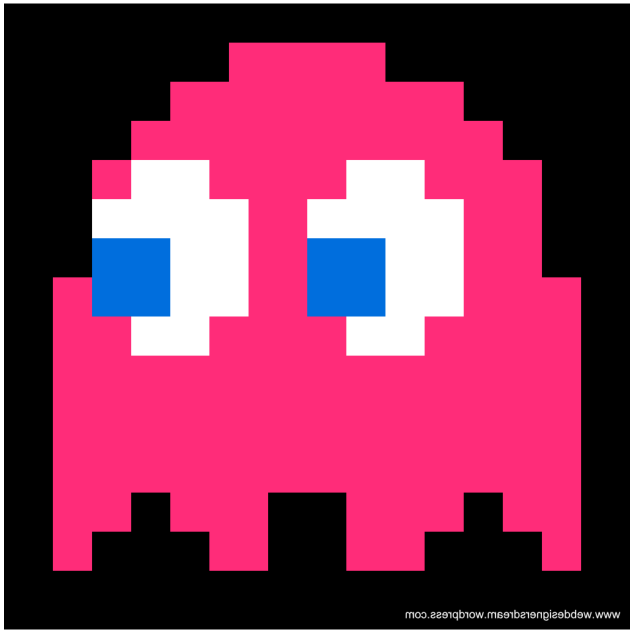
CS 188: Artificial Intelligence

Propositional Logic I



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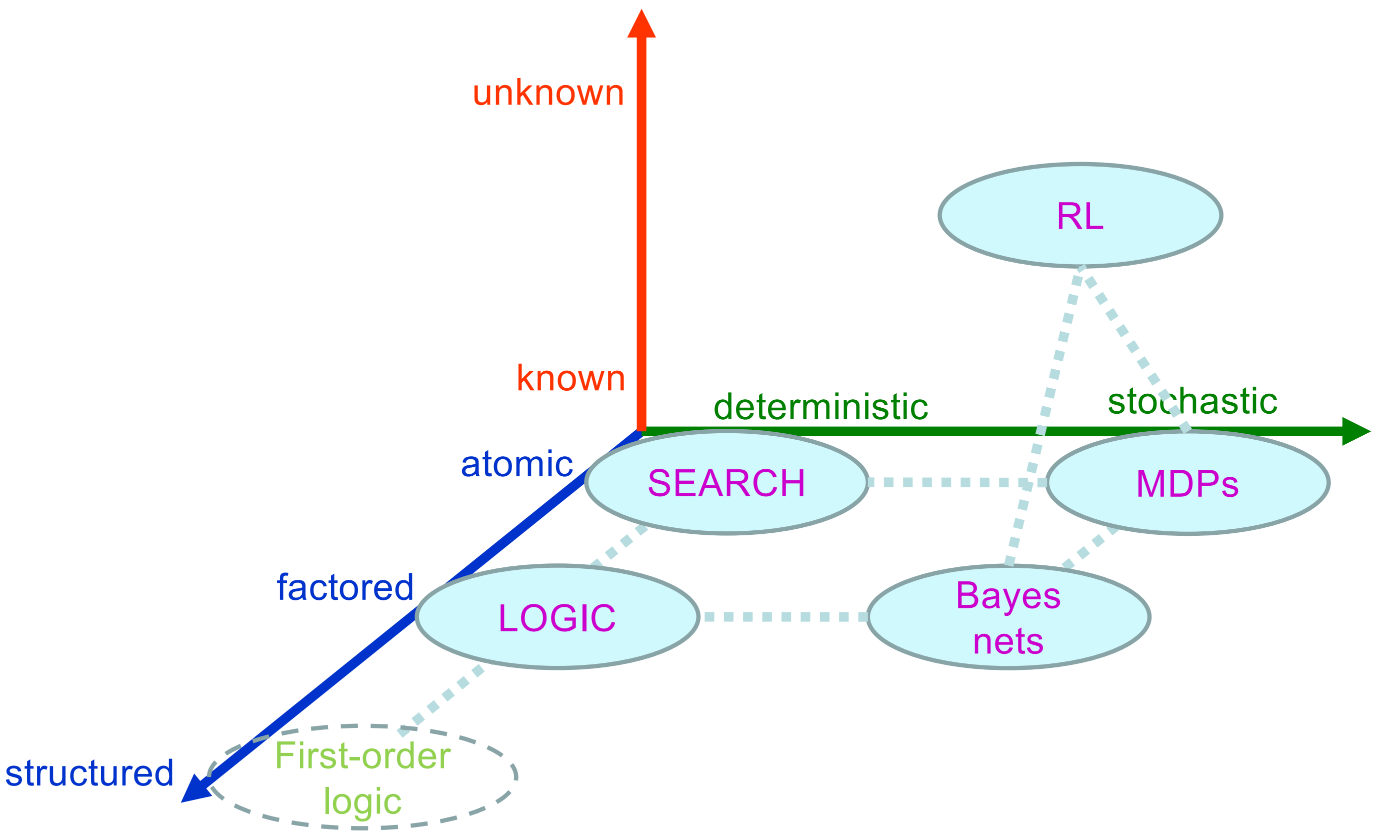
Outline of the course



Outline

1. Propositional Logic I
   - Basic concepts of knowledge, logic, reasoning
   - Propositional logic: syntax and semantics, Pacworld example
   - Inference by theorem proving
2. Propositional logic II
   - Inference by model checking
   - A Pac agent using propositional logic
3. First-order logic

Agents that know things

- Agents acquire knowledge through perception, learning, language
  - Knowledge of the effects of actions (“transition model”)
  - Knowledge of how the world affects sensors (“sensor model”)
  - Knowledge of the current state of the world
- Can keep track of a partially observable world
- Can formulate plans to achieve goals
- Can design and build gravitational wave detectors…..

Knowledge, contd.

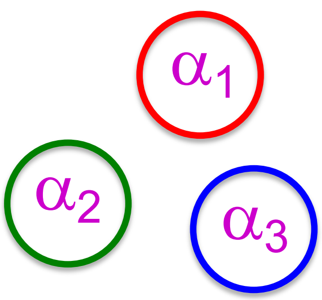
- Knowledge base = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  - ***Tell*** it what it needs to know (or have it ***Learn*** the knowledge)
  - Then it can ***Ask*** itself what to do—answers should follow from the KB
- Agents can be viewed at the ***knowledge level***
  i.e., what they ***know***, regardless of how implemented
- A single inference algorithm can answer any answerable question

Knowledge base	Domain-specific facts
Inference engine	Generic code

Logic

- ***Syntax***: What sentences are allowed?
- ***Semantics***:
  - What are the ***possible worlds***?
  - Which sentences are ***true*** in which worlds? (i.e., ***definition*** of truth)

Sentence: $x > y$
World1: $x = 5$; $y = 2$
World2: $x = 2$; $y = 3$



Syntaxland

Semanticsland

Different kinds of logic

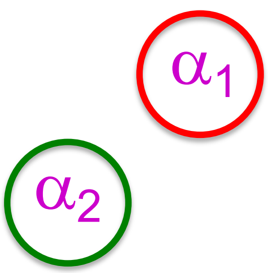
- Propositional logic
  - Syntax: $P \vee (\neg Q \wedge R)$; $X_1 \Leftrightarrow (\text{Raining} \Rightarrow \neg \text{Sunny})$
  - Possible world: {P=true,Q=true,R=false,S=true} or 1101
  - Semantics: $\alpha \wedge \beta$ is true in a world iff is α true and β is true (etc.)
- First-order logic
  - Syntax: $\forall x\ \exists y\ P(x,y) \wedge \neg Q(Joe,f(x)) \Rightarrow f(x)=f(y)$
  - Possible world: Objects o_1, o_2, o_3; P holds for <o_1,o_2>; Q holds for <o_3,o_2 >; $f(o_1)=o_1$; $Joe=o_3$; etc.
  - Semantics: $\phi(\sigma)$ is true in a world if $\sigma=o_j$ and ϕ holds for o_j; etc.

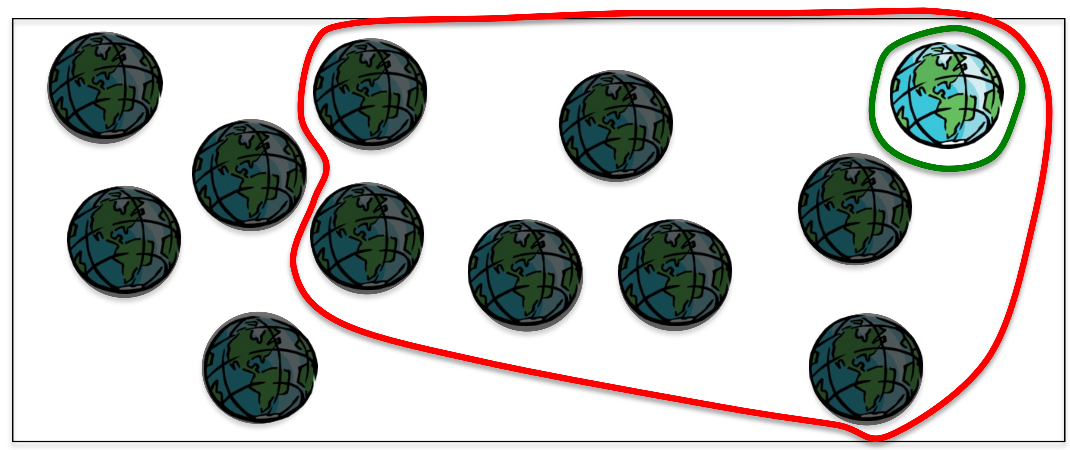
Different kinds of logic, contd.

- Relational databases:
  - Syntax: ground relational sentences, e.g., *Sibling*(*Ali*,*Bo*)
  - Possible worlds: (typed) objects and (typed) relations
  - Semantics: sentences in the DB are true, everything else is false
    - Cannot express disjunction, implication, universals, etc.
    - Query language (SQL etc.) typically some variant of first-order logic
    - Often augmented by first-order rule languages, e.g., Datalog
  - Knowledge graphs (roughly: relational DB + ontology of types and relations)
    - Google Knowledge Graph: 5 billion entities, 500 billion facts, >30% of queries
    - Facebook network: 2.8 billion people, trillions of posts, maybe quadrillions of facts

Inference: entailment

- ***Entailment***: $\alpha \models \beta$ ("α entails β" or "β follows from α") iff in every world where α is true, β is also true
  - I.e., the α-worlds are a subset of the β-worlds [***models***(α) $\subseteq$ ***models***(β)]
- In the example, $\alpha_2 \models \alpha_1$
- (Say α_2 is $\neg Q \wedge R \wedge S \wedge W$
  α_1 is $\neg Q$)





Inference: proofs

- A proof is a ***demonstration*** of entailment between α and β
- ***Sound*** algorithm: everything it claims to prove is in fact entailed
- ***Complete*** algorithm: every that is entailed can be proved

Inference: proofs

- Method 1: ***model-checking***
  - For every possible world, if α is true make sure that is β true too
  - OK for propositional logic (finitely many worlds); not easy for first-order logic
- Method 2: ***theorem-proving***
  - Search for a sequence of proof steps (applications of ***inference rules***) leading from α to β
  - E.g., from $P \wedge (P \Rightarrow Q)$, infer Q by ***Modus Ponens***

Propositional logic syntax

- Given: a set of proposition symbols $\{X_1, X_2, \ldots, X_n\}$
  - (we often add True and False for convenience)
- X_i is a sentence
- If α is a sentence then $\neg\alpha$ is a sentence
- If α and β are sentences then $\alpha \wedge \beta$ is a sentence
- If α and β are sentences then $\alpha \vee \beta$ is a sentence
- If α and β are sentences then $\alpha \Rightarrow \beta$ is a sentence
- If α and β are sentences then $\alpha \Leftrightarrow \beta$ is a sentence
- And p.s. there are no other sentences!

Propositional logic semantics

- Let *m* be a model assigning true or false to $\{X_1, X_2, \ldots, X_n\}$
- If α is a symbol then its truth value is given in *m*
- $\neg\alpha$ is true in *m* iff α is false in *m*
- $\alpha \wedge \beta$ is true in *m* iff α is true in *m* and β is true in *m*
- $\alpha \vee \beta$ is true in *m* iff α is true in *m* or β is true in *m*
- $\alpha \Rightarrow \beta$ is true in *m* iff α is false in *m* or β is true in *m*
- $\alpha \Leftrightarrow \beta$ is true in *m* iff $\alpha \Rightarrow \beta$ is true in *m* and $\beta \Rightarrow \alpha$ is true in *m*

Propositional logic semantics in code

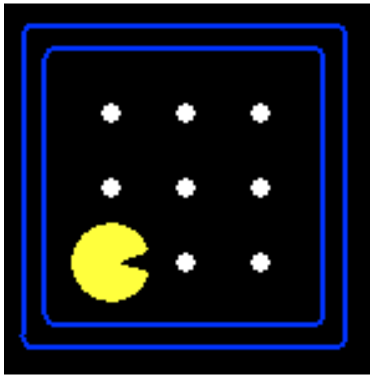
```
function PL-TRUE?(α,model) returns true or false
   if α is a symbol then return Lookup(α, model)
   if Op(α) = ¬ then return not(PL-TRUE?(Arg1(α),model))
   if Op(α) = ∧ then return  and(PL-TRUE?(Arg1(α),model),
                                 PL-TRUE?(Arg2(α),model))
   etc.
```

(Sometimes called "recursion over syntax")

- Sentence: $P \wedge (\neg Q \vee R)$
- Model/possible-world/assignment-of-values-variables: {P=true,Q=true,R=false} or 110

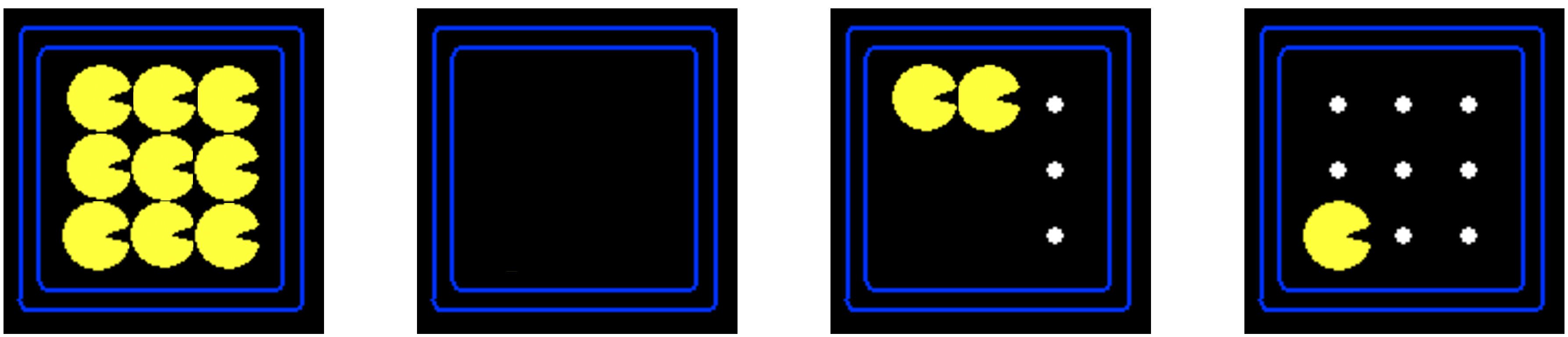
Example: Partially observable Pacman

- Pacman knows the map but perceives just wall/gap to NSEW
- Formulation: ***what variables do we need?***
  - Wall locations
    - Wall_0,0 there is a wall at [0,0]
    - Wall_0,1 there is a wall at [0,1], etc. (*N* symbols for *N* locations)
  - Percepts
    - ~~Blocked_W (blocked by wall to my West) etc.~~
    - Blocked_W_0 (blocked by wall to my West ***at time 0***) etc. (4*T* symbols for *T* time steps)
  - Actions
    - W_0 (Pacman moves West at time 0), E_0 etc. (4*T* symbols)
  - Pacman's location
    - At_0,0_0 (Pacman is at [0,0] at time 0), At_0,1_0 etc. (*NT* symbols)



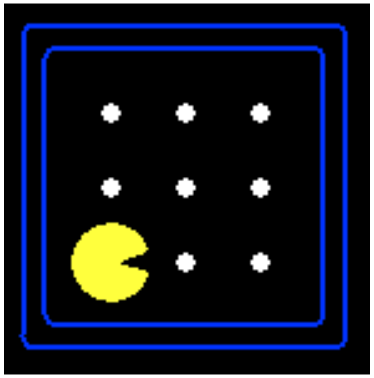
How many possible worlds?

- N locations, T time steps => $N + 4T + 4T + NT = O(NT)$ variables
- $2^{O(NT)}$ possible worlds!
- N=200, T=400 => ~10^{24000} worlds
- Each world is a complete "history"
  - But most of them are pretty weird!



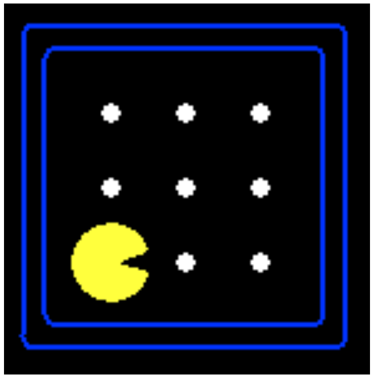
Pacman's knowledge base: Map

- Pacman knows where the walls are:
  - Wall_0,0 ∧ Wall_0,1 ∧ Wall_0,2 ∧ Wall_0,3 ∧ Wall_0,4 ∧ Wall_1,4 ∧ …
- Pacman knows where the walls aren't!
  - ¬Wall_1,1 ∧ ¬Wall_1,2 ∧ ¬Wall_1,3 ∧ ¬Wall_2,1 ∧ ¬Wall_2,2 ∧ …



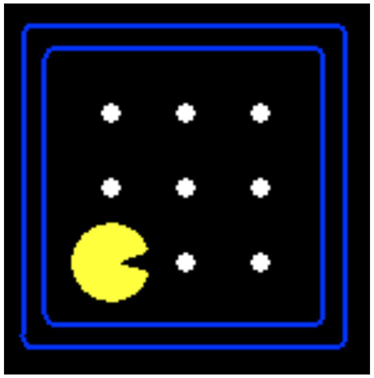
Pacman's knowledge base: Initial state

- Pacman doesn't know where he is
- But he knows he's somewhere!
  - At_1,1_0 ∨ At_1,2_0 ∨ At_1,3_0 ∨ At_2,1_0 ∨ …
- And he knows he's not in more than one place!
  - ¬ (At_1,1_0 ∧ At_1,2_0) ∧ ¬ (At_1,1_0 ∧ At_1,3_0) …



Pacman's knowledge base: Sensor model

- State facts about how Pacman's percepts arise…
  - <Percept variable at t> ⇔ <some condition on world at t>
- Pacman perceives a wall to the West at time *t* ***if and only if*** he is in *x,y* and there is a wall at *x-1,y*
  - Blocked_W_0 ⇔ ((At_1,1_0 ∧ Wall_0,1) v
    (At_1,2_0 ∧ Wall_0,2) v
    (At_1,3_0 ∧ Wall_0,3) v ….)
  - 4T sentences, each of size *O*(*N*)
  - Note: these are valid for any map



Pacman's knowledge base: Transition model

- How does each ***state variable*** at each time gets its value?
  - Here we care about location variables, e.g., At_3,3_17
- A state variable X gets its value according to a ***successor-state axiom***
  - X_t ⇔ [X_t-1 ∧ ¬(some action_t-1 made it false)] v

    [¬X_t-1 ∧ (some action_t-1 made it true)]
- For Pacman location:
  - At_3,3_17 ⇔ [At_3,3_16 ∧ ¬((¬Wall_3,4 ∧ N_16) v (¬Wall_4,3 ∧ E_16) v …)]

    v [¬At_3,3_16 ∧ ((At_3,2_16 ∧ ¬Wall_3,3 ∧ N_16) v

    (At_2,3_16 ∧ ¬Wall_3,3 ∧ E_16) v …)]

How many sentences?

- Vast majority of KB occupied by O(*NT*) transition model sentences
  - Each about 10 lines of text
  - *N*=200, *T*=400 => ~800,000 lines of text, or 20,000 pages
- This is because propositional logic has limited expressive power
- Are we really going to write 20,000 pages of logic sentences???
- No, but your code will generate all those sentences!
- In first-order logic, we need O(1) transition model sentences
- (State-space search uses atomic states: how do we keep the transition model representation small???)

Entails vs. Implies

- Entails: $\alpha \models \beta$
- Implies: $\alpha \Rightarrow \beta$
- One is a well-formed sentence in proposition logic
- One is a fact about sets of models where sentences are true
- Intuitive connection?
- KB is a set of sentences (or KB is one sentence with lots of $\wedge$s)
- If $\alpha \Rightarrow \beta \in$ KB, then $\alpha \wedge$ KB $\models \beta$ (Modus ponens)
- If you want $\alpha \wedge$ KB $\models \beta$, good idea to put $\alpha \Rightarrow \beta \in$ KB

Some reasoning tasks

- ***Localization*** with a map and local sensing:
  - Given an initial KB, plus a sequence of percepts and actions, where am I?
- ***Mapping*** with a location sensor:
  - Given an initial KB, plus a sequence of percepts and actions, what is the map?
- ***Simultaneous localization and mapping***:
  - Given …, where am I and what is the map?
- ***Planning***:
  - Given …, what action sequence is guaranteed to reach the goal?
- ***ALL OF THESE USE THE SAME KB AND THE SAME ALGORITHM!!***

Summary

- One possible agent architecture: knowledge + inference
- Logics provide a formal way to encode knowledge
  - A logic is defined by: syntax, set of possible worlds, truth condition
- A simple KB for Pacman covers the initial state, sensor model, and transition model
- Logical inference computes entailment relations among sentences, enabling a wide range of tasks to be solved