Announcements

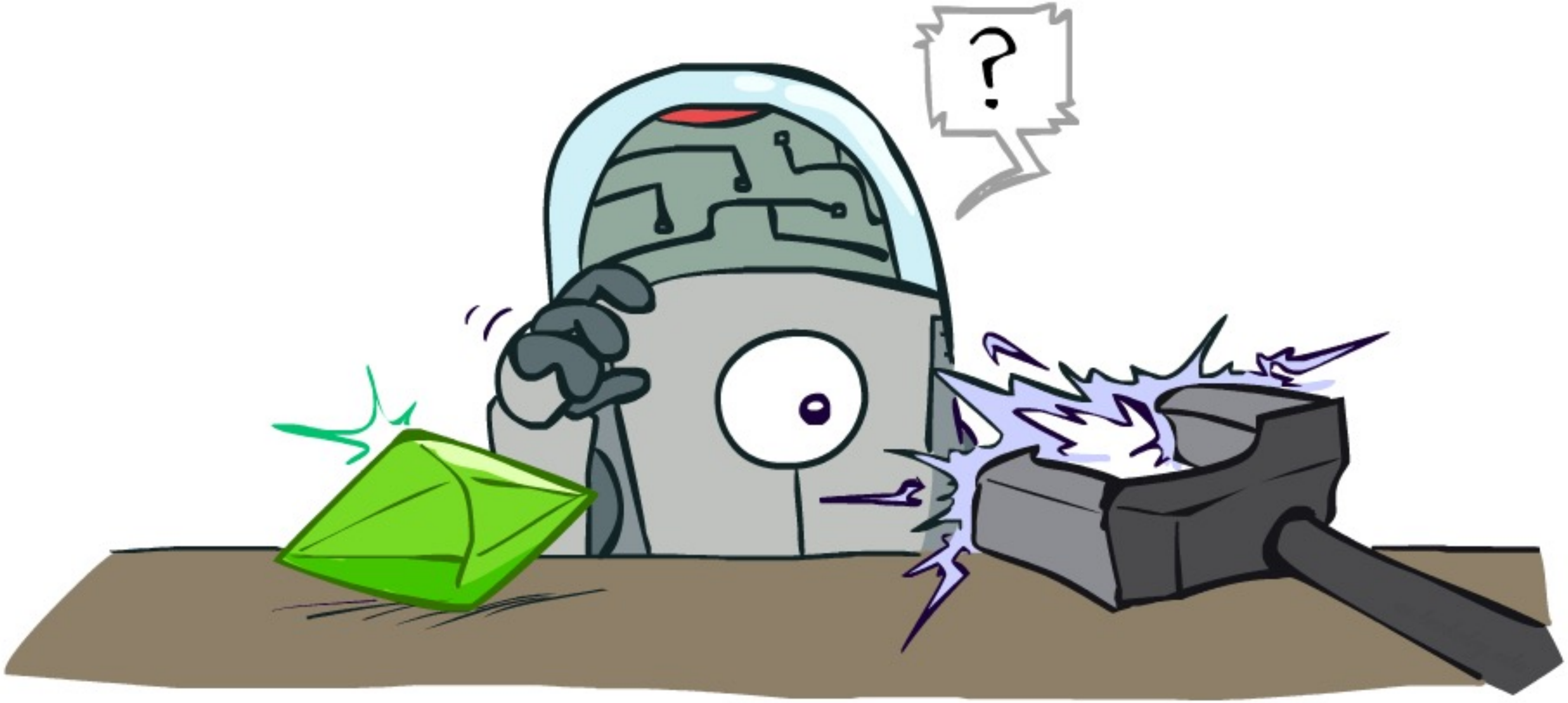
- HW 9 and Project 5 are due **tonight, April 16, 11:59 PM PT**
- HW 10 will be released soon, due **Tuesday, April 23, 11:59 PM PT**
- Project 6 out later this week, due **Friday, April 26, 11:59 PM PT**

- **Course evaluations are live!**
  - Log in at course-evaluations.berkeley.edu

Pre-scan attendance QR code now!

CS 188: Artificial Intelligence

Reinforcement Learning



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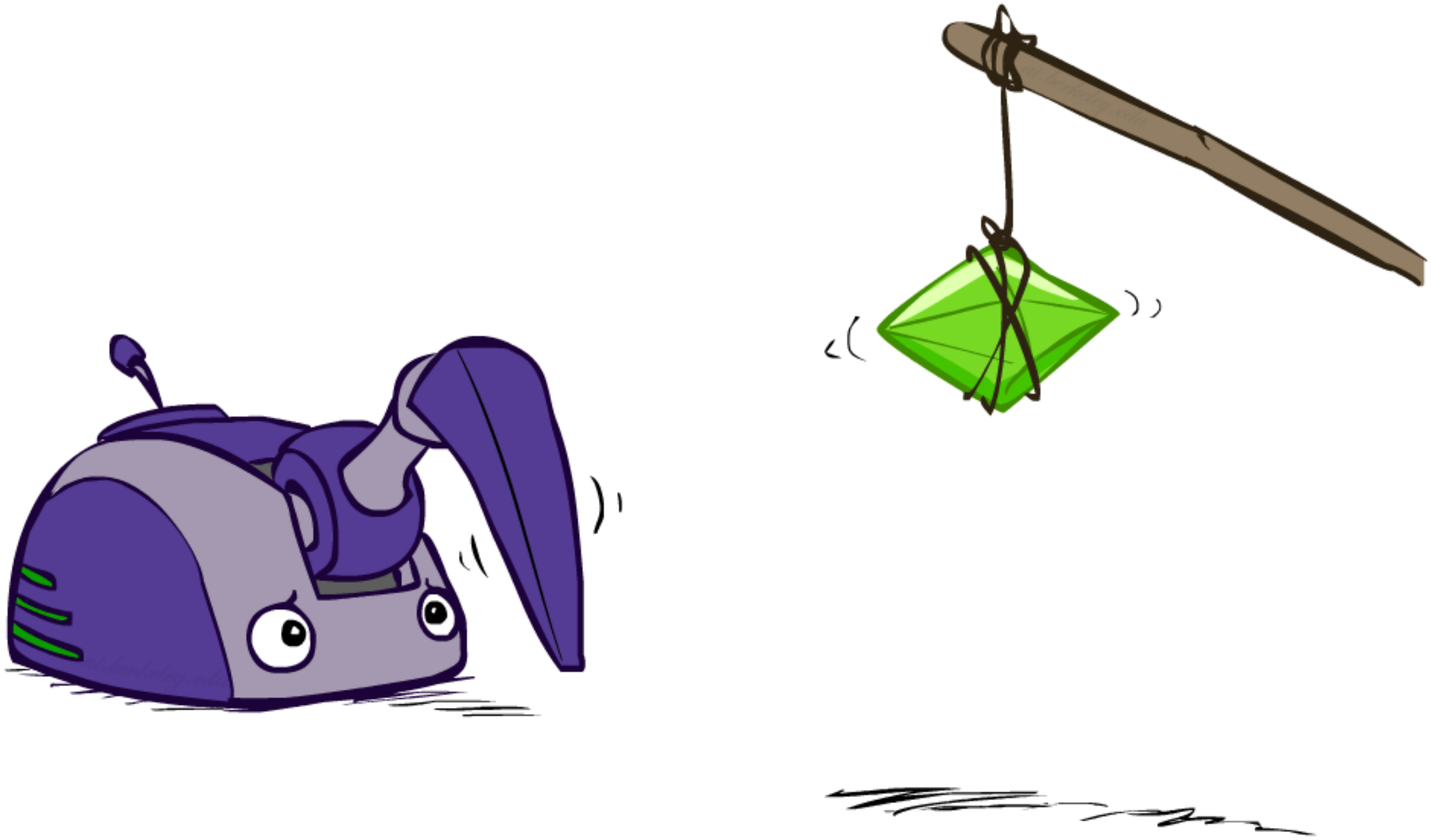
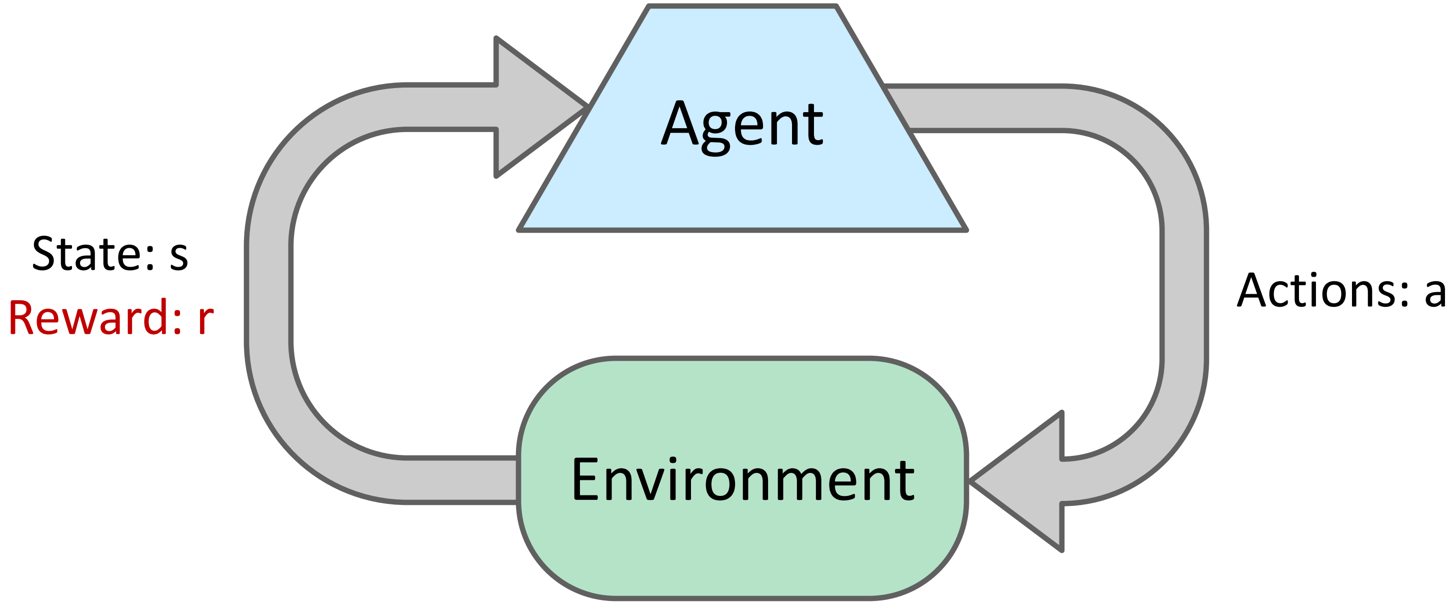
University of California, Berkeley

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]

Reinforcement Learning



Reinforcement Learning



- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

Example: Samuel's checker player (1956-67)

Example: Learning to Walk

Initial

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk

Finished

[Kohl and Stone, ICRA 2004]

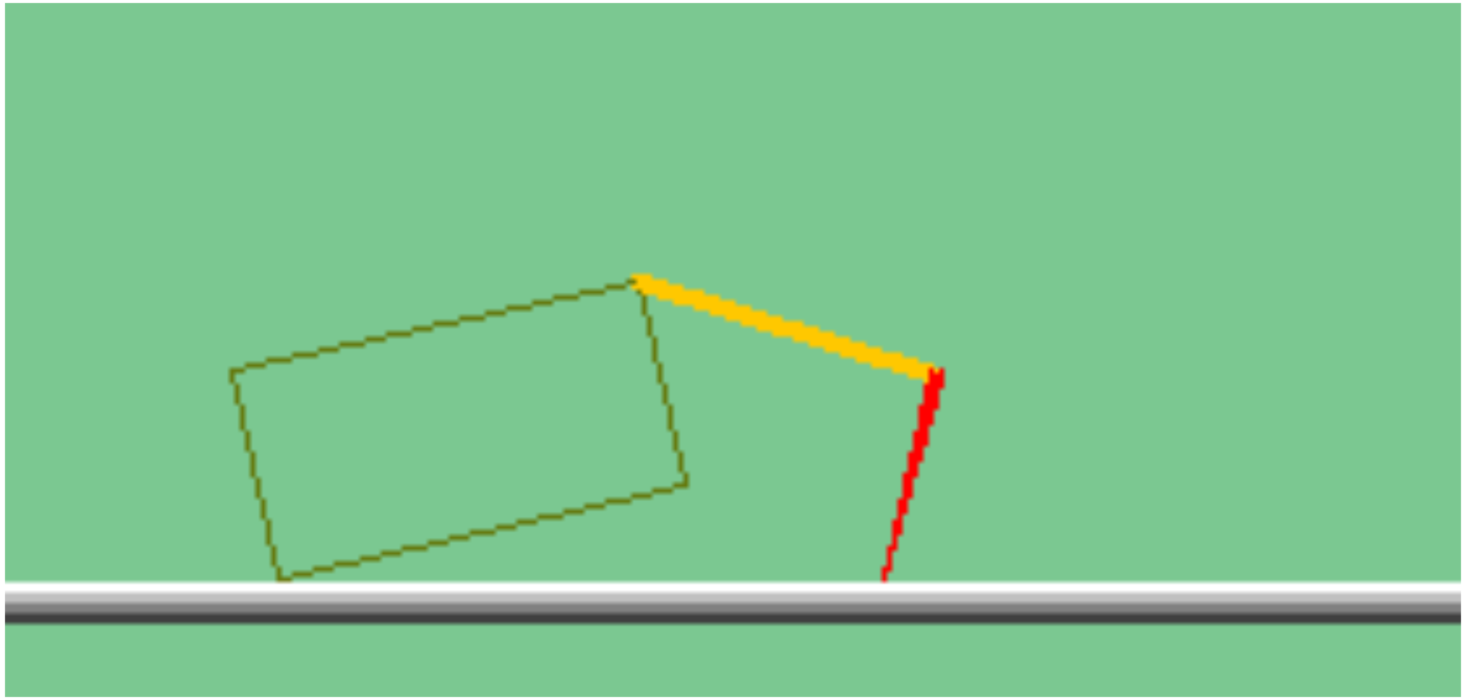
[Video: AIBO WALK – finished]

Example: Breakout (DeepMind)

[© TwoMinuteLectures]

Example: AlphaGo (2016)

The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project 6]

Video of Demo Crawler Bot

Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states $s \in S$
  - A set of actions (per state) *A*(*s*)
  - A transition model *T*(*s*,*a*,*s'*)
  - A reward function *R*(*s*,*a*,*s'*)
- Still looking for a policy $\pi(s)$

- New twist: don't know *T* or *R*
  - I.e. we don't know which states are good or what the actions do
  - Must explore new states and actions to discover how the world works

Reinforcement Learning

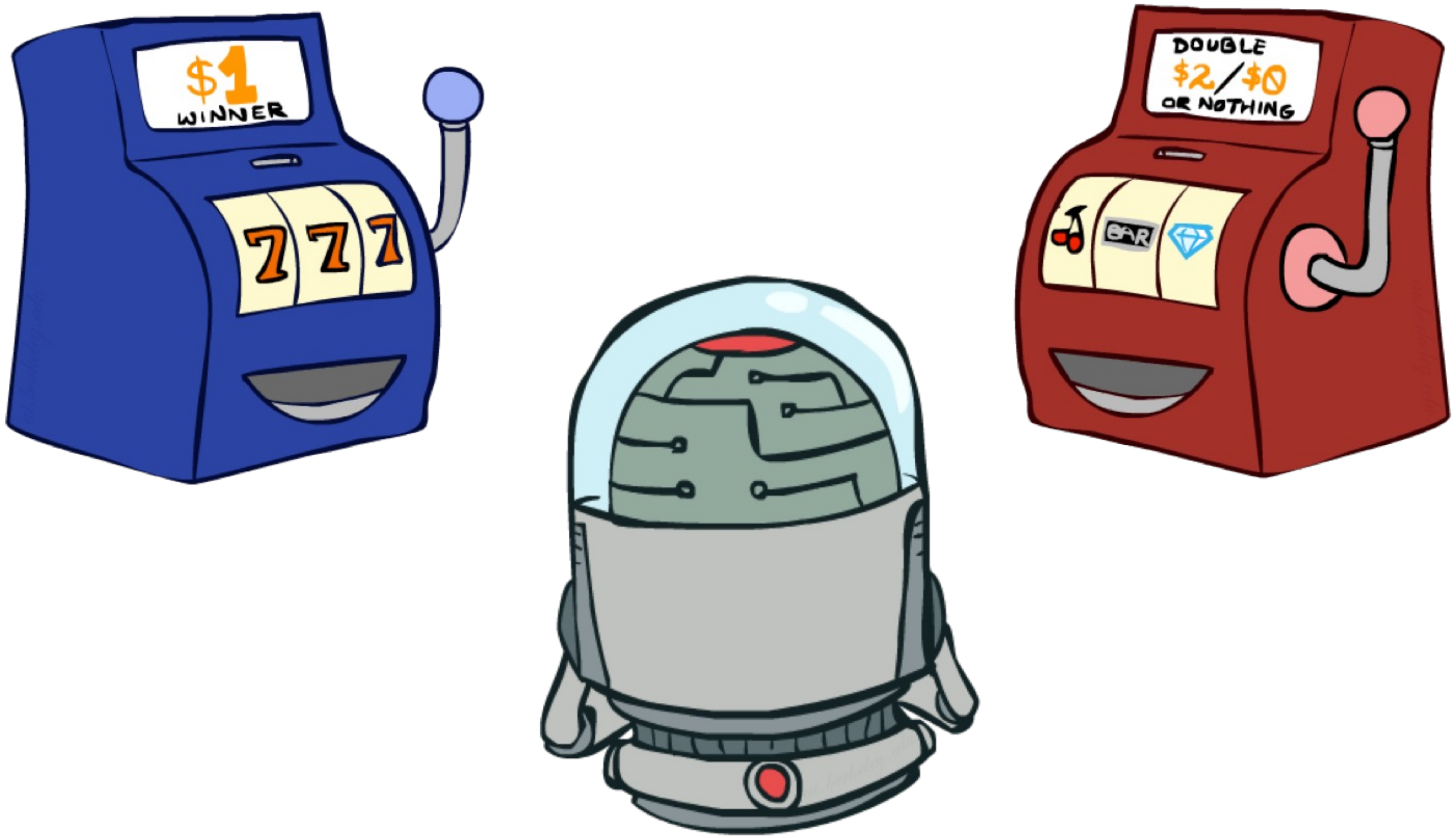
- What if the MDP is initially unknown? Lots of things change!
  - ***Exploration***: you have to ***try unknown actions*** to get information
  - ***Exploitation***: eventually, you have to use what you know
  - ***Regret***: early on, you inevitably “make mistakes” and lose reward
  - ***Sampling***: you may need to repeat many times to get good estimates
  - ***Generalization***: what you learn in one state may apply to others too

Bandits

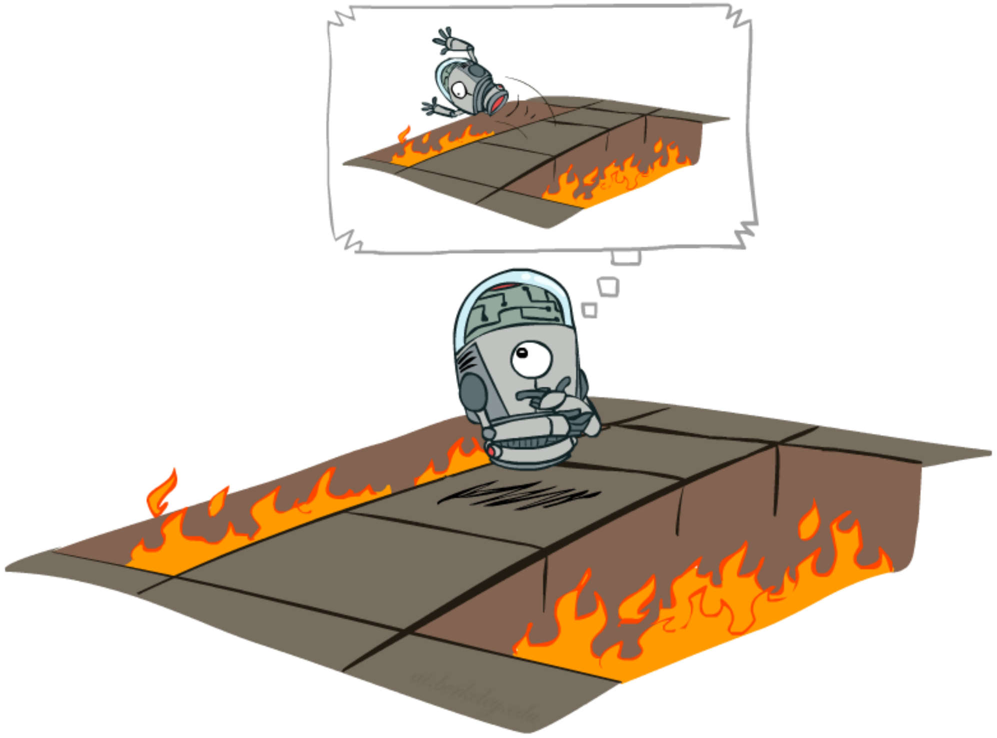
- Exactly one state
- Set of actions: A
- Stochastic reward function: $P(r|a)$

Contextual Bandits:

- Set of states $s \in S$
- Transitions always return to start state distribution $P(s'|s, a) = P_0(s')$



Offline (MDPs) vs. Online (RL)



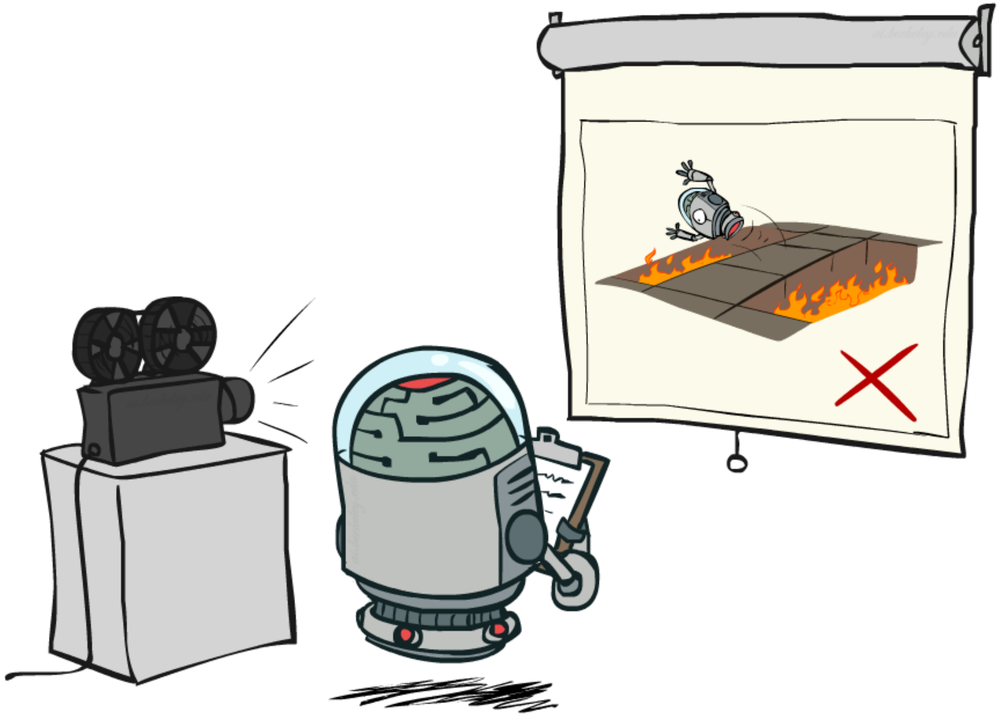
Offline Solution

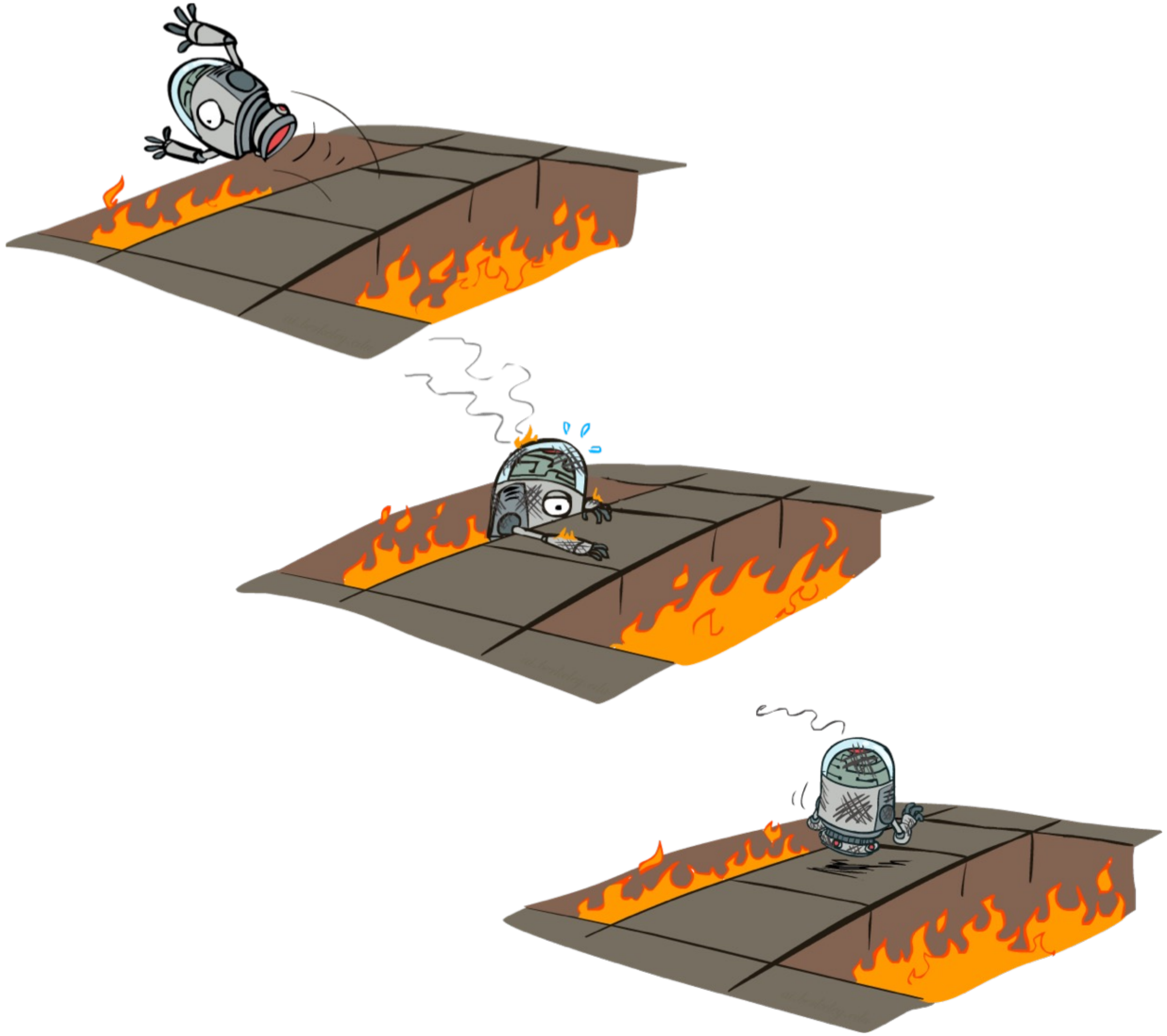
Online Learning

Approaches to reinforcement learning

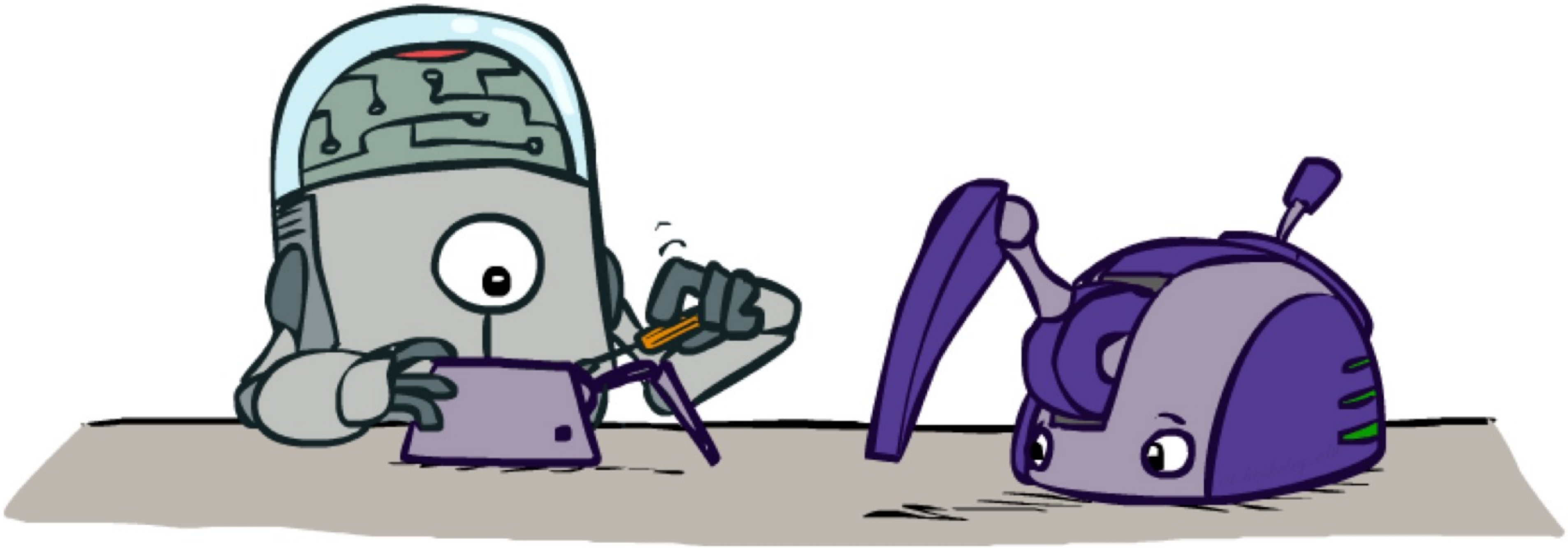
1. Model-based: Learn the model, solve it, execute the solution
2. Learn values from experiences, use to make decisions
   a. Direct evaluation
   b. Temporal difference learning
   c. Q-learning
3. Optimize the policy directly

Passive vs Active Reinforcement Learning





Model-Based RL

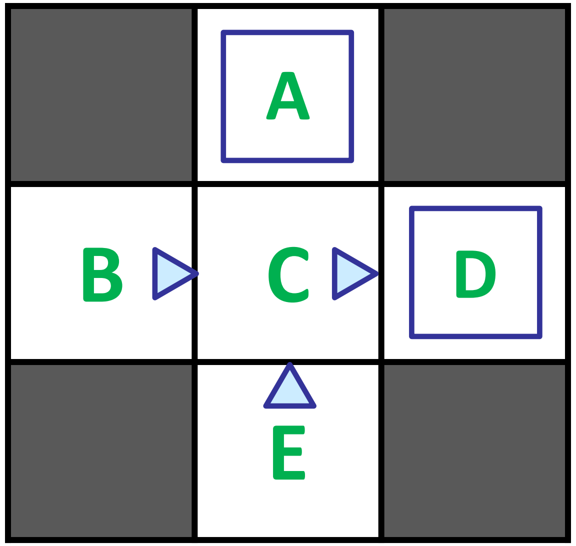


Model-Based Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
  - Count outcomes *s'* for each *s*, *a*
  - Directly estimate each entry in *T(s,a,s')* from counts
  - Discover each *R(s,a,s')* when we experience the transition
- Step 2: Solve the learned MDP
  - Use, e.g., value or policy iteration, as before

Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

T(s,a,s')

T(B, east, C) = 1.00
P(C, east, D) = 0.75
P(C, east, A) = 0.25
…

R(s,a,s')

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Pros and cons

- Pro:
  - Makes efficient use of experiences (low ***sample complexity***)
- Con:
  - May not scale to large state spaces
    - Solving MDP is intractable for very large $|S|$
  - RL feedback loop tends to magnify small model errors
  - Much harder when the environment is partially observable

Basic idea of model-free methods

- To approximate expectations with respect to a distribution, you can either
  - Estimate the distribution from samples, compute an expectation
  - Or, bypass the distribution and estimate the expectation from samples directly

Example: Expected Age

Goal: Compute expected age of cs188 students

Known $P(A)$

$$E[A] = \sum_a P(a) \cdot a \;=\; 0.35 \times 20 + \ldots$$

Without $P(A)$, instead collect samples $[a_1, a_2, \ldots a_N]$

"Model Based": estimate $P(A)$:

$$\hat{P}(A{=}a) = N_a/N$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Why does this work? Because eventually you learn the right model.

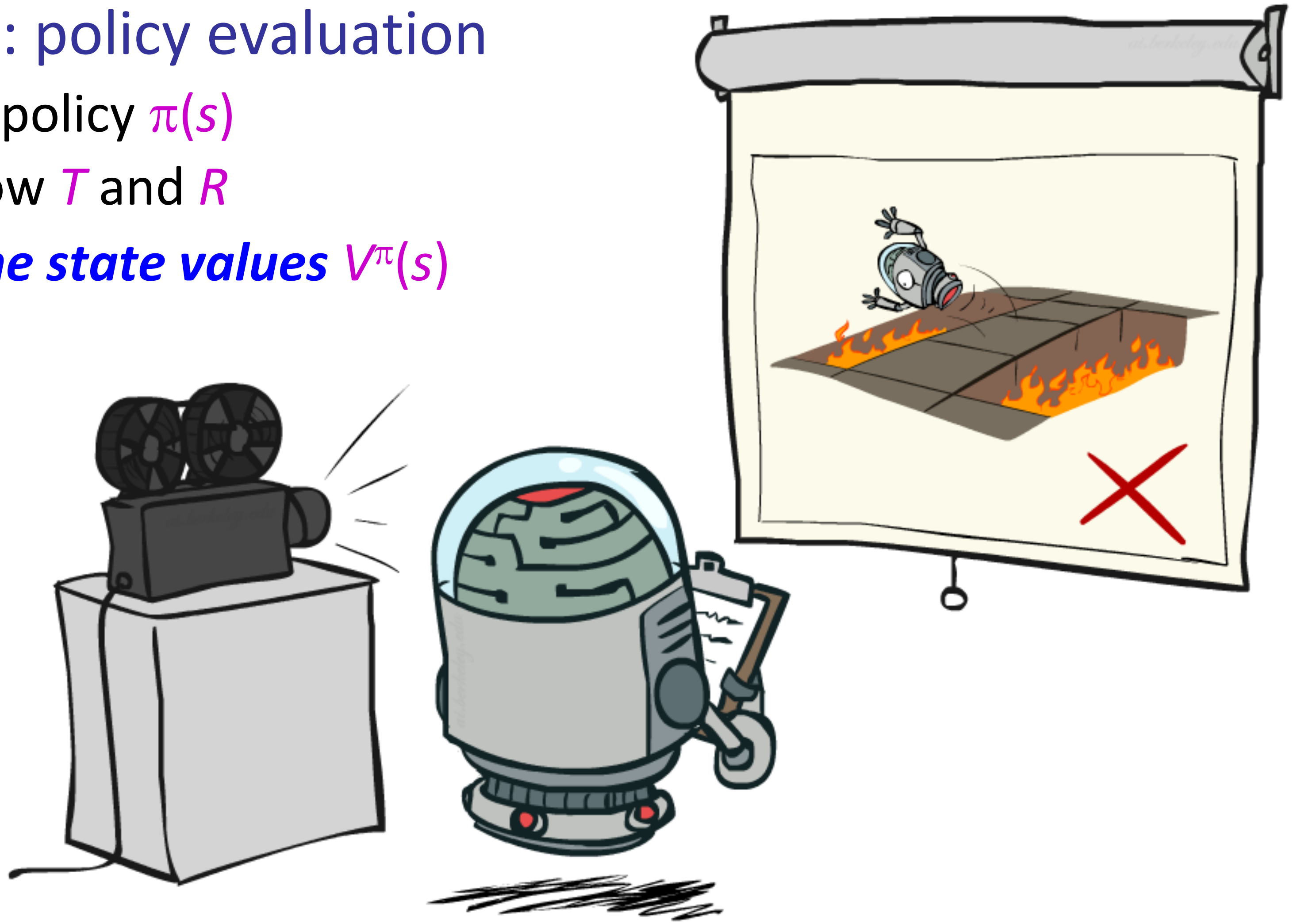
"Model Free": estimate expectation

$$E[A] \approx 1/N \; \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know T and R
  - ***Goal: learn the state values*** $V^{\pi}(s)$



Direct evaluation

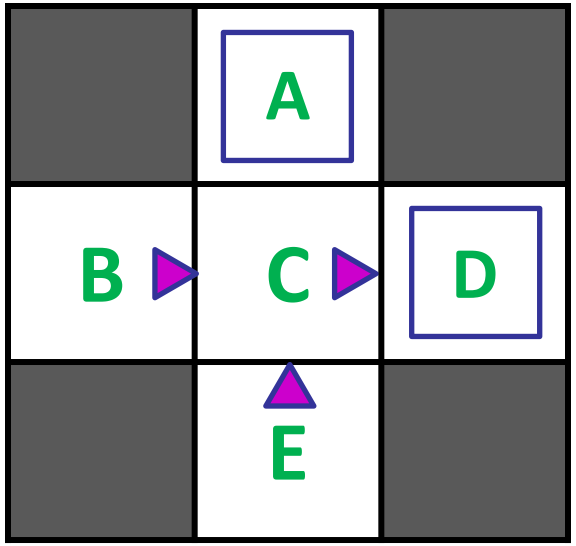
- Goal: Estimate $V^{\pi}(s)$, i.e., expected total discounted reward from s onwards

- Idea:
  - Use ***returns***, the *actual* sums of discounted rewards from s
  - Average over multiple trials and visits to s

- This is called ***direct evaluation*** (or direct utility estimation)

Example: Direct Estimation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

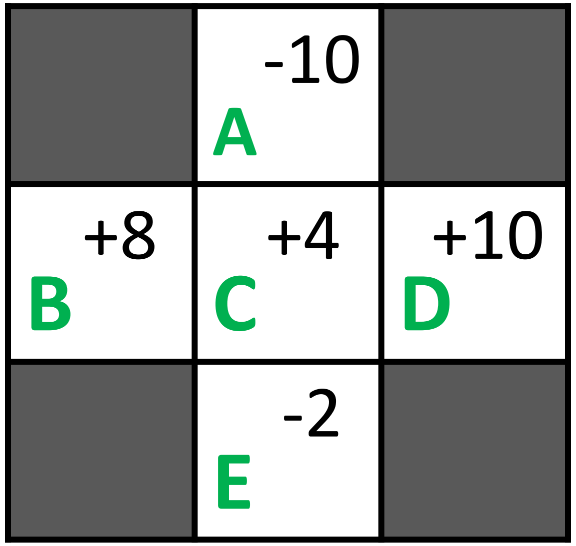
Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values

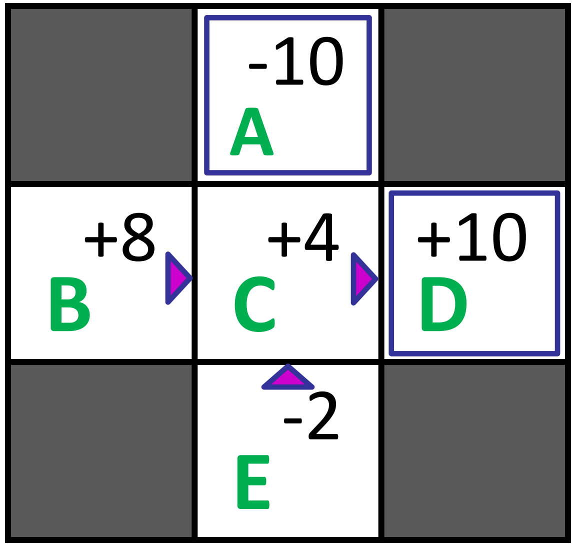


Problems with Direct Estimation

- What's good about direct estimation?
  - It's easy to understand
  - It doesn't require any knowledge of *T* and *R*
  - It converges to the right answer in the limit

- What's bad about it?
  - Each state must be learned separately (fixable)
  - It ***ignores information about state connections***
  - So, it takes a long time to learn

Output Values

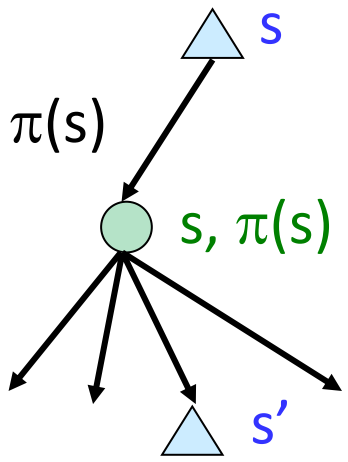


If B and E both go to C under this policy, how can their values be different?

E.g., B=at home, study hard
E=at library, study hard
C=know material, go to exam

Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often



- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average

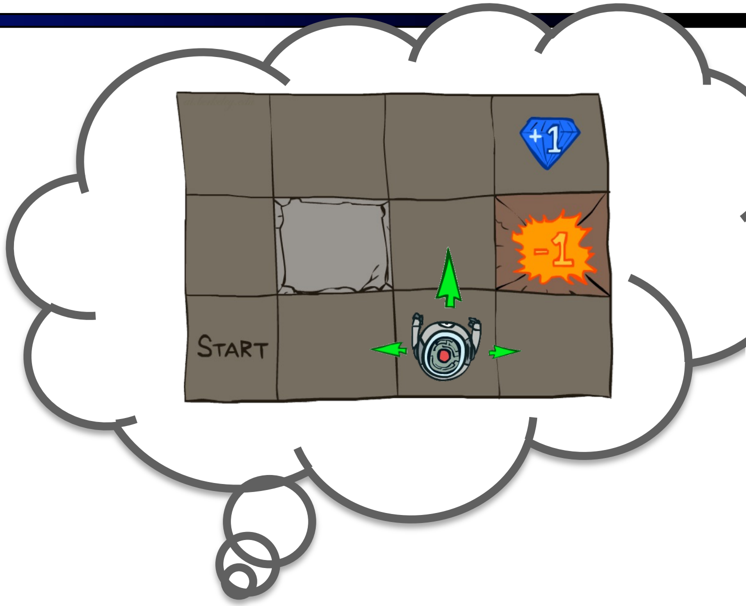
Sample of V(s): $$sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$$

Update to V(s): $$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha) sample$$

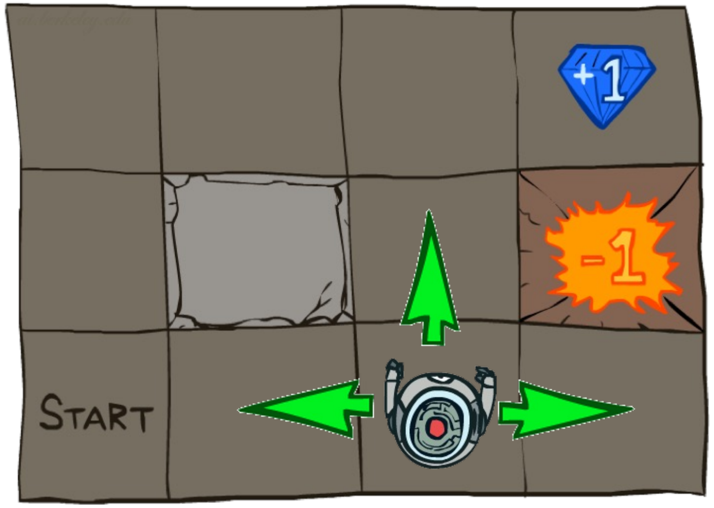
Same update: $$V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$$

TD as approximate Bellman update

- Given a fixed policy, the value of a state is an expectation over next-state values:
  - $V^{\pi}(s) = \sum_{s'} T(s,\pi(s),s') [R(s,\pi(s),s') + \gamma V^{\pi}(s')]$



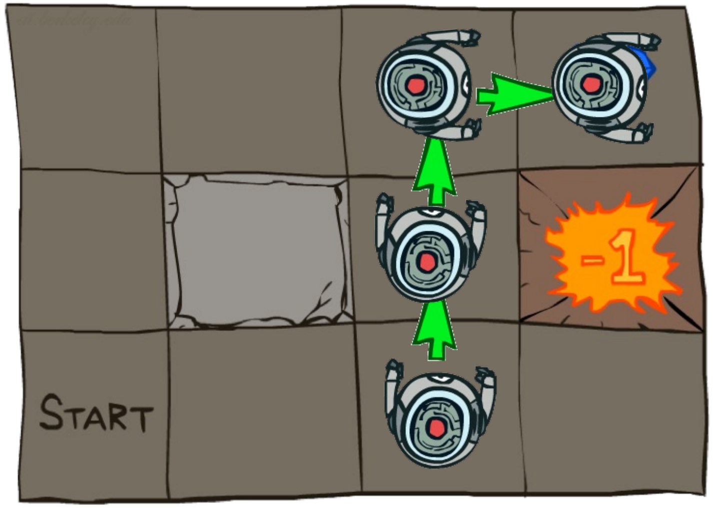
- Idea 1: Use actual samples to estimate the expectation:
  - $\text{sample}_1 = R(s,\pi(s),s_1') + \gamma V^{\pi}(s_1')$
  - $\text{sample}_2 = R(s,\pi(s),s_2') + \gamma V^{\pi}(s_2')$
  - …
  - $\text{sample}_N = R(s,\pi(s),s_N') + \gamma V^{\pi}(s_N')$
  - $V^{\pi}(s) \leftarrow 1/N \sum_i \text{sample}_i$



TD as approximate Bellman update

- Idea 2: Update value of s after each transition s,a,s',r :

- Update V^{π} ([3,1]) based on R([3,1],up,[3,2]) and γV^{π}([3,2])
- Update V^{π} ([3,2]) based on R([3,2],up,[3,3]) and γV^{π}([3,3])
- Update V^{π} ([3,3]) based on R([3,3],right,[4,3]) and γV^{π}([4,3])



TD as approximate Bellman update

- Idea 3: Update values by maintaining a ***running average***

Running averages

- How do you compute the average of 1, 4, 7?
- Method 1: add them up and divide by N
  - $1+4+7 = 12$
  - average $= 12/N = 12/3 = 4$
- Method 2: keep a running average μ_n and a running count n
  - $n=0 \quad \mu_0=0$
  - $n=1 \quad \mu_1 = (0 \cdot \mu_0 + x_1)/1 = (0 \cdot 0 + 1)/1 = 1$
  - $n=2 \quad \mu_2 = (1 \cdot \mu_1 + x_2)/2 = (1 \cdot 1 + 4)/2 = 2.5$
  - $n=3 \quad \mu_3 = (2 \cdot \mu_2 + x_3)/3 = (2 \cdot 2.5 + 7)/3 = 4$
  - General formula: $\mu_n = ((n-1) \cdot \mu_{n-1} + x_n)/n$
  - $= [(n-1)/n]\, \mu_{n-1} + [1/n]\, x_n$ (weighted average of old mean, new sample)

Running averages contd.

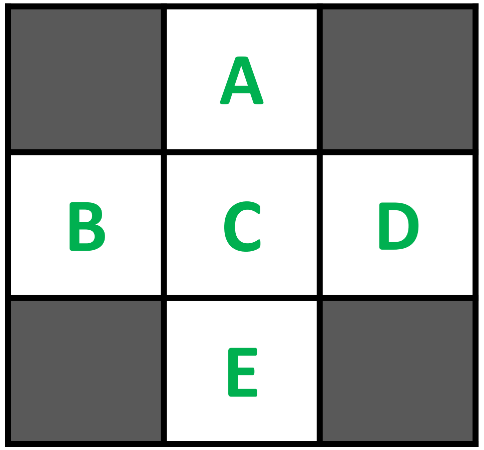
- What if we use a weighted average with a fixed weight?
  - $\mu_n = (1-\alpha)\,\mu_{n-1} + \alpha\, x_n$
  - n=1 $\quad \mu_1 = x_1$
  - n=2 $\quad \mu_2 = (1-\alpha)\cdot\mu_1 + \alpha x_2 = (1-\alpha)\cdot x_1 + \alpha x_2$
  - n=3 $\quad \mu_3 = (1-\alpha)\cdot\mu_2 + \alpha x_3 = (1-\alpha)^2\cdot x_1 + \alpha(1-\alpha)x_2 + \alpha x_3$
  - n=4 $\quad \mu_4 = (1-\alpha)\cdot\mu_3 + \alpha x_4 = (1-\alpha)^3\cdot x_1 + \alpha(1-\alpha)^2 x_2 + \alpha(1-\alpha)x_3 + \alpha x_4$
- I.e., ***exponential forgetting*** of old values
- μ_n is a convex combination of sample values (weights sum to 1)
- $E[\mu_n]$ is a convex combination of $E[X_i]$ values, hence unbiased

TD as approximate Bellman update

- Idea 3: Update values by maintaining a ***running average***
  - sample = $R(s,\pi(s),s') + \gamma V^{\pi}(s')$
  - $V^{\pi}(s) \leftarrow (1-\alpha) \cdot V^{\pi}(s) + \alpha \cdot \text{sample}$
  - $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha \cdot [\text{sample} - V^{\pi}(s)]$
  - This is the ***temporal difference learning rule***
  - $[\text{sample} - V^{\pi}(s)]$ is the "TD error"
  - α is the ***learning rate***
- Observe a sample, move $V^{\pi}(s)$ a little bit to make it more consistent with its neighbor $V^{\pi}(s')$

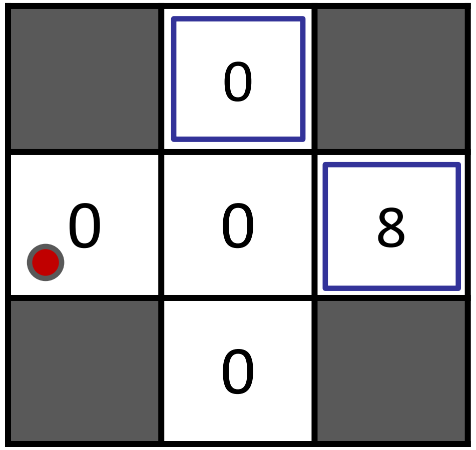
Example: Temporal Difference Learning

States

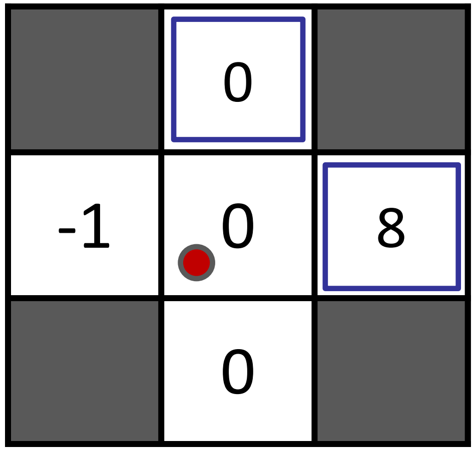


Assume: $\gamma = 1$, $\alpha = 1/2$

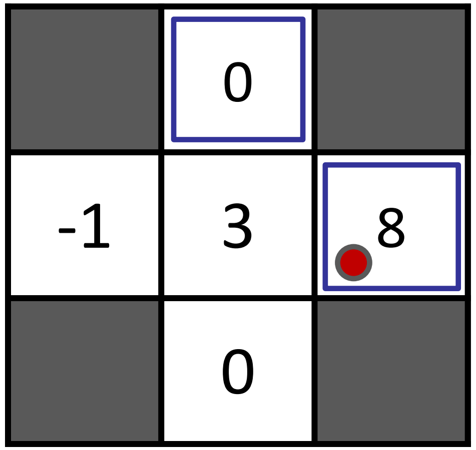
Observed Transitions



B, east, **C**, -2



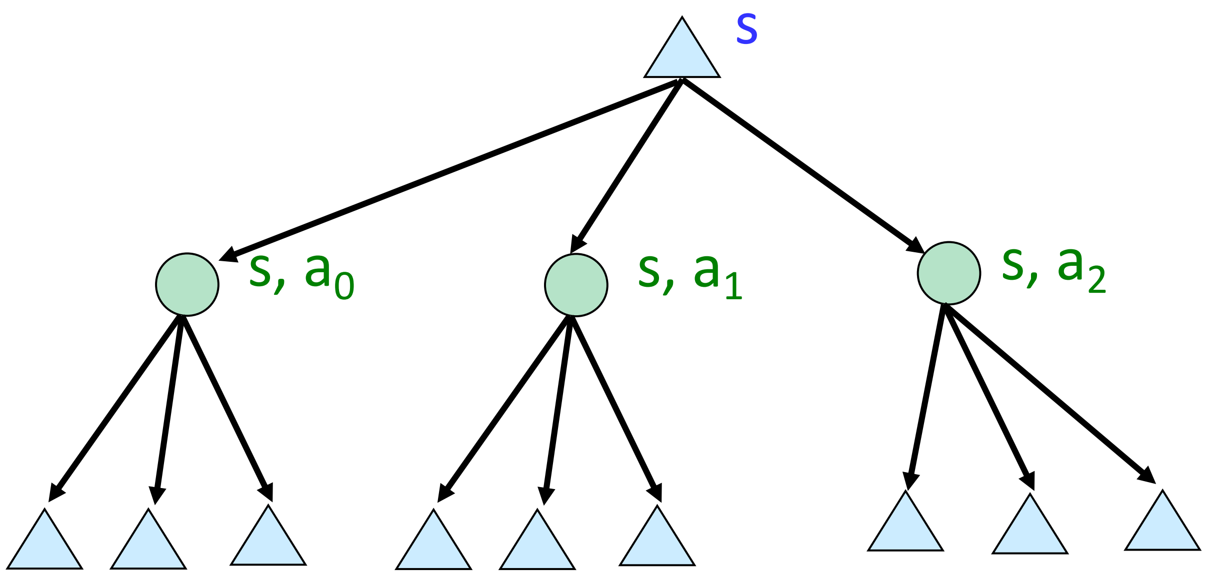
C, east, **D**, -2



$$V^{\pi}(s) \leftarrow (1-\alpha)\, V^{\pi}(s) + \alpha \cdot [R(s,\pi(s),s') + \gamma V^{\pi}(s')]$$

Problems with TD Value Learning

- Model-free policy evaluation! 🎉
- Bellman updates with running sample mean! 🎉



- Need the transition model to improve the policy! 😱

Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Q_k, calculate the depth (k+1) q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

Q-learning as approximate Q-iteration

- Recall the definition of Q values:
  - $Q^*(s,a)$ = expected return from doing a in s and then behaving optimally thereafter; and $\pi^*(s) = \max_a Q^*(s,a)$
- Bellman equation for Q values:
  - $Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')]$
- Approximate Bellman update for Q values:
  - $Q(s,a) \leftarrow (1-\alpha) \cdot Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a')]$
- We obtain a policy from learned Q(s,a), with no model!
  - (No free lunch: Q(s,a) table is |A| times bigger than V(s) table)

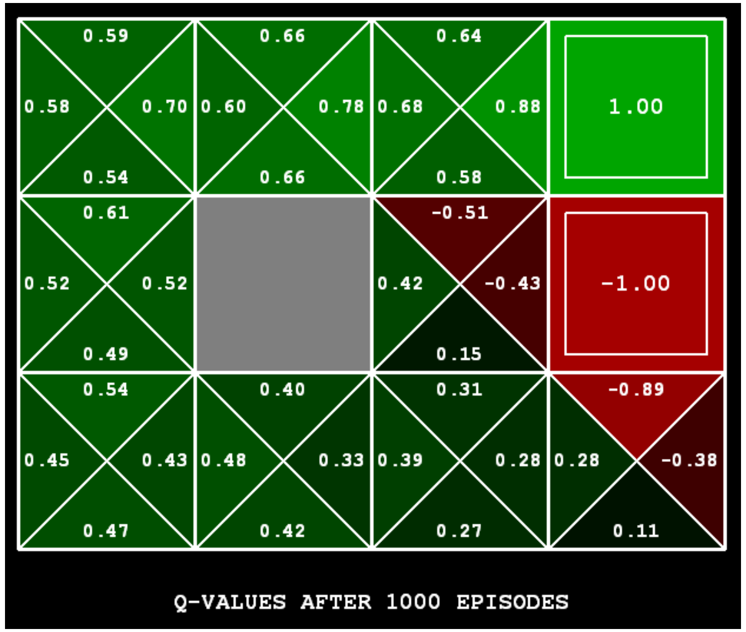
Q-Learning

- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

    $$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Incorporate the new estimate into a running average:

    $$Q(s,a) \leftarrow (1-\alpha)\, Q(s,a) + \alpha \cdot [sample]$$



[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Video of Demo Q-Learning -- Gridworld

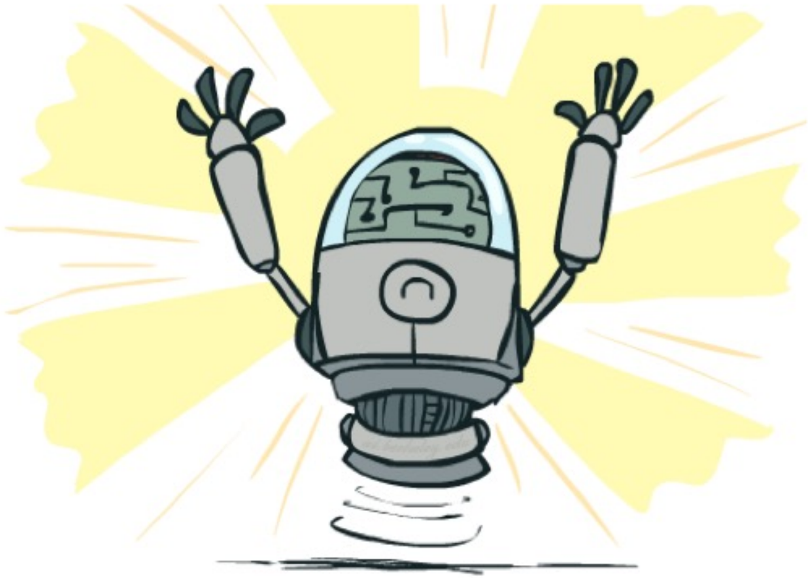
Video of Demo Q-Learning -- Crawler

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if samples are generated from a suboptimal policy!

- This is called off-policy learning

- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly
  - Basically, in the limit, it doesn’t matter how you select actions (!)



Summary

- RL solves MDPs via direct experience of transitions and rewards
- There are several approaches:
  - Learn the MDP model and solve it
  - Learn V directly from sums of rewards, or by TD local adjustments
    - Still need a model to make decisions by lookahead
  - Learn Q by local Q-learning adjustments, use it directly to pick actions
  - (and about 100 other variations)
- Big missing pieces:
  - How to explore without too much regret?
  - How to scale this up to Tetris (10^{60}), Go (10^{172}), StarCraft ($|A|=10^{26}$)?