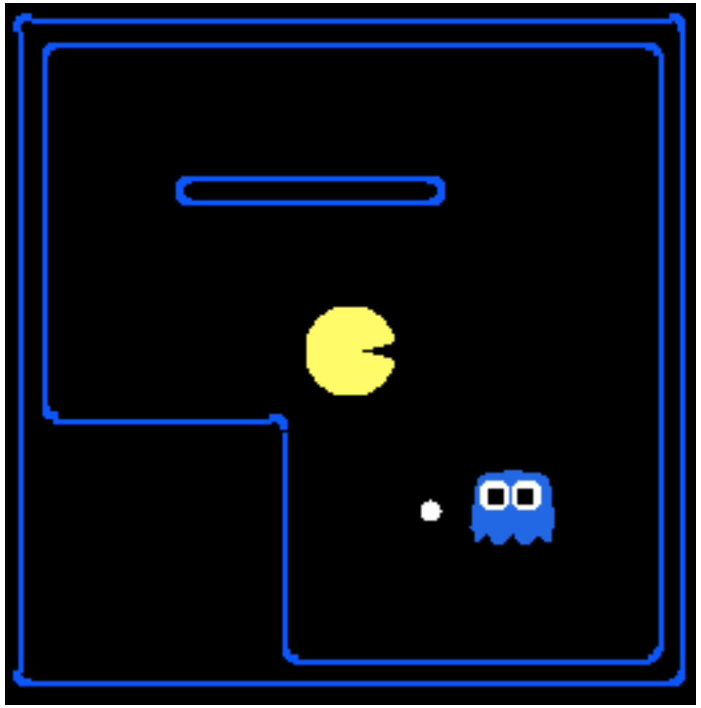
| | Adversarial Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | | |
| Expectimax Pacman | | |

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

[Demos: world assumptions (L7D3,4,5,6)]

# Assumptions vs. Reality



| | Adversarial Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | Won 5/5<br>Avg. Score: 483 | Won 5/5<br>Avg. Score: 493 |
| Expectimax Pacman | Won 1/5<br>Avg. Score: -303 | Won 5/5<br>Avg. Score: 503 |

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

[Demos: world assumptions (L7D3,4,5,6)]

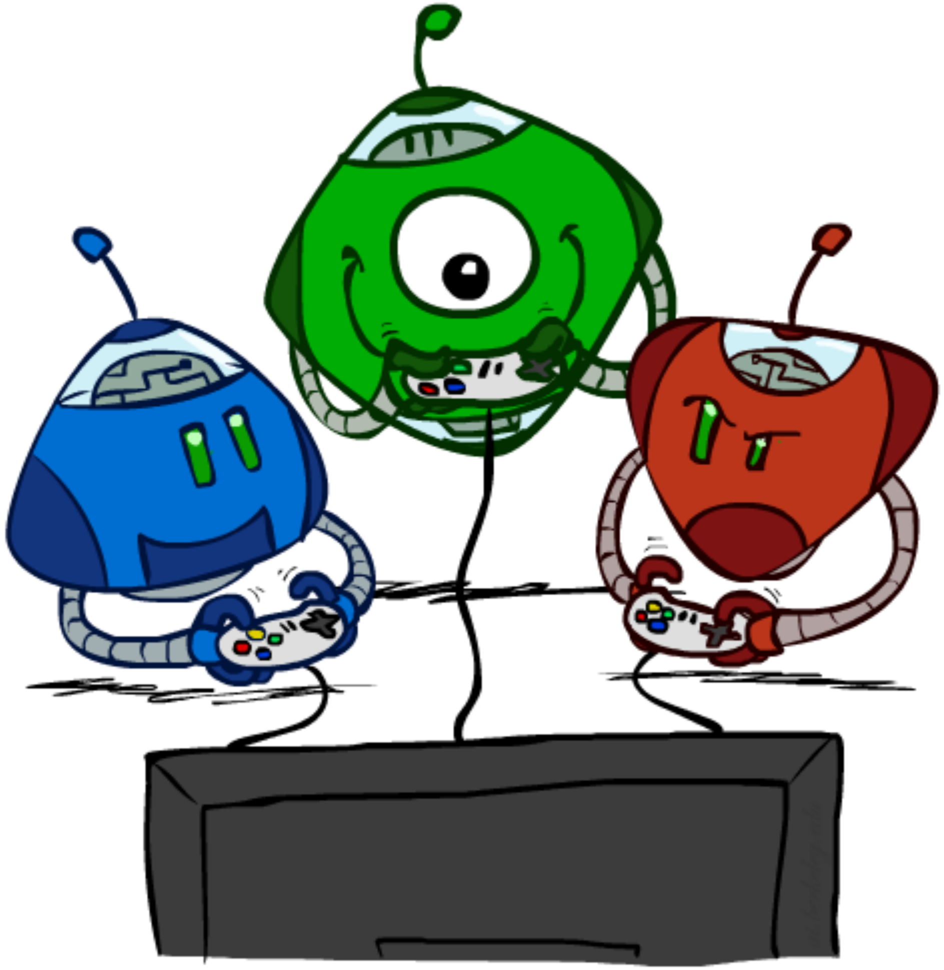
# Video of Demo World Assumptions
# Random Ghost – Expectimax Pacman

# Video of Demo World Assumptions
# Adversarial Ghost – Minimax Pacman

# Video of Demo World Assumptions
# Adversarial Ghost – Expectimax Pacman

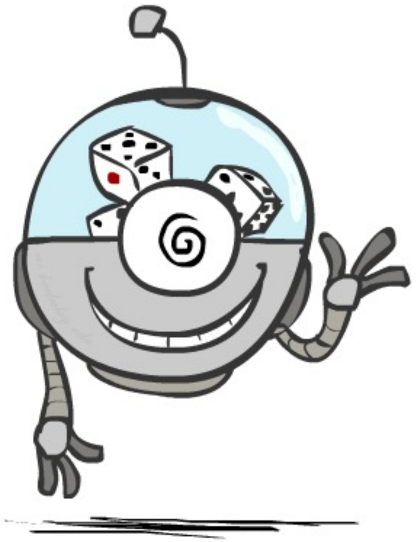
# Video of Demo World Assumptions
# Random Ghost – Minimax Pacman

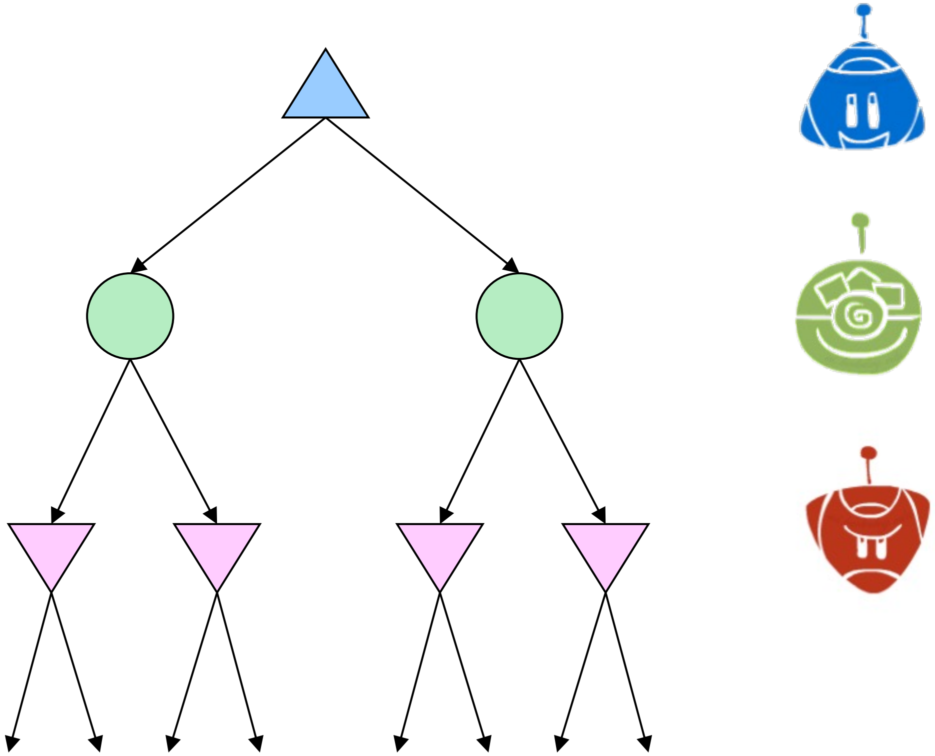
# Other Game Types



# Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra "random agent" player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



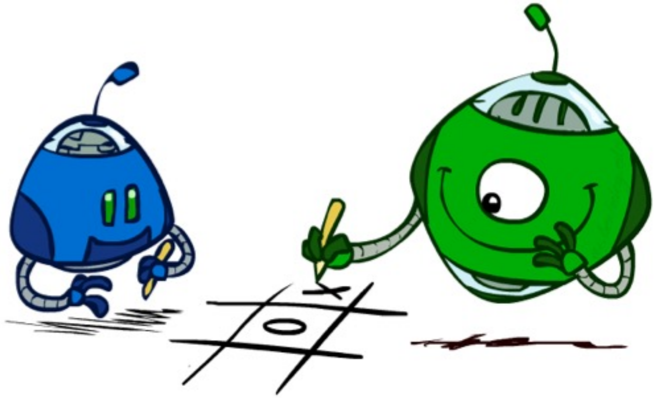


# Example: Backgammon

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1st AI world champion* in any game!

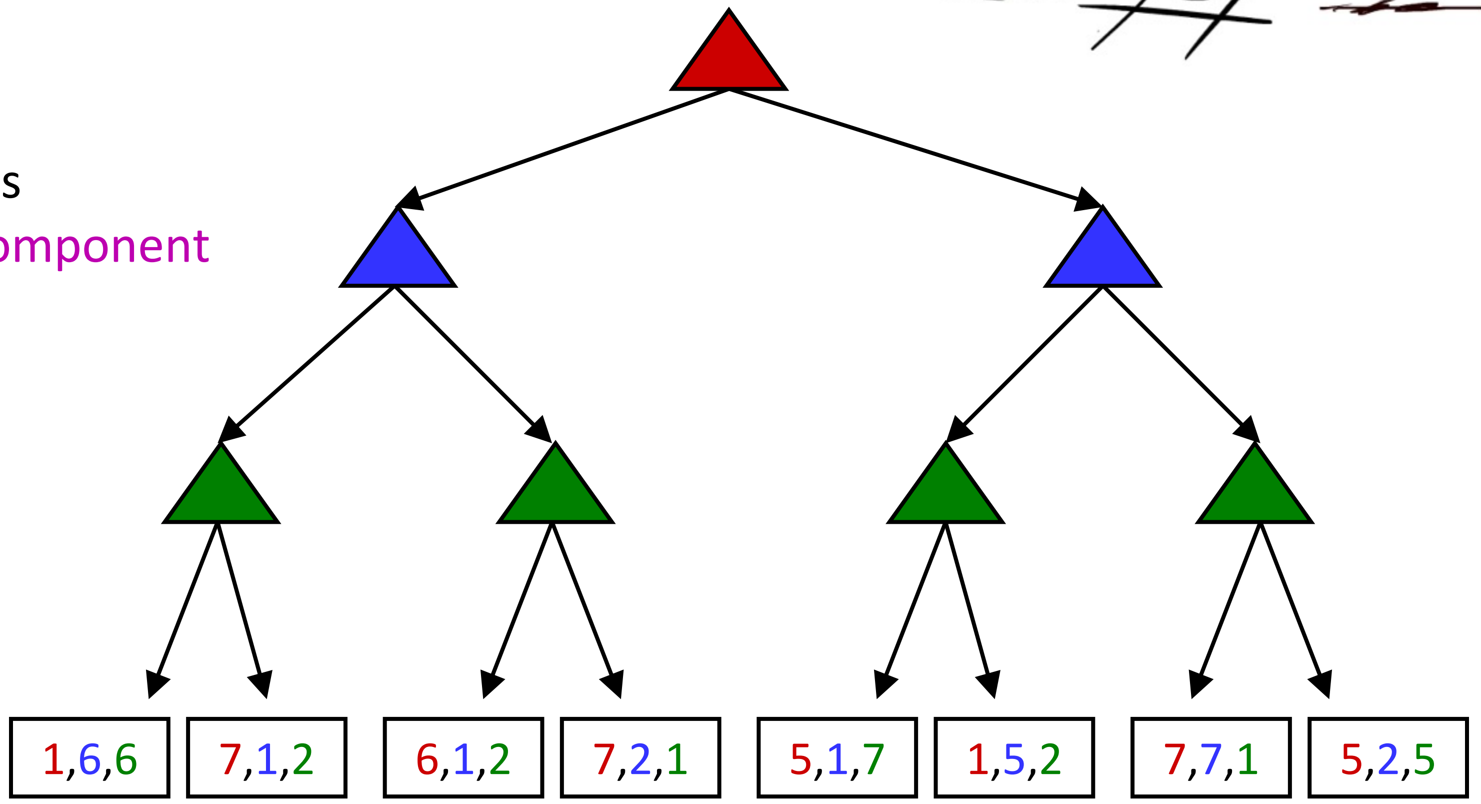
Image: Wikipedia

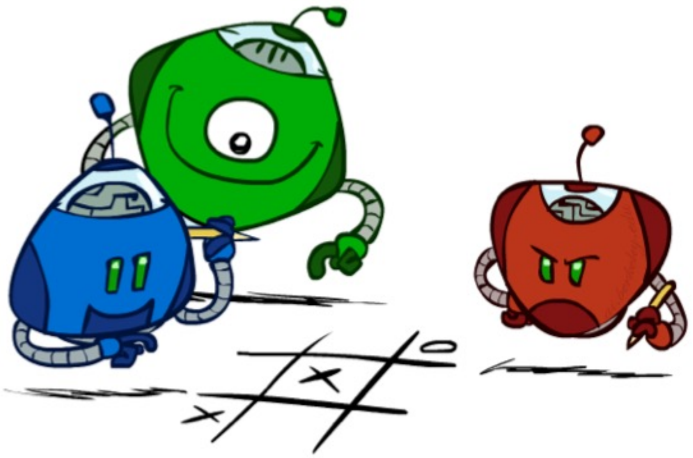
# Multi-Agent Utilities



- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



# Monte Carlo Tree Search

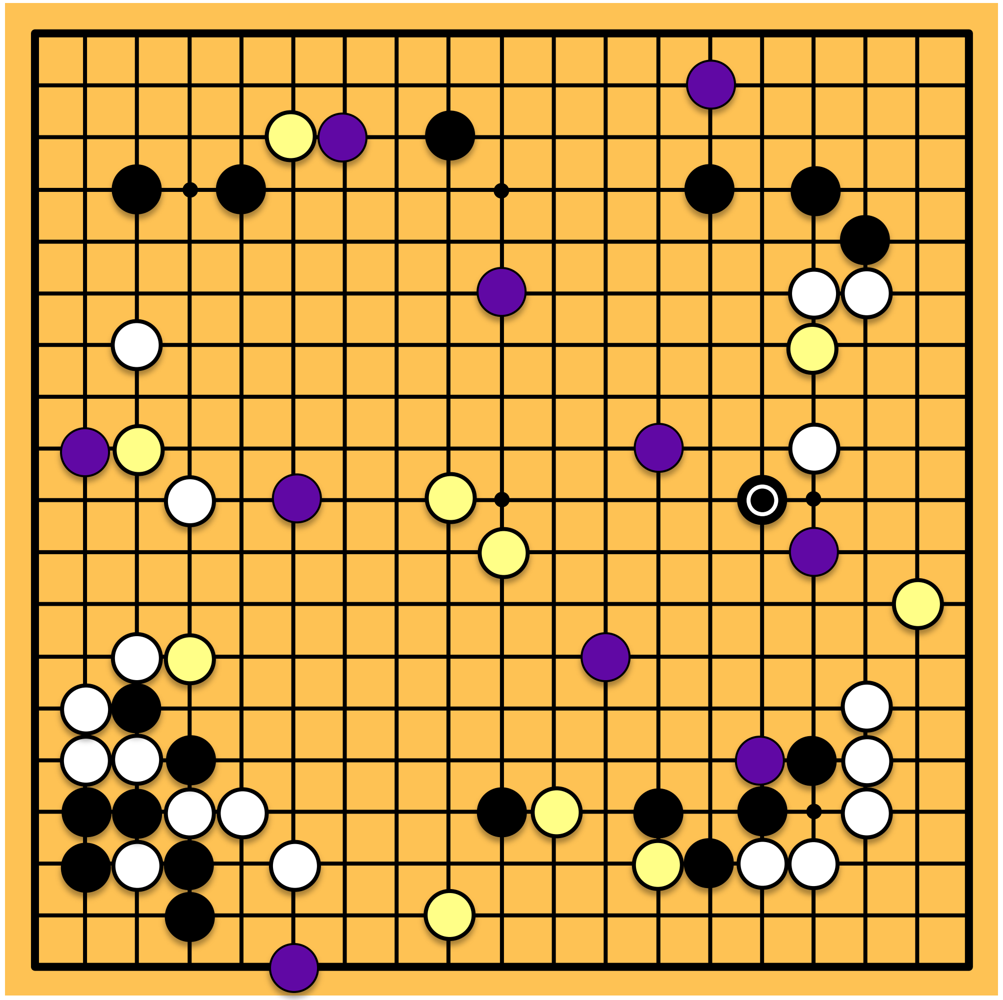
# Monte Carlo Tree Search

- Methods based on alpha-beta search assume a fixed horizon
  - Pretty hopeless for Go, with $b$ > 300
- MCTS combines two important ideas:
  - ***Evaluation by rollouts*** – play multiple games to termination from a state $s$ (using a simple, fast rollout policy) and count wins and losses
  - ***Selective search*** – explore parts of the tree that will help improve the decision at the root, regardless of depth

# Rollouts

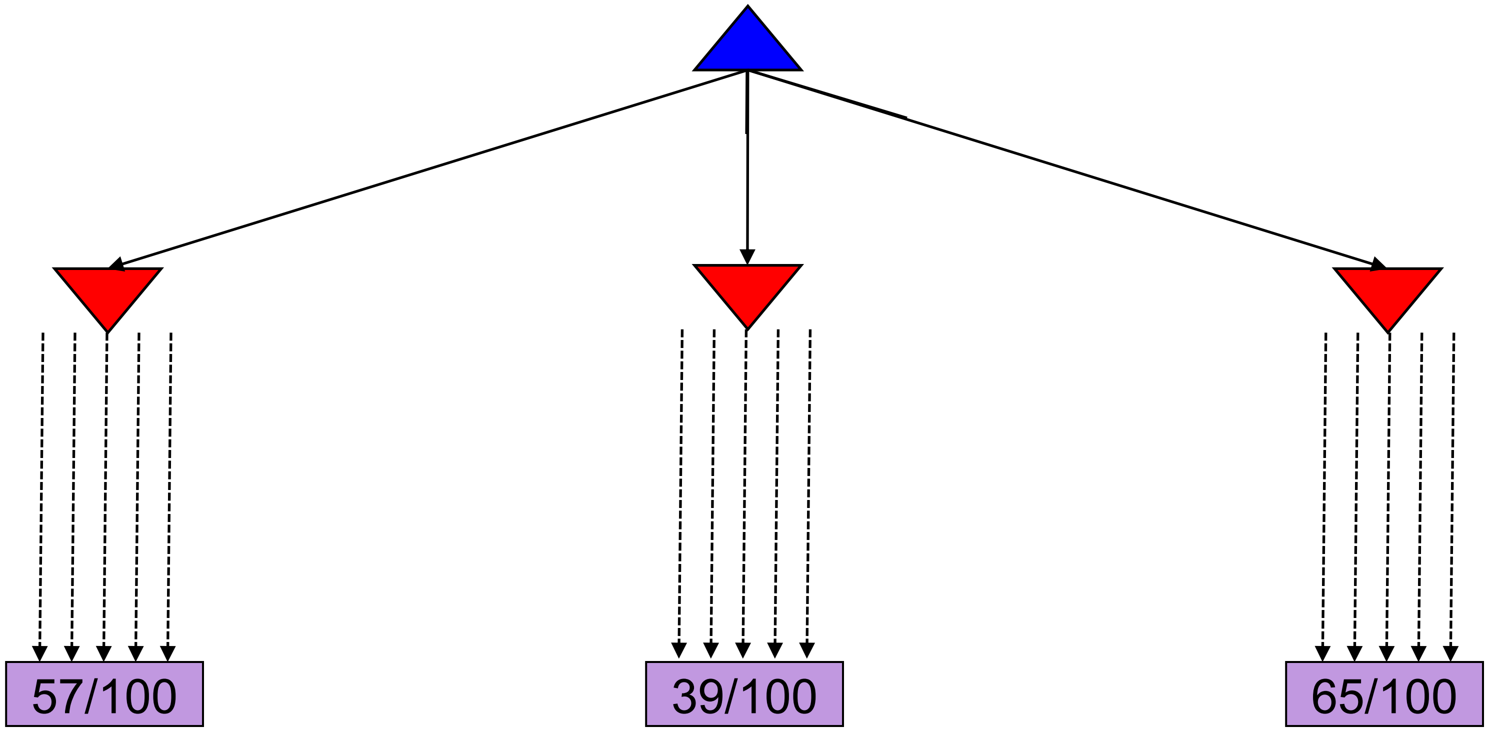
- For each rollout:
  - Repeat until terminal:
    - Play a move according to a fixed, fast rollout policy
  - Record the result
- Fraction of wins correlates with the true value of the position!
- Having a "better" rollout policy helps

"Move 37"



# MCTS Version 0

- Do *N* rollouts from each child of the root, record fraction of wins
- Pick the move that gives the best outcome by this metric



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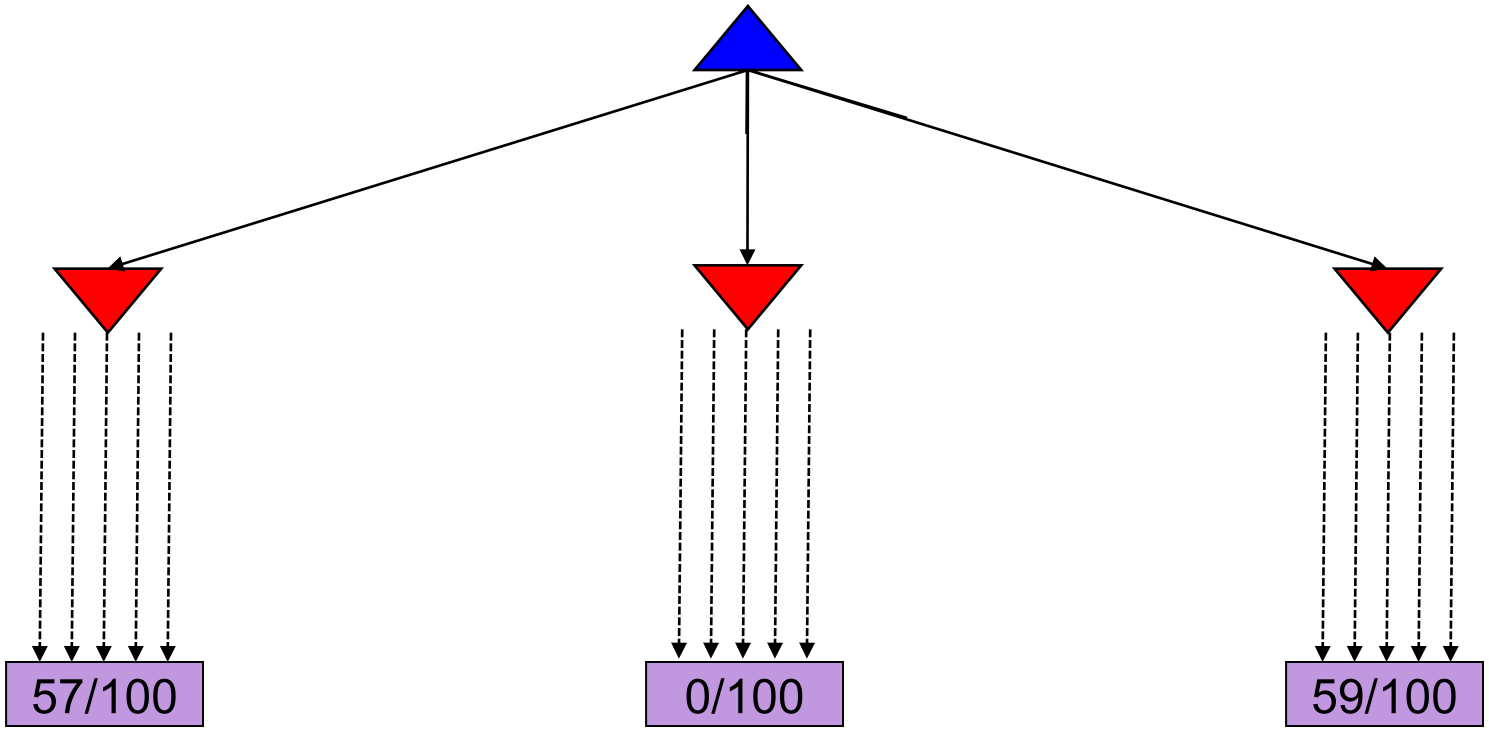
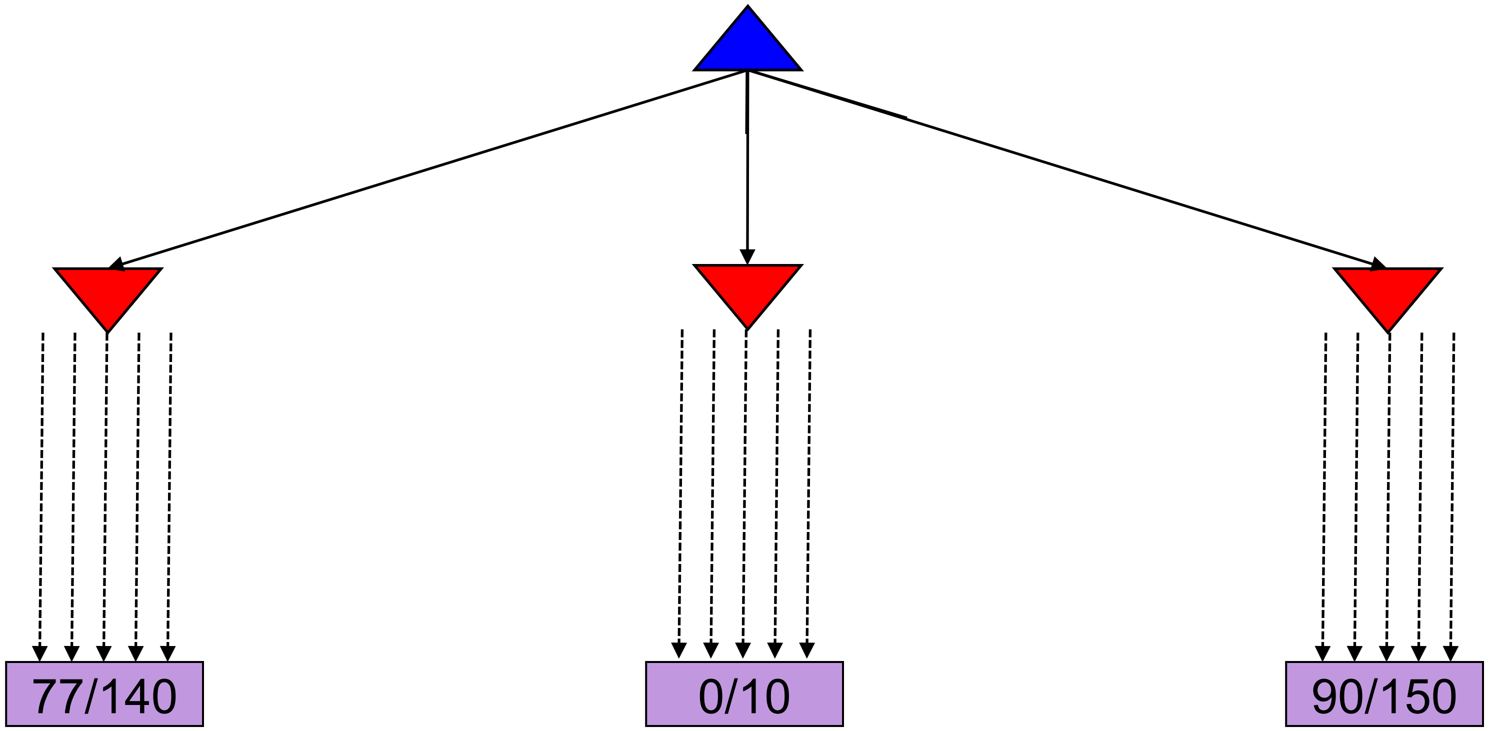


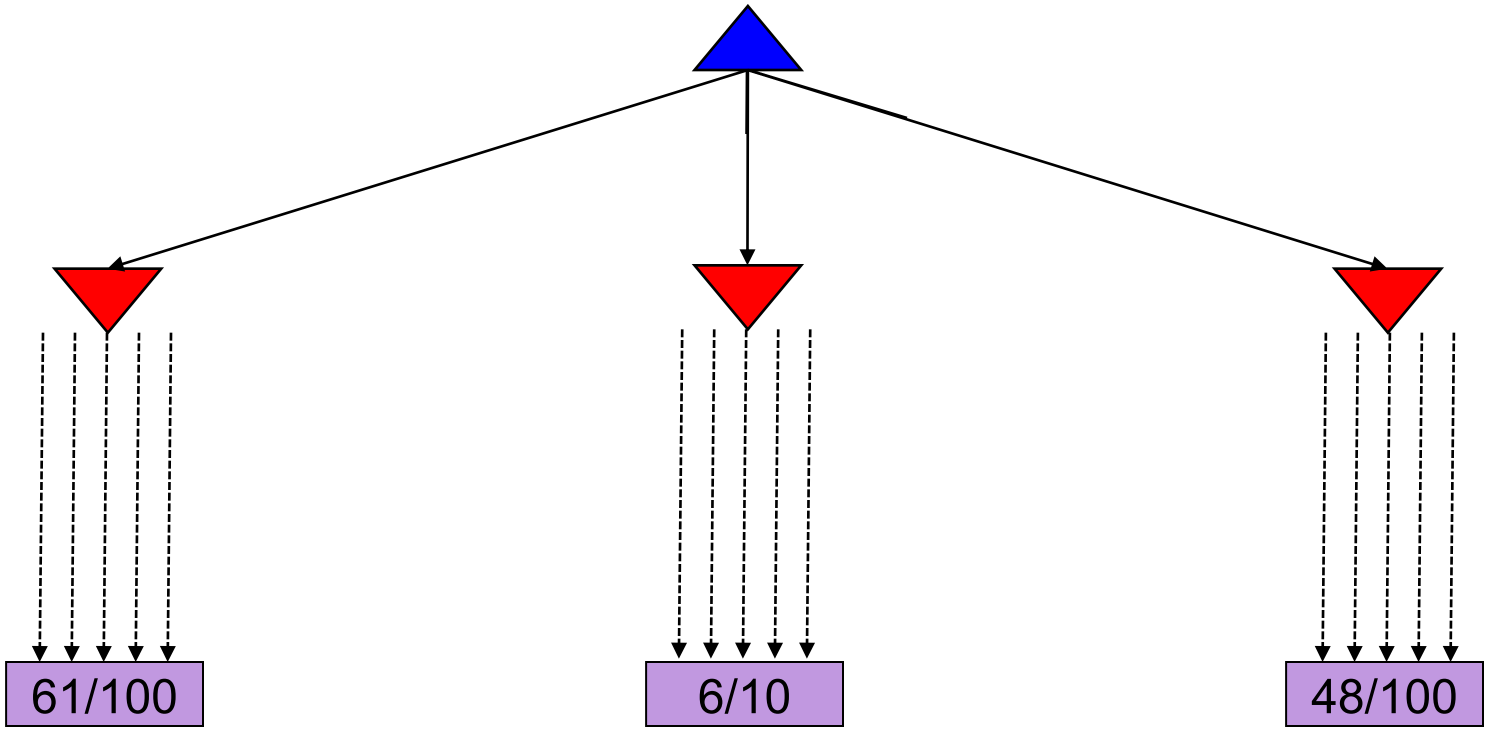
# MCTS Version 0.9

- Allocate rollouts to more promising nodes



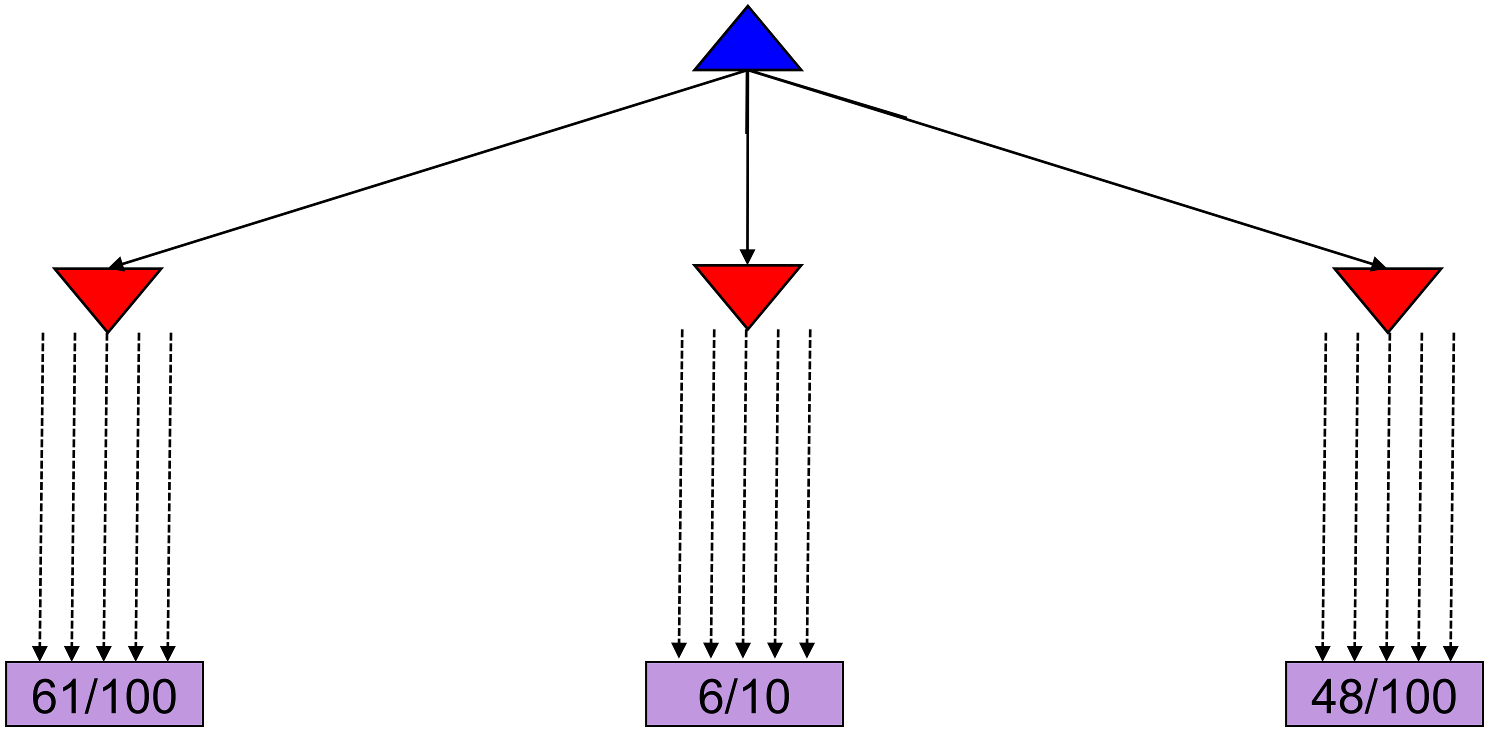
# MCTS Version 0.9

- Allocate rollouts to more promising nodes



# MCTS Version 1.0

- Allocate rollouts to more promising nodes
- Allocate rollouts to more uncertain nodes



# UCB heuristics

- UCB1 formula combines "promising" and "uncertain":

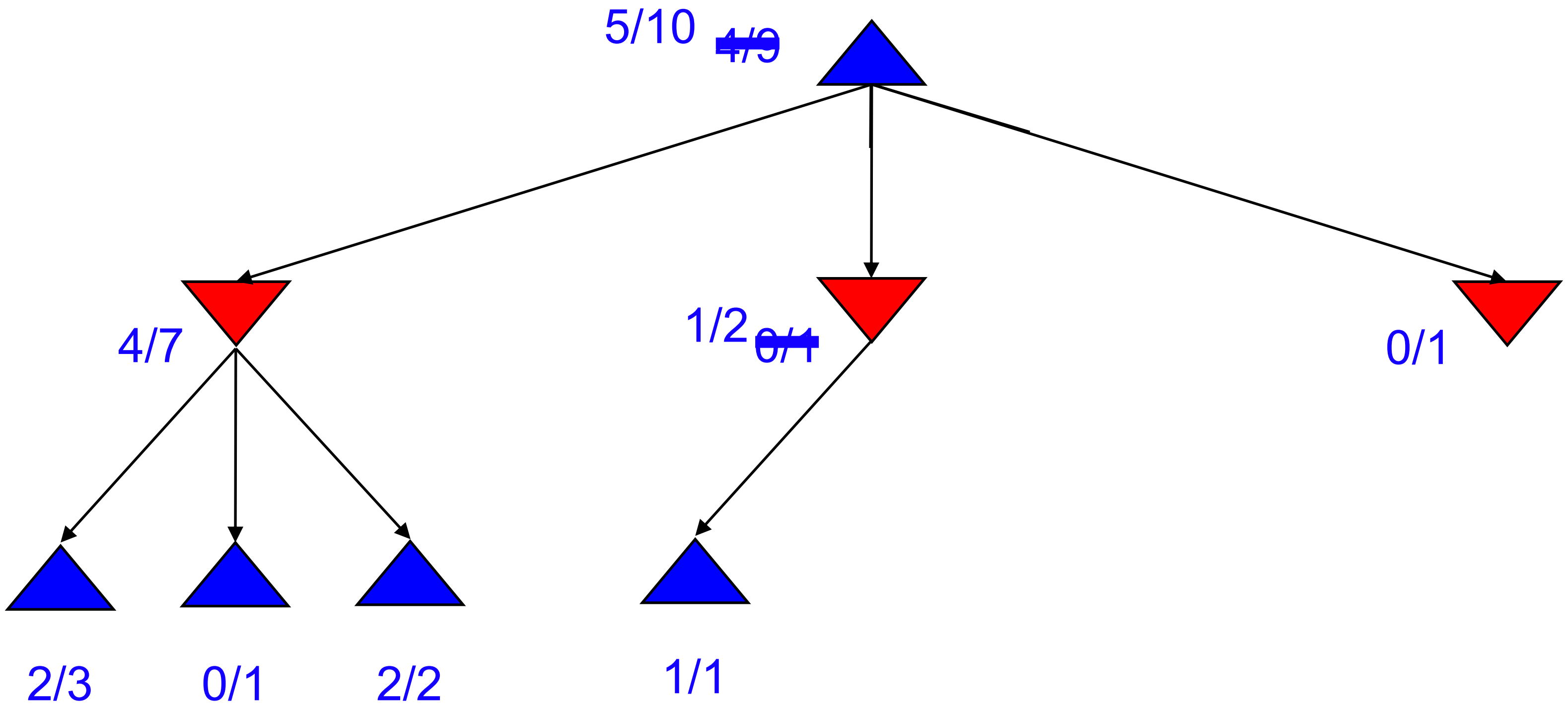
$$UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\textsc{Parent}(n))}{N(n)}}$$

- $N(n)$ = number of rollouts from node $n$
- $U(n)$ = total utility of rollouts (e.g., # wins) for Player(Parent($n$))
- A provably not terrible heuristic for ***bandit problems***
  - (which are not the same as the problem we face here!)

# MCTS Version 2.0: UCT

- Repeat until out of time:
  - Given the current search tree, recursively apply UCB to choose a path down to a leaf (not fully expanded) node *n*
  - Add a new child *c* to *n* and run a rollout from *c*
  - Update the win counts from *c* back up to the root
- Choose the action leading to the child with highest *N*

# UCT Example



# Why is there no min or max?

- "Value" of a node, $U(n)/N(n)$, is a weighted ***sum*** of child values!
- Idea: as $N \rightarrow \infty$ , the vast majority of rollouts are concentrated in the best child(ren), so weighted average $\rightarrow$ max/min
- Theorem: as $N \rightarrow \infty$ UCT selects the minimax move
  - (but $N$ never approaches infinity!)

# Summary

- Games require decisions when optimality is impossible
  - Bounded-depth search and approximate evaluation functions
- Games force efficient use of computation
  - Alpha-beta pruning, MCTS
- Game playing has produced important research ideas
  - Reinforcement learning (checkers)
  - Iterative deepening (chess)
  - Rational metareasoning (Othello)
  - Monte Carlo tree search (chess, Go)
  - Solution methods for partial-information games in economics (poker)
- Video games present much greater challenges – lots to do!
  - $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$, partially observable, often > 2 players

# Next Time: Logic!